

(3 Hours)

[Total Marks : 100]

- N.B.:** (1) Question No. 1 is **compulsory**.
 (2) Attempt any **four** questions out of the remaining **six** questions.
 (3) **Figures** to the **right** indicate **full** marks.

1. (a) Prove that the eigen values of $\begin{bmatrix} \frac{(1+i)}{2} & \frac{-(1-i)}{2} \\ \frac{(1+i)}{2} & \frac{(1-i)}{2} \end{bmatrix}$ are of unit modulus. 5
- (b) If $u = -r^3 \sin 3\theta$ find the analytic function $f(z)$ in terms of z where u is the real part of $f(z)$. 5
- (c) Evaluate $\int_C \frac{2z+3}{z} dz$ where C is the lower half of the circle $|z| = 2$. 5
- (d) Determine all basic feasible solutions of the equation $\begin{aligned} 2x_1 + 6x_2 + 2x_3 + x_4 &= 3 \\ 6x_1 + 4x_2 + 4x_3 + 6x_4 &= 2 \end{aligned}$ 5
2. (a) Find the characteristics equation of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1} . 6
- (b) Find the analytic function and $(z) = u + iv$ if $3u + 2v = y^2 - x^2 + 16xy$. 6
- (c) Using Duality solve the L.P.P. 8
- Minimise $z = 4x_1 + 3x_2 + 6x_3$
 Subject to $x_1 + x_3 \geq 2$
 $x_2 + x_3 \geq 5$
 $x_1, x_2, x_3 \geq 0$

* Corrections Attached

3. (a) If $A = \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \end{bmatrix}$ prove that $e^A = e^\alpha \begin{bmatrix} \cosh\alpha & \sinh\alpha \\ \sinh\alpha & \cosh\alpha \end{bmatrix}$

6

(b) Solve the LPP by simplex method.

6

Minimise $z = 6x_1 - 2x_2 + 3x_3$

Subject to $2x_1 - x_2 + 2x_3 \leq 2$

$x_1 + 4x_3 \leq 4$

$x_1, x_2, x_3 \geq 0$

(c) Evaluate $\int_0^{2\pi} \frac{d\theta}{25 - 16\cos^2 \theta}$.

8

4. (a) If $w = f(z)$ prove that $\frac{dw}{dz} = (\cos\theta - i\sin\theta) \frac{\partial w}{\partial r}$.

6

(b) Obtain two distinct Laurent's series for $f(z) = \frac{2z-3}{z^2-4z-3}$

6

(c) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

8

5. (a) Show that $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory.

6

(b) Find the image of the circle $|z| = 2$ under the transformation $w = z+3+2i$

8

(c) Solve the NLPP using the method of Lagrangian multipliers.

8

Optimise $z = 4x_1^2 + 2x_2^2 + 4x_3^2 - 4x_1x_2$

Subject to $x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20$

$x_1, x_2, x_3 \geq 0$

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6. (a) Find the orthogonal trajectory of the family of curves given by $2x - x^3 + 3xy^2 = a$ 6
 (b) Solve the NLPP using Kuhn-Tucker conditions 6

$$\text{Maximise } z = 2x_1^2 - 7x_2^2 + 12x_1x_2$$

$$\text{Subject to } \begin{aligned} 2x_1 + 5x_2 &\leq 98 \\ x_1, x_2 &\geq 0 \end{aligned}$$

- (c) Evaluate $\int_C \frac{z+1}{z^3 - 2z^2} dz$ where C is 8

- (a) the circle $|z| = 1$
- (b) the circle $|z-2-i| = 2$
- (c) the circle $|z-1-2i| = 2$

7. (a) Find the bilinear transformation which maps the points $2, i, -2$ onto the points $1, i, -1$. 6

- (b) Using residue theorem evaluate $\oint_C \frac{e^{2z}}{(z-\pi i)^3} dz$ where C is $|z-2i|=2$ 6

- (c) Use the dual simplex method solve the LPP 8

$$\text{Minimise } z = 6x_1 + x_2$$

$$\text{Subject to } 2x_1 + x_2 \geq 3$$

$$x_1 - x_2 \geq 0$$

$$x_1, x_2 \geq 0$$

Course : Prog. 652 to 666 S.E. (ALL BRANCH) (SEM IV) (OLD)

Q.P Code : 14312

Correction :

4 (b) Obtain two laurent's series for $f(z) = \frac{2z - 3}{z^2 - 4z + 3}$

5. (c) Solve the NLPP using the method of Lagrangian multipliers

Optimize $z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$

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