Additional Examples of Chapter 8: Digital Filter Structures

Example E8.1: Determine by inspection whether or not the digital filter structure of Figure E8.1 has delay-free loops. Identify these loops if they exist. Develop an equivalent structure without delay-free loops.

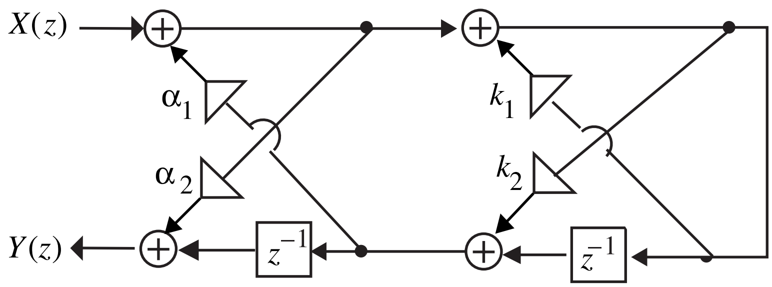
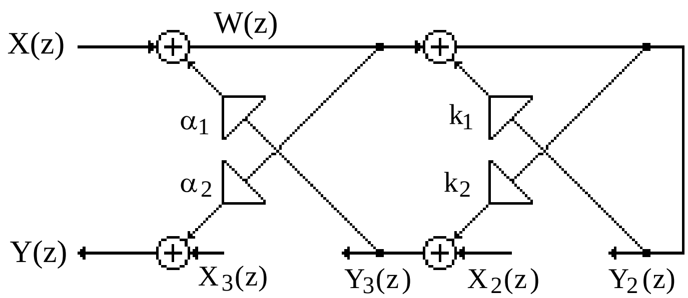


Figure E8.1

Answer:



Note from the above figure, there is a delay-free loop through the multipliers α_1 and k_2, and another one through the multiplier k_1.

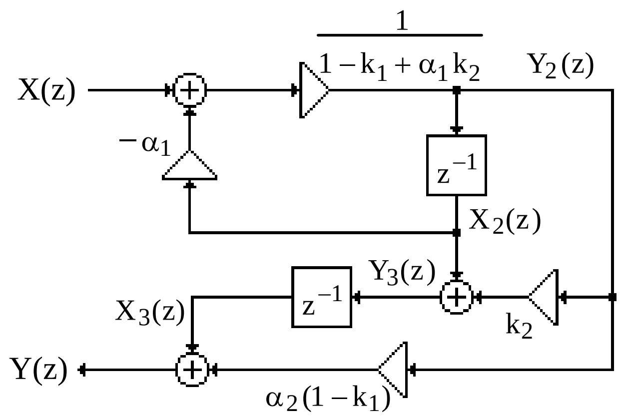
Analyzing the above figure we get (1): $W(z) = X(z) - \alpha_1 Y_3(z)$, (2): $Y_2(z) = W(z) + k_1 Y_2(z)$, (3): $Y_3(z) = k_2 Y_2(z) + X_2(z)$, and (4): $Y(z) = \alpha_2 W(z) + X_3(z)$.

From Eq. (2) we get (5): $W(z) = (1 - k_1)Y_2(z)$. Substituting Eqs. (3) and (5) in Eq. (1) we get $(1 - k_2)Y_2(z) = X(z) - \alpha_1\big(k_2 Y_2(z) + X_2(z)\big)$ or (6): $Y_2(z) = \dfrac{1}{1 - k_1 + \alpha_1 k_2}\big(X(z) - \alpha_1 X_2(z)\big)$.

Substituting Eq. (5) in Eq. (4) we get (7): $Y(z) = \alpha_2(1 - k_1)Y_2(z) + X_3(z)$.

A realization based on Eqs. (6), (7) and (3) shown below has no delay-free loops.

Additional Examples of Chapter 8:
Digital Filter Structures



Example E8.2: Analyze the digital filter structure of Figure E8.2 and determine its transfer function H(z) = Y(z)/X(z).

