Experiment 6
Overlap and Add Algorithm

Aim

To execute overlap and add algorithm for linear filtering of a long data sequence.

Theory

The input sequence $x(n)$ is often a very long sequence when real-time signal processing is done. The method for real-time filtering of long data sequences is already covered in Experiment 5.

For linear filtering of a long sequence, we make use of FFT to execute circular convolution of the two sequences. The long sequence is divided in small segments of size L. If the size of each block of data is L and the size of filter length is M then the convolved sequence will have length equal to $L + M - 1$. Therefore, we have to append zeros at the end of each data block so as to make the processing block size of $L + M - 1$. We will use overlap and add algorithm.

For detailed theory of overlap and add algorithm refer to section 5.9 on fast convolution.

Experiment

Let the input sequence be

$$x(n) = [1\ 2\ 3\ 4\ 5\ 1\ 2\ 3\ 4\ 5\ 1\ 2\ 3\ 4\ 5]$$

Let the impulse response of the filter be $h(n) = [3\ 2\ 1]$. The block diagram for circular convolution using FFT is shown in Figure 1.

We will use block size for the data as $L = 5$ and $M = 4$. We have to append three zeros at the end of each data block and four zeros to $h(n)$ so that the length of both the sequences is $L + M - 1 = 8$. Note that if the last block is of shorter length, extra zeros are to be appended.

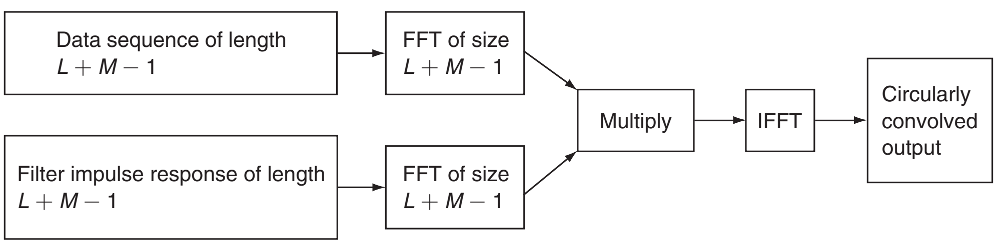


Figure 1 Block diagram for circular convolution using FFT.

The data blocks will be

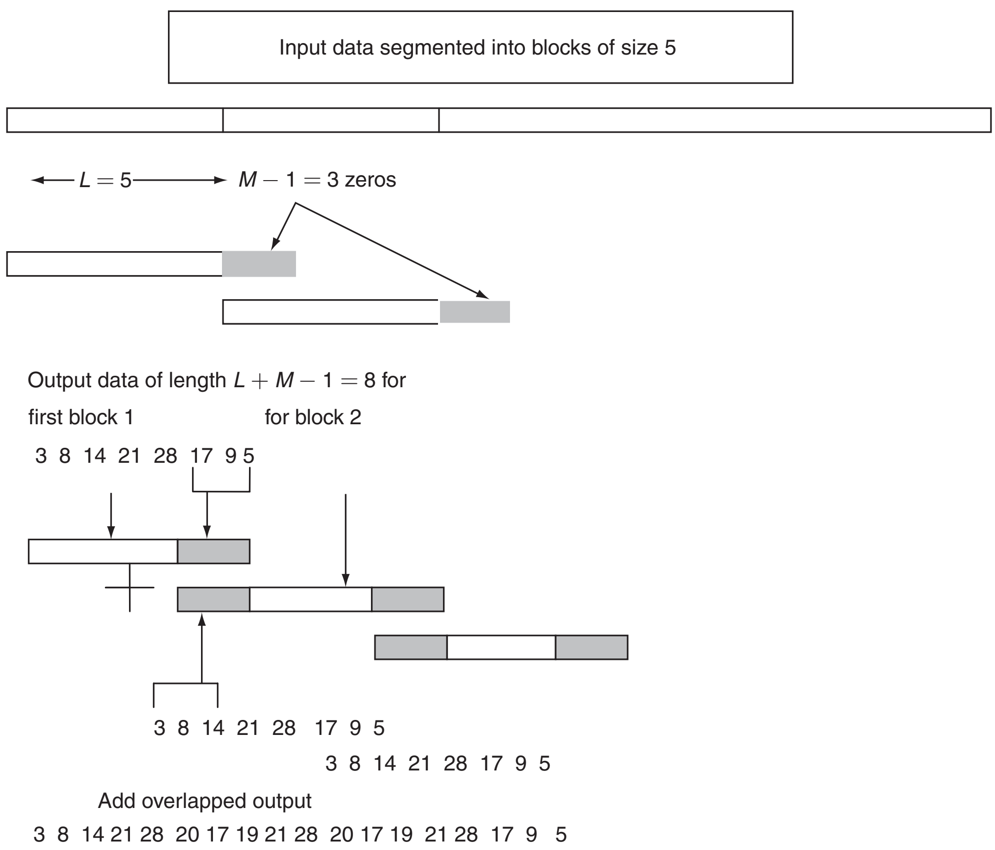
$$x_1(n) = \{1\ 2\ 3\ 4\ \ 5\ 0\ 0\ 0\}$$
$$x_2(n) = \{1\ 2\ 3\ 4\ 5\ 0\ 0\ 0\}$$
$$x_3(n) = \{1\ 2\ 3\ 4\ 5\ 0\ 0\ 0\}$$

