Additional Examples of Chapter 10: FIR Digital Filter Design

Example E10.1: A lowpass FIR filter of order N = 65 and a transition band of width $\omega_s - \omega_p = 0.1\pi$ is to designed using the Parks-McClellan method. Determine the approximate values of the corresponding peak stopband ripple δ_s and the minimum stopband attenuation α_s if the filter order is estimated using the Kaiser's formula of Eq. (10.3). Assume the peak passband ripple δ_p and peak stopband ripple δ_s to be the same.

Answer: From Eq. (10.3) we get $100 = -\dfrac{20\log_{10}(\delta_s)+13}{14.6(0.1\pi)/2\pi} = -\dfrac{20\log_{10}(\delta_s)+13}{0.73}$. Solving this equation we arrive at $20\log_{10}(\delta_s) = -60.45$ or $\delta_s = 10^{-3.0225} = 9.495\times 10^{-4}$. The corresponding minimum stopband attenuation is $\alpha_s = -20\log_{10}(\delta_s) = 60.45$ dB.

Example E10.2: Repeat Example E10.1 if the filter order is estimated using the Bellanger's formula of Eq. (10.4).

Answer: From Eq. (10.3) we get $66 = -\dfrac{2\log_{10}(10\delta_s^2)}{3(0.1\pi)/2\pi} = -\dfrac{2\log_{10}(10\delta_s^2)}{0.15}$. Solving this equation we then arrive at $\log_{10}(10\delta_s^2) = -4.95$ or $\delta_s = 0.0011$. The corresponding minimum stopband attenuation is $\alpha_s = -20\log_{10}(\delta_s) = 59.5$ dB.

Example E10.3: Repeat Example E10.1 if the filter is designed using the Kaiser's window-based method.

Answer: From Eq. (10.42) we get $65 = \dfrac{\alpha_s - 8}{2.285(0.1\pi)}$ or $\alpha_s = 228.5\times 0.1\pi + 8 = 54.6605$ dB. The corresponding peak passband ripple is $\delta_s = 10^{-\alpha_s/20} = 0.0018$.

Example E10.4: Show that the length-(2M+1) Bartlett window of Eq. (10.29) can be obtained by a linear-convolution of two scaled length-L rectangular windows. Determine L and the scale factor. From this relation, determine the expression for the frequency response of the length-(2M+1) Bartlett window. Determine the main lobe width Δ_{ML} and the relative sidelobe level $A_{s\ell}$ of the Bartlett window sequence.

Answer: Let $w_R[n] = \frac{1}{k}$ represent the scaled rectangular window. Since the convolution of two length L sequences produces a sequence of length 2L– 1, therefore 2L –1 = 2M + 1 which gives L = M + 1. Therefore, $w_R[n] = \frac{1}{k}$, $-\frac{M}{2} \le n \le \frac{M}{2}$. Now,

$$w[n] = w_R[n] \circledast w_R[n] = \begin{cases} \frac{1}{k^2}\left(M+1-|n|\right), & -M \le n \le M, \\ 0, & \text{elsewhere,} \end{cases}$$

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or, $w[n] = \begin{cases} \frac{M+1}{k^2}\left(1 - \frac{|n|}{M+1}\right), & -M \le n \le M, \\ 0, & \text{elsewhere.} \end{cases}$,

Now $\frac{M+1}{k^2} = 1$ which yields $k = \sqrt{M+1}$. Hence a Bartlett window of length 2M + 1 is obtained by the linear convolution of 2 length M + 1 rectangular windows scaled by a factor of $\frac{1}{\sqrt{M+1}}$ each. The DTFT $\Psi_{BART}(e^{j\omega})$ of the Bartlett window is thus given by

$\Psi_{BART}(e^{j\omega}) = \left(\Psi_R(e^{j\omega})\right)^2 = \frac{1}{M+1} \frac{\sin^2\left(\frac{\omega(M+1)}{2}\right)}{\sin^2\left(\frac{\omega}{2}\right)}$, where $\Psi_R(e^{j\omega})$ is the DTFT of the rectangular window. Hence $\Psi_{BART}(e^{j\omega}) = 0$ at $\omega = \pm\frac{2\pi}{M+1}, \pm\frac{4\pi}{M+1}, ...$

Therefore $\Delta_{ML,BART} = \frac{4\pi}{M+1}$. It should be noted that the main lobe width given here is for a Bartlett window of length 2M + 1 = 2N – 1 as compared to that of a rectangular window of length N = M + 1. The maximas of the DTFT $\Psi_{BART}(e^{j\omega})$ of the Bartlett window occur at the same location as the DTFT $\Psi_R(e^{j\omega})$ of the rectangular window. Since $\Psi_{BART}(e^{j\omega}) = \left(\Psi_R(e^{j\omega})\right)^2$, it follows then $A_{s\ell,BART} = 2 \times A_{s\ell,R} = 2 \times 13.3 = 26.6$ dB.

