Experiment 1
Sampling Theorem in Time Domain

Aim

To study the sampling theorem and aliasing effects by sampling an analog signal with various sampling frequencies.

Theory

Consider analog sinusoidal signal with frequency of $F = 10$ Hz. The analog signal can be represented as

$$\begin{aligned} x(t) &= A\cos(2\pi Ft) \\ &= A\cos(2\pi 10t) \\ &= A\cos(20\pi t) \end{aligned} \tag{1}$$

Let the signal be sampled using sampling frequency of $F_S = 100$ Hz. The sampled signal is represented as

$$\begin{aligned} x(n) &= A\cos(2\pi Fn/F_S) \\ &= A\cos(2\pi 10n/100) \\ &= A\cos(2n\pi/10) \end{aligned} \tag{2}$$

The general form of the DT signal with frequency f is

$$x(n) = A\cos(2\pi fn) \tag{3}$$

Comparing Eqs. (2) and (3), we get the frequency of the DT signal as $f = 1/10$. The reconstructed frequency will be same as 10 Hz. Now sample the signal of 90 Hz with F_S of 100 Hz. The reader can verify that $f = 1/10$. The reconstructed frequency is -10 Hz. (Refer to Chapter 2 on Sampling; Case II.)

The reconstructed signal of 10 Hz sampled using sampling frequency of 100 Hz and its spectrum are shown in Figures 1 and 2, respectively. The reconstructed signal of 90 Hz sampled using sampling frequency of 100 Hz and its spectrum are shown in Figures 3 and 4, respectively. In Figure 3 we can observe the phase shift of 180°.

A MATLAB program to show aliasing in frequency domain using sampling theorem in time domain is as follows.

```
clear all;
f1=10;F1=100;f2=90;F2=100;
```

```
for n=1:50,
    x1(n)=10*sin(2*pi*f1*n/F1);
    fr1(n)=n/25*F1/2;
end
y1=fft(x1,50);
plot(fr1,abs(y1));title ('spectrum of signal of 10 Hz sampled
using Sampling frequency of 100 Hz');
xlabel('frequency in Hz');
ylabel('amplitude');
figure;
plot(x1);title ('10 Hz signal sampled using Sampling frequency
of 100 Hz');
xlabel('sample number');
ylabel('amplitude');
for n=1:50,
    x2(n)=-10*sin(2*pi*10*n/100);
    fr2(n)=n/25*F2/2;
end
y2=fft(x2,50);
figure;
plot(fr2,abs(y2));title ('spectrum of signal of 90 Hz sampled
using Sampling frequency of 100 Hz');
xlabel('frequency in Hz');
ylabel('amplitude');
figure;
plot(x2);title ('90 Hz signal sampled using Sampling frequency
of 100 Hz');
xlabel('sample number');
ylabel('amplitude');
```

Teaser *The reader is encouraged to use different sampling frequencies to find aliases of 50 Hz. The reader may take up a signal containing three different frequencies, say, 35 Hz, 50 Hz and 100 Hz. Sample it using sampling frequency, F_S of 150 Hz.*

The equation for the signal can be written as

$$\begin{aligned} x(n) &= A[\cos(2\pi 35n/150) + \cos(2\pi 50n/150) + \cos(2\pi 100n/150)] \\ &= A[\cos(2\pi 35n/150) + \cos(2\pi n/3) + \cos(2\pi 2n/3)] \\ &= A[\cos(2\pi 35n/150) + \cos(2\pi n/3) + \cos(2\pi - 2\pi n/3)] \\ &= A[\cos(2\pi 35n/150) + \cos(2\pi n/3) + \cos(2\pi n/3)] \qquad (4) \end{aligned}$$

Equation (4) indicates that frequencies 50 Hz and 100 Hz are reconstructed as 50 Hz. The frequency of 100 Hz is aliased as 50 Hz. Verify that the reconstructed signal will have only two frequencies using MATLAB plots. Figures 5 and 6 show the plot of reconstructed signal and its spectrum, respectively.

