AM-III-CO-SEM-III 20th Nov 2014

(OLD COURSE) **QP Code :12210**

(3 Hours) **Total Marks : 100**

N. B. : (1) **Question 1 is compulsory.**
(2) Attempt any **FOUR** of the remaining.
(3) **Figures** to the right indicate **full** marks.

1. (a) If a matrix $A=\begin{bmatrix} -4 & -3 & -2 \\ -1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ then find adjA **5**

(b) Find $L\left\{\frac{\sin^2 2t}{t}\right\}$ **5**

(c) Find Z-transform of $f(K) = 3^k$, $k \geq 0$ **5**

(d) Find half range Fourier cosine series for $f(x) = x$, $0 < x < \pi$ **5**

2. (a) Find $L^{-1}\left\{\frac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}\right\}$ **6**

(b) Find Laplace Transform of $f(t) = a$, $0 < t < b$
$= 0$, $t > b$ **6**

(c) Test the consistency of following equations and solve them if consistent **8**
$x - 2y + 3t = 2$, $2x + y + z + t = -4$, $4x - 3y + z + 7t = 8$

3. (a) Evaluate $\int_0^\infty e^{-t}\left(\int_0^t e^{-4u}\cos u\,du\right)dt$ **6**

(b) Find inverse Z-transform of $f(z) = \frac{1}{(z-1)(z-2)}$ for $1 < |z| < 2$ **6**

(c) Find Fourier series for $f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$ in $(0, 2\pi)$ **8**

Hence, deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$

4. (a) Show that the set of functions cosnx, n = 1,2,3,... is orthogonal on $(0,2\pi)$ **6**

(b) Express the following matrix A as sum of symmetric and skew-symmetric matrix **6**

Where $A = \begin{bmatrix} 2 & -4 & 9 \\ 14 & 7 & 13 \\ 3 & 5 & 11 \end{bmatrix}$

(c) Solve $(D^2-3D+2)y = 4e^{2t}$, with $y(0) = -3$ and $y'(0) = 5$ **8**

5. (a) Solve following using Convolution theorem 6

$$L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$$

(b) Find Fourier Series expansion for $f(x) = 1-x^2$ on $(-1,1)$ 6

(c) Find Fourier Cosine integral representation for $f(x) = e^{-ax}$, $x > 0$ 8

Hence show that $\int_0^\infty \frac{\cos ws}{1+w^2} dw = \frac{\pi}{2} e^{-x}$

6. (a) Find $L^{-1}\left\{\frac{e^{-\pi s}}{s^2+2s+2}\right\}$ 6

(b) Find rank of $A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$ converting to normal form. 6

(c) Obtain Fourier expansion of $f(x) = x^2$ where $-\pi \le x \le \pi$ 8

Hence, deduced that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

7. (a) Show that the matrix $A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is Unitary and hence, find A^{-1} 6

(b) Find $Z\{k^2 a^{k-1} U(k-1)\}$ 6

(c) Obtain the complex form of Furier series of $f(x) = e^{ax}$ in $(-L,L)$ 8