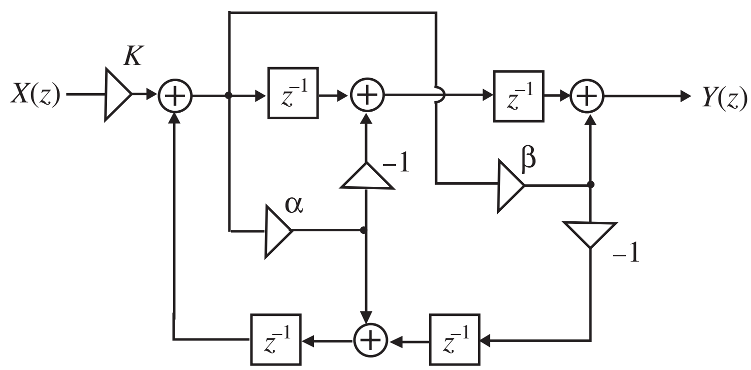
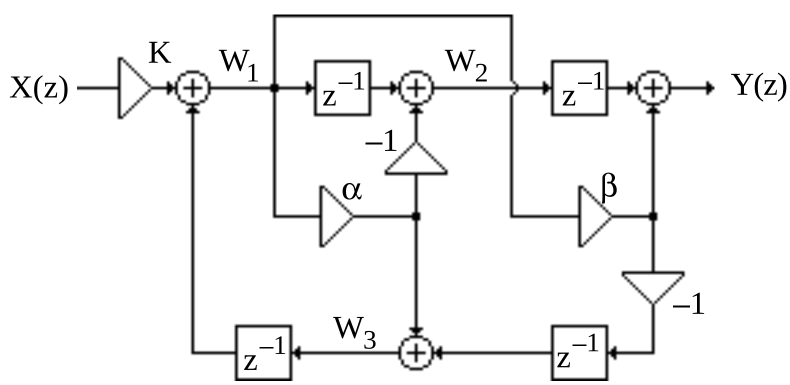


Figure E8.2

Answer:



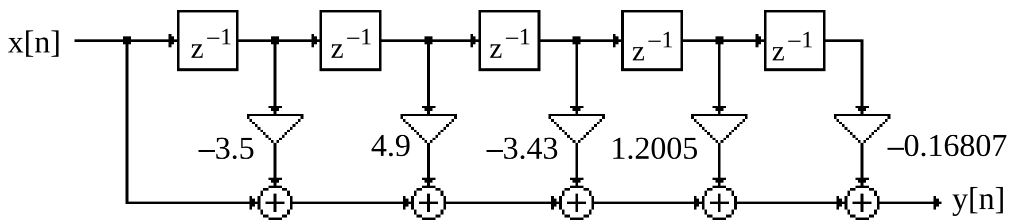
From the above figure, we get $W_1 = KX + z^{-1}W_3$, $W_2 = (z^{-1} - \alpha)W_1$, $W_3 = \alpha W_1 - \beta z^{-1}W_1 = (\alpha - \beta z^{-1})W_1$, and $Y = z^{-1}W_2 + \beta W_1$. Substituting the third equation in the first we get $W_1 = KX + z^{-1}(\alpha - \beta z^{-1})W_1$, or $[1 - \alpha z^{-1} + \beta z^{-2}]W_1 = KX$. Next, substituting the second equation in the last one we get $Y = [z^{-1}(z^{-1} - \alpha) + \beta]W_1$. From the last two equations we finally arrive at $H(z) = \dfrac{Y}{X} = K\left(\dfrac{\beta - \alpha z^{-1} + z^{-2}}{1 - \alpha z^{-1} + \beta z^{-2}}\right)$.

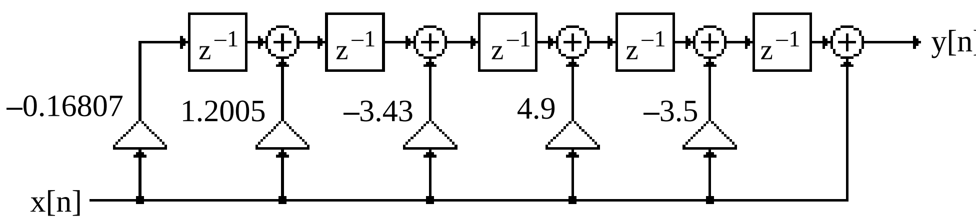
Example E8.3: Realize the transfer function $H(z) = (1 - 0.7z^{-1})^5$ in the following forms:
(a) Two different direct forms, (b) cascade of 5 first-order sections, (c) cascade of one first-order and 2 second-order sections, and (d) cascade of one second-order and one third-order section.

