SE Sem III (CBSGS) 279

App. Maths III

(EE & ET) 26/11/2014

QP Code : 14571

(3 Hours) **[Total Marks : 80**

N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

1. (a) Prove that $f(z) = x^2 - y^2 + 2ixy$ is analytic and find $f'(z)$ 5

(b) Find the Fourier series expansion for $f(x) = |x|, \; in \quad (-\pi, \pi)$ 5

(c) Using laplace transform solve the following differential equation with given condition $\frac{d^2y}{dt^2} + y = t, \; given\ that\ y(0) = 1\ \&\ y'(0) = 0$ 5

(d) If $\overline{A} = \nabla(xy + yz + zx), \; find\ \nabla \cdot \overline{A}\ and\ \nabla \times \overline{A}$ 5

2. (a) If $L[J_0(t)] = \frac{1}{\sqrt{s^2+1}}$, prove that $\int_0^\infty e^{-6t} t\, J_0(4t)\, dt = 3/500$ 6

(b) Find the directional derivative of $\phi = x^4 + y^4 + z^4$ $at\ A\ (1, -2, 1)$ in the direction of AB where $B\ is\ (2, 6, -1)$. Also find the maximum directional derivative of ϕ at $(1, -2, 1)$. 6

(c) Find the Fourier series expansion for $f(x) = 4 - x^2, in\ (0, 2)$
Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \cdots\cdots$ 8

3. (a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ 6

(b) Using Green's theorem evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where 'c' is the boundary of the surface enclosed by the lines $x = 0,\ y = 0,\ x = 2,\ y = 2$. 6

(c) i) Find Laplace Transform of $e^{-3t}\int_0^t u \sin 3u\, du$

ii) Find the Laplace transform of $\frac{d}{dt}\left(\frac{1 - \cos 2t}{t}\right)$ 8

4. (a) Obtain complex form of Fourier series for the functions $f(x) = \sin ax \quad in \quad (-\pi, \pi)$, where a is not an integer. 6

(b) Find the analytic function whose imaginary part is $v = \frac{x}{x^2 + y^2} + \cosh y \cdot \cos x$ 6

(c) Find inverse Laplace Transform of following
i) $\log\left(\frac{s^2 + a^2}{\sqrt{s + b}}\right)$ ii) $\frac{1}{s^3(s-1)}$ 8

5. (a) Obtain half-range cosine series for $f(x) = x(2 - x)\ in\ 0 < x < 2$ 6

(b) Prove that $\overline{F} = \frac{\overline{r}}{r^3}$ is both irrotational and solenoidal 6

(c) Show that the function $u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$ satisfies

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Correction Attached.