We have to take FFT of each block, multiply it with FFT of $h(n)$ and take IFFT of the multiplication output. This is the convolution result. The result of convolution of the data blocks is obtained using a MATLAB program.

The output convolved sequences obtained are

$$y_1(n) = \{3\ 8\ 14\ 21\ 28\ 17\ 9\ 5\}$$
$$y_2(n) = \{3\ 8\ 14\ 21\ 28\ 17\ 9\ 5\}$$
$$y_3(n) = \{3\ 8\ 14\ 21\ 28\ 17\ 9\ 5\}$$

Accumulate the result by adding the overlapped samples of the convolved output for successive blocks. We are adding the overlapped samples of the result, hence the name of the algorithm is overlap-add algorithm. (Refer to Figure 22 in Chapter 5 to add the overlapping samples. The figure is repeated here for ready reference.)



The result of convolution is

[3 8 14 21 28 20 17 19 21 28 20 17 19 21 28 17 9 5]

The MATLAB program is as follows.

```
%overlap & add algorithm
clear all;
a=[1 2 3 4 5 1 2 3 4 5 1 2 3 4 5];
b=[3 2 1 1 0 0 0 0];
c=conv(a,b);
disp(c);
b1=fft(b,8);
a1=[1 2 3 4 5 0 0 0];
a11=fft(a1,8);
for i=1:8,
   a111(i)=a11(i)*b1(i);
end
c1=ifft(a111,8);
disp(c1);
a2=[1 2 3 4 5 0 0 0];
a22=fft(a2,8);
for i=1:8,
   a222(i)=a22(i)*b1(i);
end
c2=ifft(a222,8);
disp(c2);
a3=[1 2 3 4 5 0 0 0];
a33=fft(a3,8);
for i=1:8,
   a333(i)=a33(i)*b1(i);
end
   c3=ifft(a333,8);
   disp(c3);
   %disp(c4);
for i=1:5,
   d(i)=c1(i);
end
for i=6:8,
   d(i)=c2(i-5)+c1(i);
end
for i=9:10,
   d(i)=c2(i-5);
end
for i=11:13,
   d(i)=c3(i-10)+c2(i-5);
end
```

```
for i=14:18,
   d(i)=c3(i-10);
end
disp(d);
```

The resultant output using MATLAB program is

```
3 8 14 21 28 20 17 19 21 28 20 17 19 21 8 17 9 5 0 0
```

Teaser *The reader is encouraged to verify that the result of direct convolution of the two sequences tallies with the result of overlap and add algorithm.*