Answer: (a) A direct form realization of

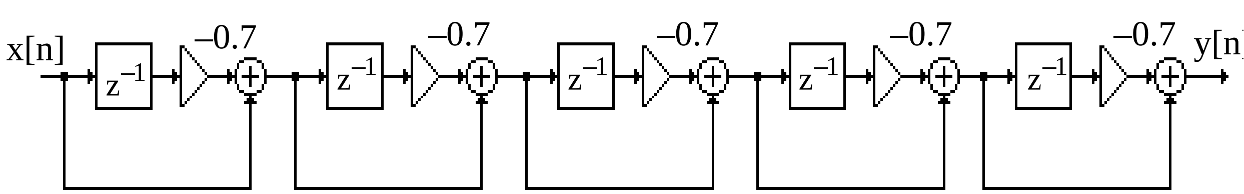
$$H(z) = 1 - 3.5z^{-1} + 4.9z^{-2} - 3.43z^{-3} + 1.2005z^{-4} - 0.16807z^{-5}$$

and its transposed structure are shown below:

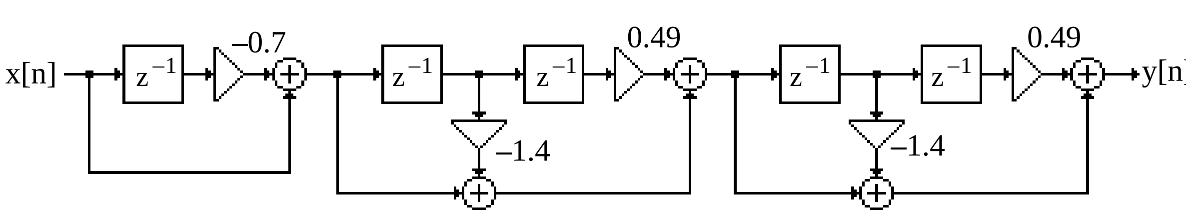




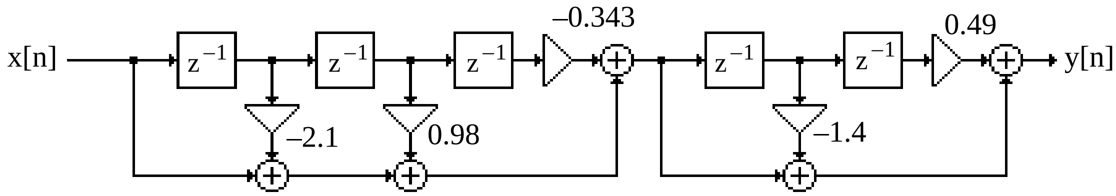
(b) A cascade realization of $H(z) = (1 - 0.7z^{-1})(1 - 0.7z^{-1})(1 - 0.7z^{-1})(1 - 0.7z^{-1})(1 - 0.7z^{-1})$ is shown below:



(c) A cascade realization of $H(z) = (1 - 0.7z^{-1})(1 - 1.4z^{-1} + 0.49z^{-2})(1 - 1.4z^{-1} + 0.49z^{-2})$ is shown below:



(d) A cascade realization of $H(z) = (1 - 2.1z^{-1} + 1.47z^{-2} - 0.343z^{-3})(1 - 1.4z^{-1} + 0.49z^{-2})$ is shown below:



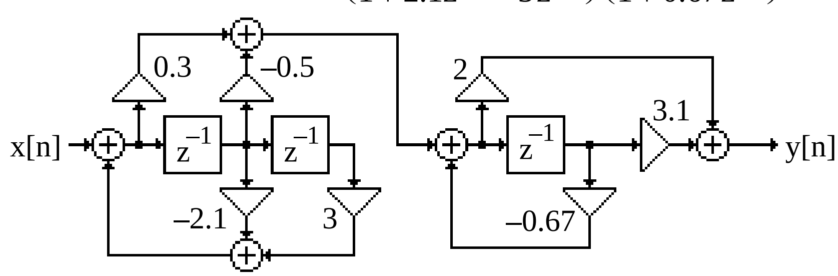
Example E8.4: Develop a three-branch polyphase realization of the FIR transfer function $H(z) = h[0] + h[1]z^{-1} + h[2]z^{-2} + h[3]z^{-3} + h[4]z^{-4} + h[5]z^{-5} + h[6]z^{-6} + h[7]z^{-7}$ and determine the expressions for the polyphase transfer functions $E_0(z)$, $E_1(z)$, and $E_2(z)$.

Answer:
$H(z) = \left(h[0] + h[3]z^{-3} + h[6]z^{-6}\right) + z^{-1}\left(h[1] + h[4]z^{-3} + h[7]z^{-6}\right) + z^{-2}\left(h[2] + h[5]z^{-3}\right)$ Hence,
$E_0(z) = h[0] + h[3]z^{-3} + h[6]z^{-6}$, $E_1(z) = h[1] + h[4]z^{-3} + h[7]z^{-6}$, $E_2(z) = h[2] + h[5]z^{-3}$.

Example E8.5: Develop two different canonic cascade realizations of the causal IIR transfer function

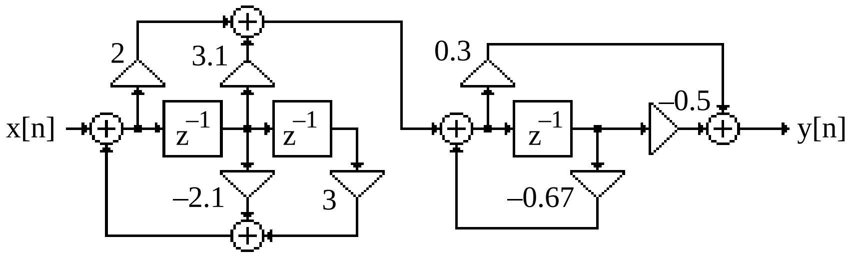
$$H(z) = \left(\frac{0.3 - 0.5z^{-1}}{1 + 2.1z^{-1} - 3z^{-2}}\right)\left(\frac{2 + 3.1z^{-1}}{1 + 0.67z^{-1}}\right)$$

Answer: A cascade realization of $H(z) = \left(\frac{0.3 - 0.5z^{-1}}{1 + 2.1z^{-1} - 3z^{-2}}\right)\left(\frac{2 + 3.1z^{-1}}{1 + 0.67z^{-1}}\right)$ is shown below:



A cascade realization of $H(z) = \left(\frac{2 + 3.1z^{-1}}{1 + 2.1z^{-1} - 3z^{-2}}\right)\left(\frac{0.3 - 0.5z^{-1}}{1 + 0.67z^{-1}}\right)$ is shown below:

Additional Examples of Chapter 8: Digital Filter Structures

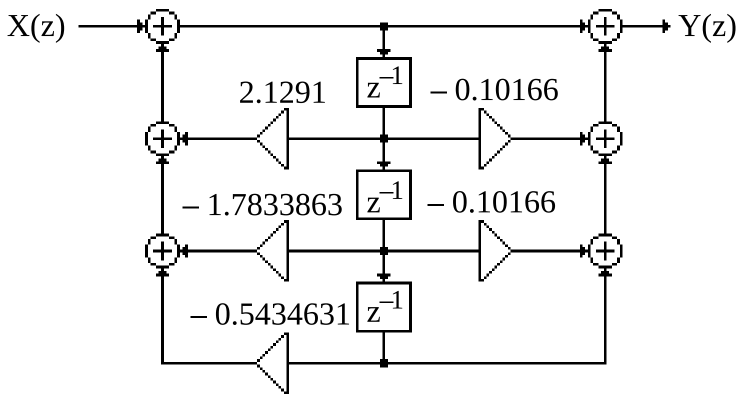


Example E8.6: Realize the causal IIR transfer function

$$H(z) = \frac{0.5634(1+z^{-1})(1-1.10166z^{-1}+z^{-2})}{(1-0.683z^{-1})(1-1.4461z^{-1}+0.7957z^{-2})}.$$

