AM-III-CE-SEM-III

20th Nov 2014

**(OLD COURSE)** **QP Code :12195**

(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Solve any **four** questions out of the **remaining**.
(3) **Each** questions carries **equal** marks.

1. (a) Find P if $f(z) = r^2\cos 2\theta + ir^2 \sin p\theta$ is analytic. 5
(b) Show that the set of functions cosx, cos2x, cos3x, .......is a set of orthogonal functions over $(-\pi, \pi)$. 5
(c) Find the Laplace Transform of $\frac{e^{-2t}\sin 2t \cdot \cosh t}{t}$ 5
(d) Show taht the matrix $A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is unitary and find $A^{-1}$. 5

2. (a) Prove that $\int_0^\infty e^{-t} \cdot \frac{\sin^2 t}{t} dt = \frac{1}{4}\log 5$. 6
(b) Reduce the following matrix to normal form and find the rank. 6

$$\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(c) Find the Bilinear transformation which maps $z = 2, 1, 0$ onto $w = 1, 0, i$. 8

3. (a) Find the inverse Laplace transform of $\frac{s+2}{(s+3)(s+1)^3}$. 6
(b) Obtain complex form of Fourier Series for $f(x) = \cosh 3x + \sinh 3x$ in $(-3, 3)$. 6
(c) Prove that $u = x^2 - y^2$, $v = -\frac{y}{x^2+y^2}$ both u and v satisfy Laplace's equation but that u + iv is not an anlytic function of z. 8

4. (a) Examine whether the given vectors are linearly independent or dependent. 6
$[1, -1, 1]$, $[2, 1, 1]$, $[3, 0, 2]$
(b) Find the analytic function whose real part is $\frac{\sin 2x}{\cosh 2y + \cos 2x}$ 6
(c) Solve $(D^2 - D - 2)y = 20\sin 2t$, with $y(0) = 1$ and $y'(0) = 2$. 8

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5. (a) Find the image of the circle $(x-3)^2 + y^2 = 2$, under the transformation $w = 1/z$. 6
   (b) Find non-singular matrices P and Q such that PAQ is in normal form. Aslso find the rank. 6

$$\begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$

   (c) Find Fourier series for $f(x) = 2x - x^2$ in $(0, 3)$. 8

6. (a) Find the Fourier series for $f(x) = e^x$ in $(0, 2\pi)$. 6
   (b) Find the characteristic equation of the matrix A given below and hence, find the matrix represented by $A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I$ 6

$$\text{where } A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

   (c) Find the inverse Laplace transform of : 8

   (i) $\dfrac{e^{4-3s}}{(s+4)^{5/2}}$  (ii) $\dfrac{(s+1)e^{-s}}{s^2+s+1}$

7. (a) Find the Fourier expansion of $f(x) = x^2$, in $(-\pi, \pi)$ and hence prove that $\dfrac{\pi^2}{6} = \sum_{1}^{\infty} \dfrac{1}{n^2}$. 6

   (b) Solve by using convolution theorem $\dfrac{s^2}{(s^2+a^2)^2}$. 6

   (c) Find the eigen values and eigen vectors for $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$. 8

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