Example E10.5: Design a linear-phase FIR lowpass filter of length 17 with a passband edge at using the frequency sampling approach. Assume an ideal brickwall characteristic for the desired magnitude response. (a) Using Eq. (10.14?) develop the exact values for the desired frequency samples. (b) Using MATLAB, plot the magnitude response of the designed filter.

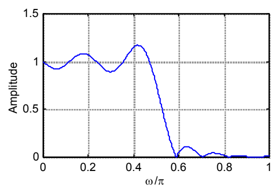
Answer: (a) The frequency spacing between two consecutive samples is given by $\frac{2\pi}{17} = 0.11765\pi$, and hence the desired passband edge $\omega_p = 0.5\pi$ is between the frequency samples at $\omega = \frac{2\pi \times 4}{17} = 0.47059\pi$ and $\omega = \frac{2\pi \times 5}{17} = 0.58824\pi$. From Eq. (10.14) the specified DFT samples are thus given by

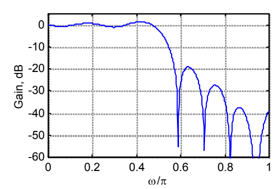
$$H[k] = \begin{cases} e^{-j(2\pi/17)8}, & k = 0,1,\ldots,4,12,\ldots,16, \\ 0, & k = 5,\ldots,11. \end{cases}$$

(b) A 17-point inverse DFT of the above DFT samples yields the impulse response coefficients given below in ascending powers of z^{-1}:

```
 Columns 1 through 5
 4.3498e-03   6.0460e-03  -2.9166e-02   8.1624e-03   5.7762e-02

Columns 6 through 10
```

```
  -6.4003e-02  -7.9934e-02   3.0267e-01   5.8824e-01   3.0267e-01

  Columns 11 through 15
   -7.9934e-02  -6.4003e-02   5.7762e-02   8.1624e-03  -2.9166e-
02

  Columns 16 through 17
    6.0460e-03   4.3498e-03
```





Example E10.6: Determine the weighting function $W(\omega)$ that is to be used to design a Type 1 linear-phase FIR lowpass filter using the Parks-McClellan method to meet the following specifications: $\omega_p = 0.36\pi$, $\omega_s = 0.58\pi$, $\alpha_p = 0.5$ dB, and $\alpha_s = 55$ dB.

Answer: We first calculate the peak passband and stopband ripples.
$\delta_p = 1 - 10^{-\alpha_p/20} = 0.0559$ and $\delta_s = -10^{-\alpha_s/20} = 0.0018$. Hence, one choice for $W(\omega)$ is

$$W(\omega) = \begin{cases} 1, & 0 \le \omega \le \omega_p, \\ \delta_p/\delta_s, & \omega_s \le \omega \le \pi, \end{cases} = \begin{cases} 1, & 0 \le \omega \le 0.36\pi, \\ 31.4569, & 0.58\pi \le \omega \le \pi. \end{cases}$$

Another choice is $W(\omega) = \begin{cases} \delta_s/\delta_p, & 0 \le \omega \le \omega_p, \\ 1, & \omega_s \le \omega \le \pi, \end{cases} = \begin{cases} 0.0318, & 0 \le \omega \le 0.36\pi, \\ 1, & 0.58\pi \le \omega \le \pi. \end{cases}$

Example E10.7: Determine the weighting function $W(\omega)$ that is to be used to design a Type 1 linear-phase FIR bandpass filter using the Parks-McClellan method to meet the following specifications: $\omega_{p1} = 0.46\pi$, $\omega_{p2} = 0.65\pi$, $\omega_{s1} = 0.35\pi$, $\omega_{s2} = 0.8\pi$, $\delta_p = 0.4$, $\delta_{s1} = 0.05$, and $\delta_{s2} = 0.008$, where δ_{s1} and δ_{s2} are, respectively, the peak ripples in the lower and upper stopbands.

Answer: $$W(\omega) = \begin{cases} \delta_p/\delta_{s1}, & 0 \le \omega \le \omega_{s1}, \\ 1, & \omega_{p1} \le \omega \le \omega_{p2}, \\ \delta_p/\delta_{s2}, & \omega_{s2} \le \omega \le \pi, \end{cases} = \begin{cases} 8, & 0 \le \omega \le 0.35\pi, \\ 1, & 0.46\pi \le \omega \le 0.65\pi, \\ 50, & 0.8\pi \le \omega \le \pi. \end{cases}$$

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