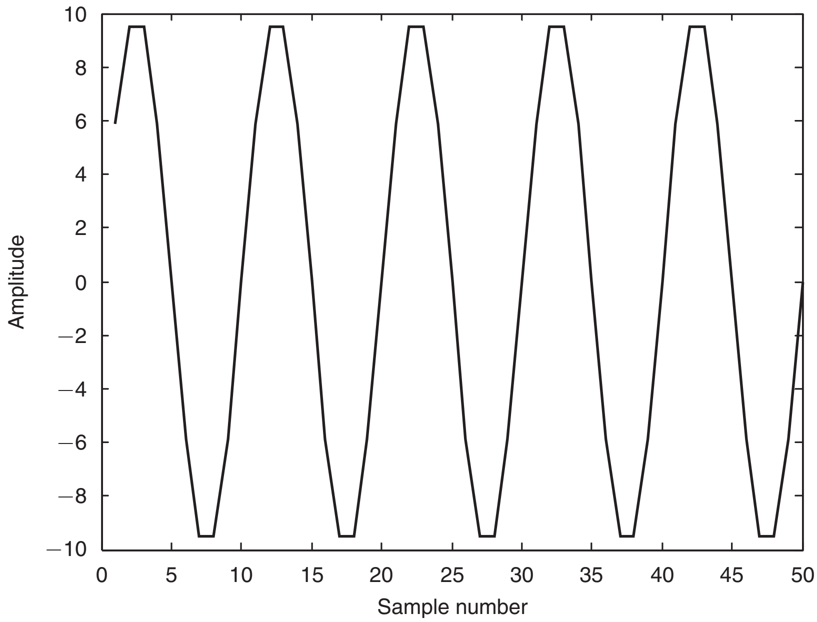

Figure 1 Reconstructed signal of 10 Hz sampled using sampling frequency of 100 Hz.

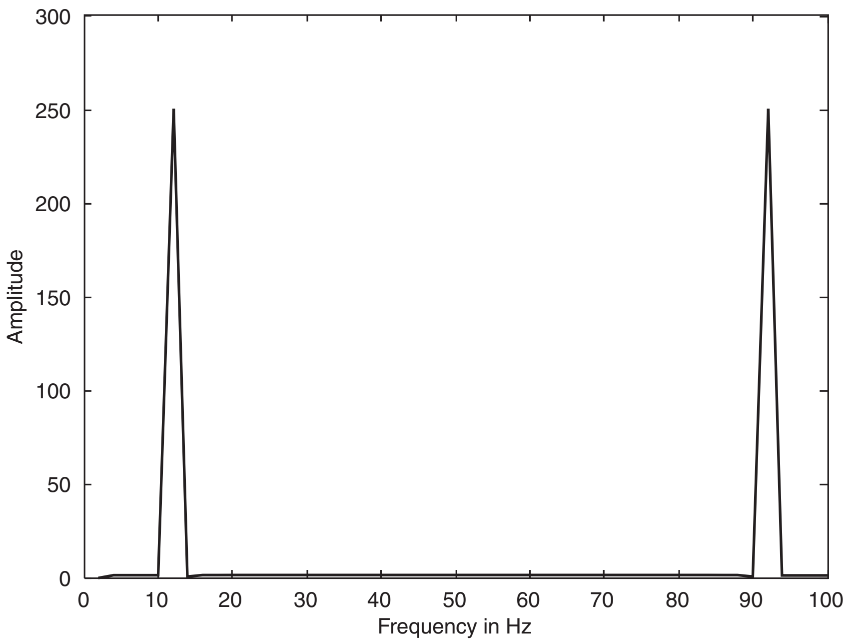

Figure 2 Spectrum of 10 Hz signal using sampling frequency of 100 Hz.