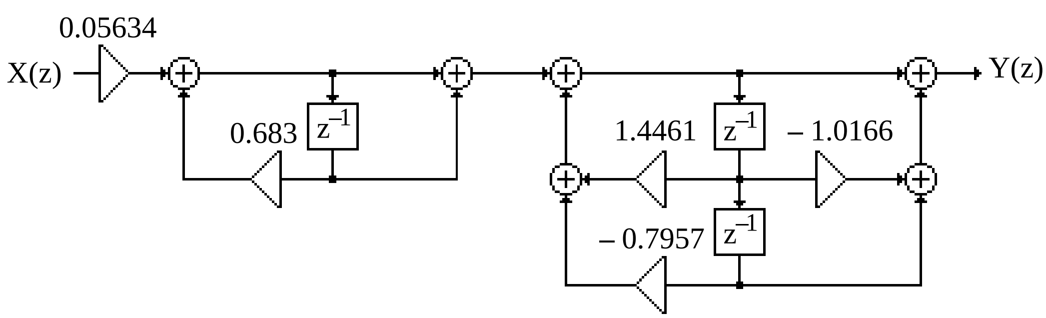
in the following forms: (a) direct canonic form, (b) cascade form, and (c) Gray-Markel form.

Answer: (a) Direct canonic form -



Hardware requirements: # of multipliers = 5, # of two-input adders = 6, # of delays = 3.

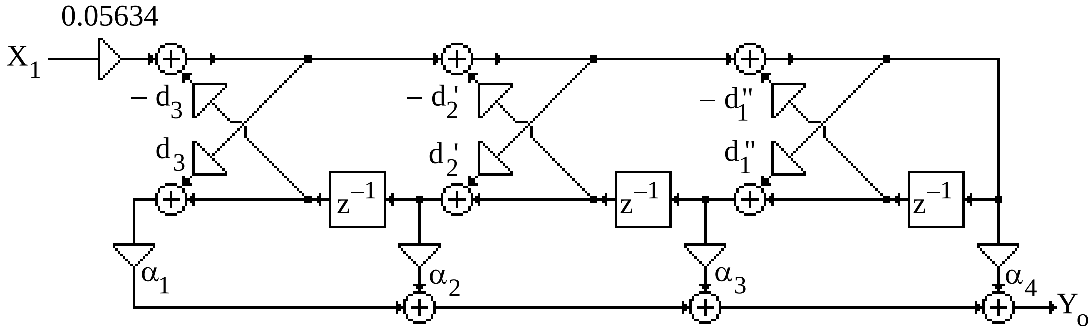
(b) Cascade Form



Hardware requirements: # of multipliers = 5, # of two-input adders = 6, # of delays = 3.

(c) Gray-Markel Form -

Additional Examples of Chapter 8:
Digital Filter Structures



$$d_3 = -0.5434631, \quad d_2^{'} = 0.8881135, \quad d_1^{''} = -0.8714813.$$

$$\alpha_1 = p_3 = 1, \quad \alpha_2 = p_2 - \alpha_1 d_1 = 2.02744, \quad \alpha_3 = p_1 - \alpha_1 d_2 - \alpha_2 d_1^{'} = 1.45224,$$

$$\alpha_4 = p_0 - \alpha_1 d_3 - \alpha_2 d_2^{'} - \alpha_3 d_1^{''} = 1.00702.$$

Hardware requirements: # of multipliers = 9, # of two-input adders = 6, # of delays = 3.

Example E8.7: Realize the causal stable IIR transfer function

$$H(z) = \frac{3 + 9z^{-1} + 9z^{-2} + 3z^{-3}}{12 + 10z^{-1} + 2z^{-2}}$$

as a parallel connection of 2 allpass filters.