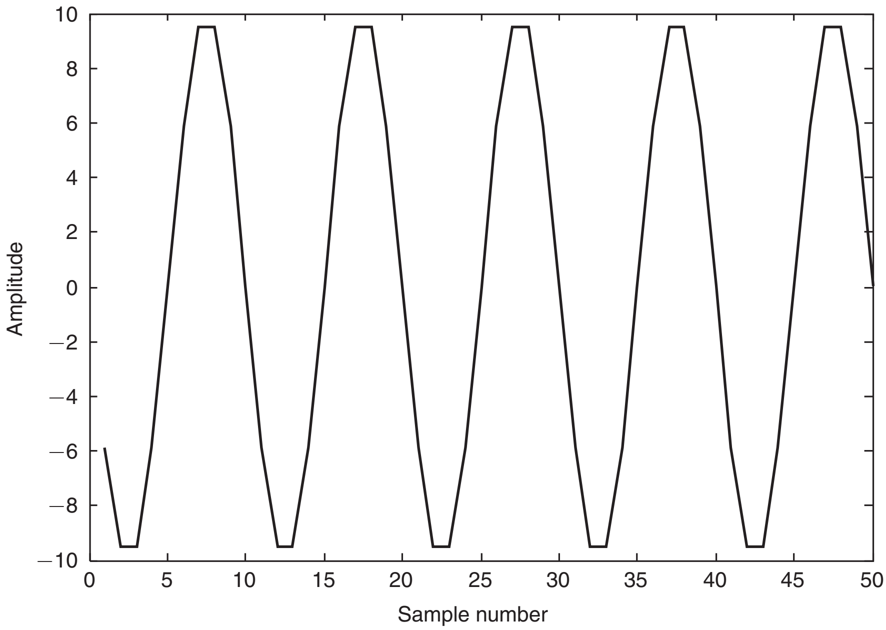

Figure 3 Reconstructed signal of 90 Hz sampled using sampling frequency of 100 Hz. (Notice the phase shift.)

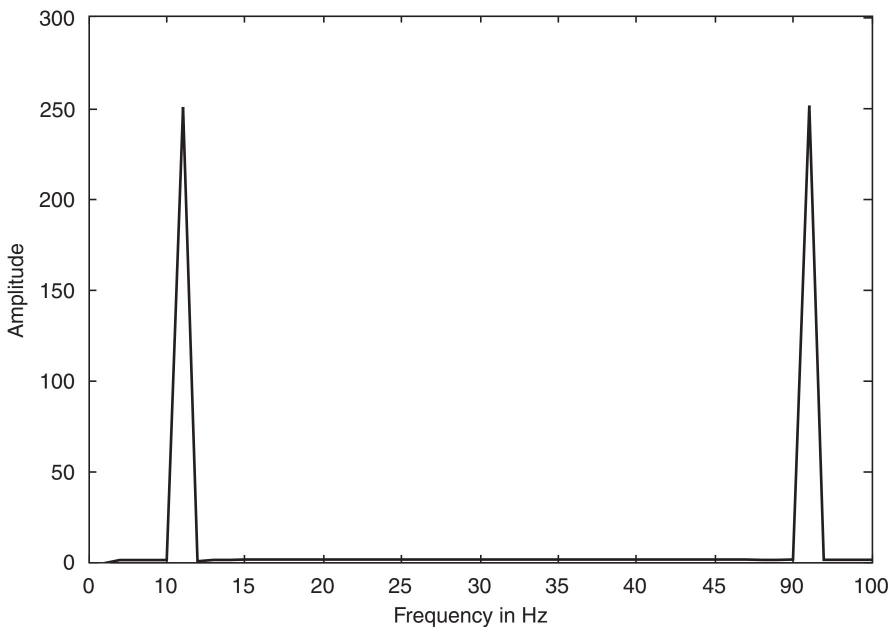

Figure 4 Spectrum of 90 Hz signal using sampling frequency of 100 Hz. (Notice that 90 Hz is aliased as 10 Hz.)

The MATLAB program for aliasing using three different frequencies is as follows.

```
%showing aliasing using 3 different frequencies
clear all;
f1=35;f2=50;f3=100;F1=150;
for n=1:50,
    x1(n)=10*cos(2*pi*f1*n/F1)+10*cos(2*pi*f2*n/
    F1)+10*cos(2*pi*f3*n/F1);
    fr1(n)=n/25*F1/2;
end
y1=fft(x1,50);
plot(fr1,abs(y1));title ('spectrum of signal with frequencies 35
Hz, 50 Hz and 100 Hz, samplng frequency of 150 Hz');
xlabel('frequency in Hz');
ylabel('amplitude');
figure;
plot(x1);title ('35 Hz, 50 Hz and 100 Hz signal sampled using
Sampling frequency of 100 Hz');
xlabel('sample number');
ylabel('amplitude');
```

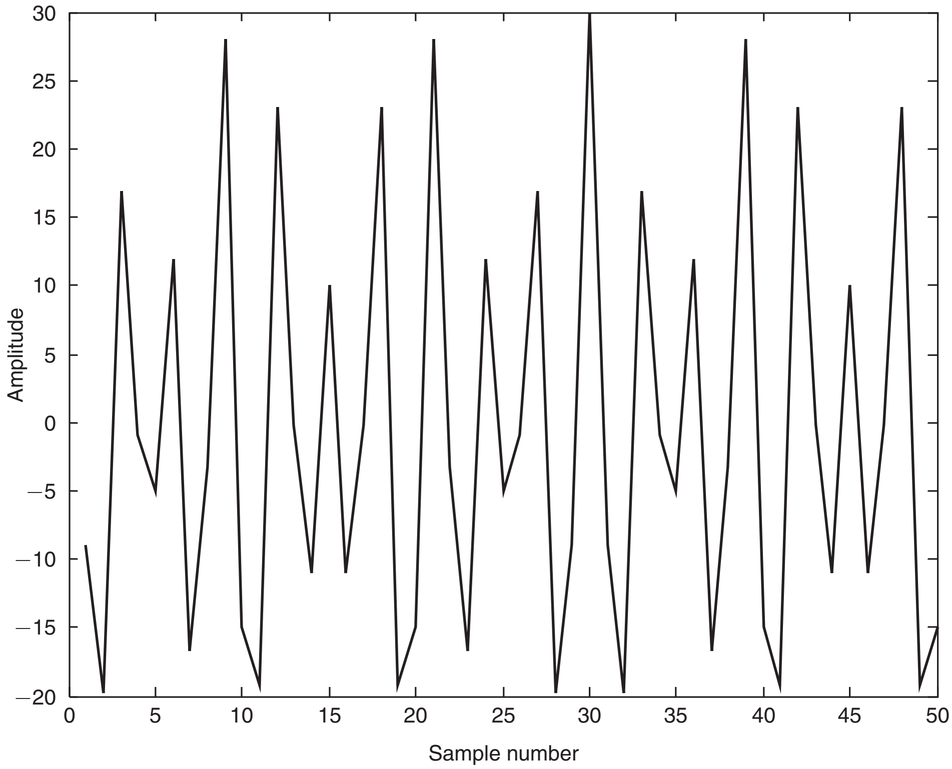

Figure 5 Reconstructed signal of 35 Hz, 50 Hz and 100 Hz sampled using sampling frequency of 150 Hz. (Notice the phase shift.)

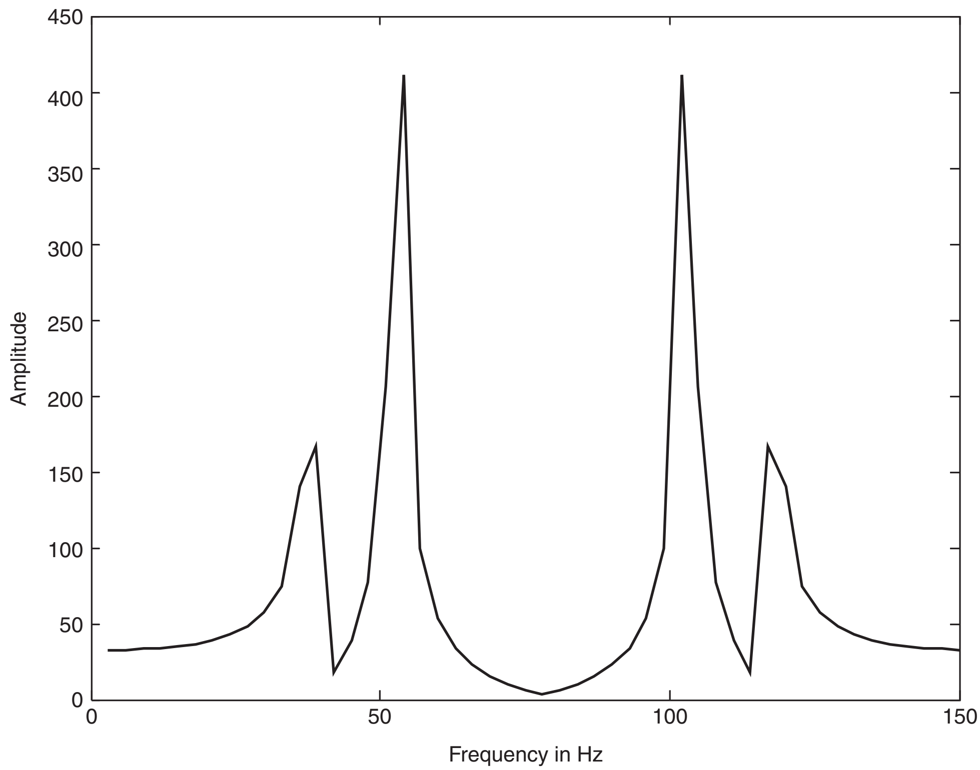

Figure 6 Spectrum of signal of 35 Hz, 50 Hz and 100 Hz sampled using sampling frequency of 150 Hz. (Notice that 100 Hz is aliased as 50 Hz and we see that there are only two peaks.)