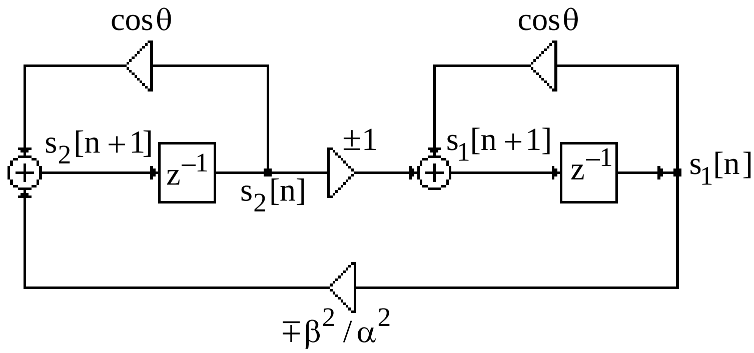
Answer: $H(z) = \dfrac{1}{2}\left(\dfrac{3 + 9z^{-1} + 9z^{-2} + 3z^{-3}}{(3+z^{-1})(2+z^{-1})}\right) = \dfrac{1}{2}\left[z^{-1}\left(\dfrac{1+3z^{-1}}{3+z^{-1}}\right) + \left(\dfrac{1+2z^{-1}}{2+z^{-1}}\right)\right]$

$$= \frac{1}{2}\big(A_0(z) + A_1(z)\big), \text{ where } A_0(z) = z^{-1}\left(\frac{1+3z^{-1}}{3+z^{-1}}\right) \text{ and } A_1(z) = \left(\frac{1+2z^{-1}}{2+z^{-1}}\right).$$

Example E8.8: Develop a 3-multiplier realization of a digital sine-cosine generator obtained by setting $\alpha \sin\theta = \pm\beta$ in Eq. (8.123).

Answer: By setting $\alpha \sin\theta = \pm\beta$ in Eq. (8.123), the state-space description of the sine-cosine generator reduces to $\begin{bmatrix} s_1[n+1] \\ s_2[n+1] \end{bmatrix} = \begin{bmatrix} \cos\theta & \pm 1 \\ \mp\dfrac{\beta^2}{\alpha^2} & \cos\theta \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix}$, which leads to the three-multiplier structure shown below:

Additional Examples of Chapter 8: Digital Filter Structures



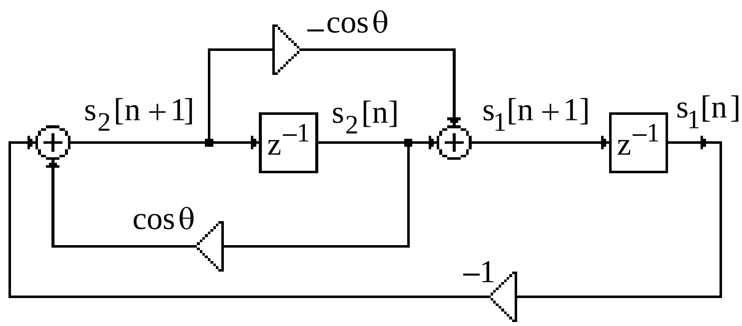
Example E8.9: Develop a 1-multiplier realization of a digital sine-cosine generator obtained by setting C = 0 in Eq. (8.123) and then choosing and properly.

Answer: $\begin{bmatrix} s_1[n+1] \\ s_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{\alpha(C-\cos\theta)}{\beta\sin\theta} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s_1[n+1] \\ s_2[n+1] \end{bmatrix} + \begin{bmatrix} C & \dfrac{\alpha(1-C\cos\theta)}{\beta\sin\theta} \\ -\dfrac{\beta}{\alpha}\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix}.$

If C = 0, choose $\alpha = \beta\sin\theta$. Then

$\begin{bmatrix} s_1[n+1] \\ s_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & -\cos\theta \\ 0 & 0 \end{bmatrix} \begin{bmatrix} s_1[n+1] \\ s_2[n+1] \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & \cos\theta \end{bmatrix} \begin{bmatrix} s_1[n] \\ s_2[n] \end{bmatrix},$ which can be realized with two

multipliers as shown below:



The above structure can be modified to yield a single multiplier realization as indicated below: