Experiment 3
Properties of DFT

Aim

To study the properties of DFT.

Complex Conjugate Property of DFT

Theory

Let $x(n)$ be a real N periodic sequence, that is

$$x(n) \xleftrightarrow{\text{DFT}} X(k)$$

then

$$X^*(N/2-k) = X^*(-k)_N = X(N/2+k)$$

Experiment

Consider the eight-point signal

$$x(n) = \{1\ 2\ 3\ 4\ 5\ 2\ 3\ 1\}$$

Calculate eight-point fast Fourier transform (FFT) and display FFT output. We can observe symmetric nature about the center value, that is $X(4)$. The DFT output using a MATLAB program is given by the following table.

DFT X	*Value*	*DFT X*	*Value*
$X(0)$	21.000		
$X(1)$	−6.1213 − 2.1213i	$X(7)$	−6.1213 + 2.1213i
$X(2)$	0 + 1.0000i	$X(6)$	0 − 1.0000i
$X(3)$	−1.8787 − 2.1213i	$X(5)$	−1.8787 + 2.1213i
$X(4)$	3.0000		

We can clearly observe the complex conjugate property.

The MATLAB program is as follows.

```
clear all;
x=[1 2 3 4 5 2 3 1];
x1=fft(x);
disp(x1);
```

Property of Time Reversal

Theory

If

$$x(n) \xleftrightarrow{\text{DFT}} X(k)$$

then

$$x((-n))_N = x(N-n) \xleftrightarrow{\text{DFT}} X((-k))_N = X(N-k)$$

Here the double parentheses indicate that it is periodic with period N.

Experiment

We will consider a DT signal $x(n)$ and a time-reversed sequence and calculate the FFT of both. Calculate eight-point DFT for the following sequences:

$$x(n) = \{1\ 2\ 3\ 4\ 5\ 2\ 3\ 1\}$$

$$x(-n) = \{1\ 1\ 3\ 2\ 5\ 4\ 3\ 2\}$$

Note the process of time reversal for a periodic sequence. Table 1 represents the time reversal for a periodic signal. Table 2 indicates how time reversal will occur for the actual signal $x(n)$.

Table 1 Process indicating time reversal of a periodic signal

Sample No.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Original sequence	$x(0)$	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$x(5)$	$x(6)$	$x(7)$
Time-reversed sequence	$x(0)$	$x(7)$	$x(6)$	$x(5)$	$x(4)$	$x(3)$	$x(2)$	$x(1)$

Table 2 Process indicating time reversal of actual signal $x(n)$

Sample No.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Original sequence	1	2	3	4	5	2	3	1
Time-reversed sequence	1	1	3	2	5	4	3	2

The MATLAB program is as follows.

```
clear all;
x=[1 2 3 4 5 2 3 1];
x1=fft(x);
disp(x1);
y=[1 1 3 2 5 4 3 2];
y1=fft(y);
disp(y1);
```

The reader has to write a program to calculate DFT of $x(n)$ using fft command in MATLAB. The eight-point DFT of $x(n)$ is obtained as

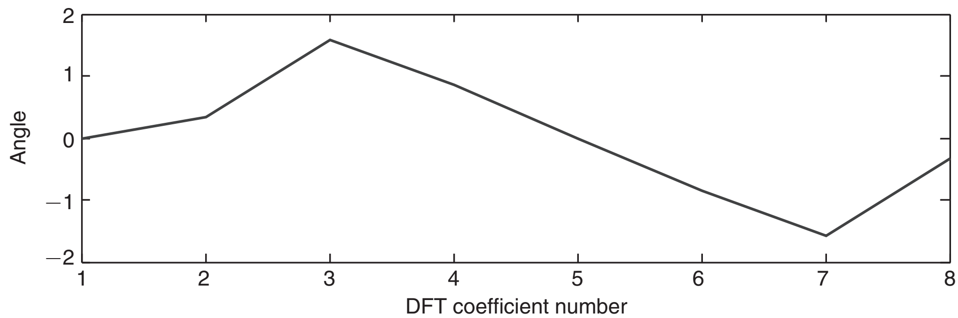
$$X(k) = \{21.0000, -6.1213 + 2.1213i, 0 - 1.0000i, -1.8787 + 2.1213i \\ 3.0000, -1.8787 - 2.1213i, 0 + 1.0000i, -6.1213 - 2.1213i\}$$

We have to now calculate DFT of a time reversed signal also. The reader has to write a program to calculate DFT of $x(-n)$. The eight-point DFT of $x(-n)$ is

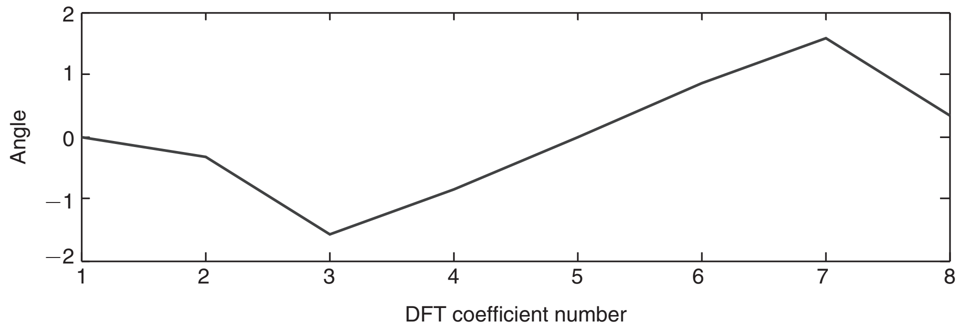
$$X(-k) = \{21.0000, -6.1213 - 2.1213i, 0 + 1.0000i, -1.8787 - 2.1213i \\ 3.0000, -1.8787 + 2.1213i, 0 - 1.0000i, -6.1213 + 2.1213i\}$$

This can be verified to be same as the reversed DFT output.

If we carefully observe the DFT coefficients of $X(-k)$, we see that for each coefficient, there is a phase reversal. The phase angle for each sample of DFT coefficient of $x(-n)$ is the negative of the phase angle for corresponding DFT coefficient for $x(n)$. The phase reversal is clearly visible in Figure 1. This is because each DFT coefficient gets replaced by its complex conjugate. Consider the second DFT value $-6.1213 - 2.1213i$. It can be represented in magnitude and phase value as $6.47844\, e^{j19.11}$. This value gets replaced by $-6.1213 - 2.1213$ and is represented as $6.47844\, e^{-j19.11}$. This indicates the phase reversal.



(a)



(b)

Figure 1 Phase value of DFT plotted for the (a) original and (b) time-reversed sequences.

Property of Circular Frequency Shifting

Theory

If $x(n)$ is an N-periodic sequence, then

$$x(n)W_N^{-nh} \xleftrightarrow{\text{DFT}} X((k-h))_N$$

Here double parenthesis is used to indicate that $x((k - h))$ is N periodic.

Experiment

Consider the following sequence

$$x(n) = \{1\ 2\ 3\ 4\ 5\ 2\ 3\ 1\}$$

We will multiply each nth sample by $W_8^{2n} = e^{-j2\pi(2n)/8}$ $(h = 2)$ to get the modified sequence $x_1(n)$ and evaluate the DFT of the resulting sequence. The reader has to write a MATLAB program to evaluate the modified signal by multiplying the nth sample by $W_8^{2n} = e^{-j2\pi(2n)/8}$. We have to calculate DFT of both $x(n)$ and modified $x(n)$ using "fft" command in MATLAB.

We have to verify that there is a shift of two samples in magnitude plot of the real sequence when it is modified. We must get the DFT output shifted by two samples, as we have used $h = 2$. The result of execution of the program using MATLAB is plotted in Figure 2. The magnitude and phase plot of the signal in time domain when it is modified is shown in Figure 3. We see that the phase angle varies (increases) linearly with the sample number.

Note that the complex conjugate property does not hold good for DFT of $x_1(n)$ as $x_1(n)$ is no longer a real sequence, even though $x(n)$ is real. The MATLAB program is as follows.

```
%frequency shifting property
clear all;
x=[1 2 3 4 5 2 3 1];
y=abs(fft(x));
y3=fft(x);
subplot(2,1,1);
plot(y);
title('DFT of x(n)');
xlabel('DFT coefficient number');
ylabel('Amplitude');
for i=1:8,
    x1(i)=x(i)*exp(-j*2*pi*i*2/8);
end
y1=abs(fft(x1));
y2=fft(x1);
subplot(2,1,2);
plot(y1);
title('DFT of x1(n)');
xlabel('DFT coefficient number');
ylabel('Amplitude');
a=[1 2 3 4 5 2 3 1];
for i= 1:8,
    a=a*exp((j*2*pi*2*i)/8);
end
figure;
subplot(2,1,1);
stem(abs(a));
title('magnitude plot of the signal');
xlabel('sample number');
ylabel('amplitude');
for i=1:8,
p(i)=2*pi*2*i/8;
p(i)=p(i)*180/pi;
end
subplot(2,1,2);
stem(p);
title('phase plot of the signal');
xlabel('sample number');
ylabel('degrees');
```

Property of Circular Time Shifting

Theory

Consider a DT signal $x(n)$ as an N periodic sequence and let $z(n)$ be a shifted sequence given by

$$z(n) = x((n - h))_N$$

Here double parenthesis is used to indicate that $x((n - h))$ is N periodic. Also if h is the amount of shift, then

$$Z(k) = W^{kh}X(k)$$

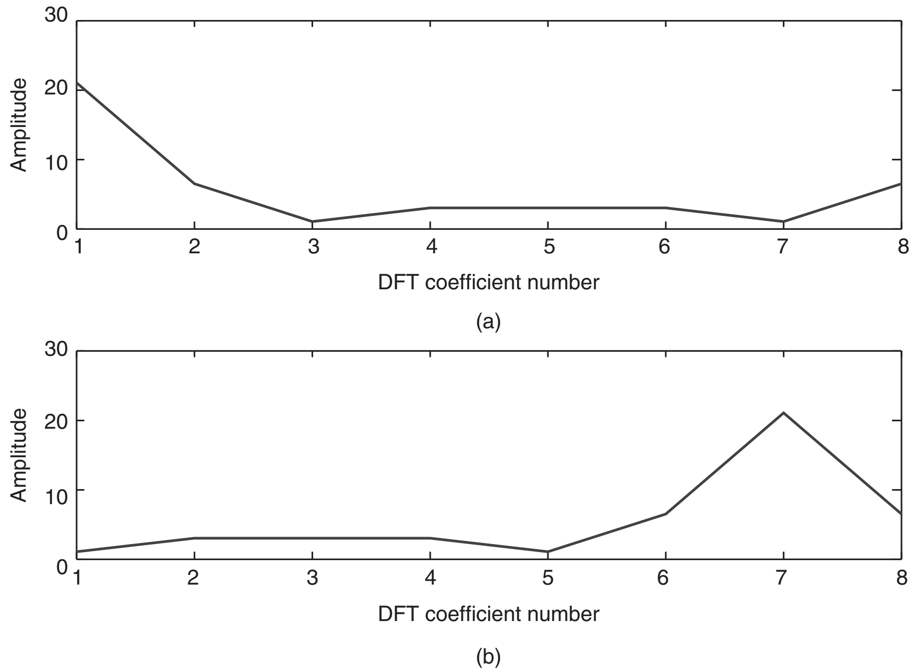


Figure 2 Magnitude plots for (a) original $x(n)$ and (b) modified $x_1(n)$ sequence.

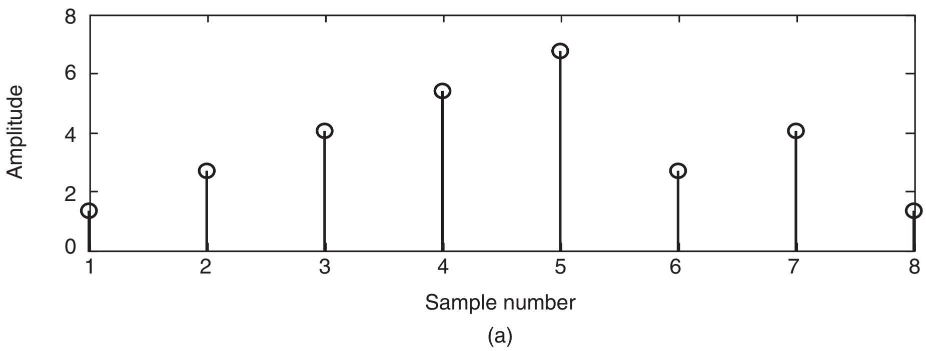
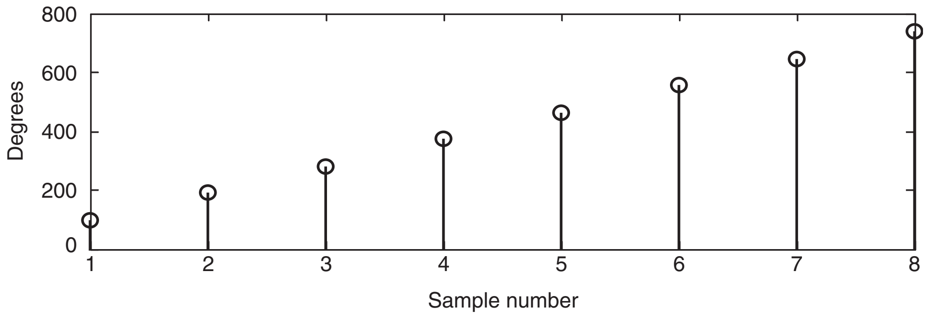
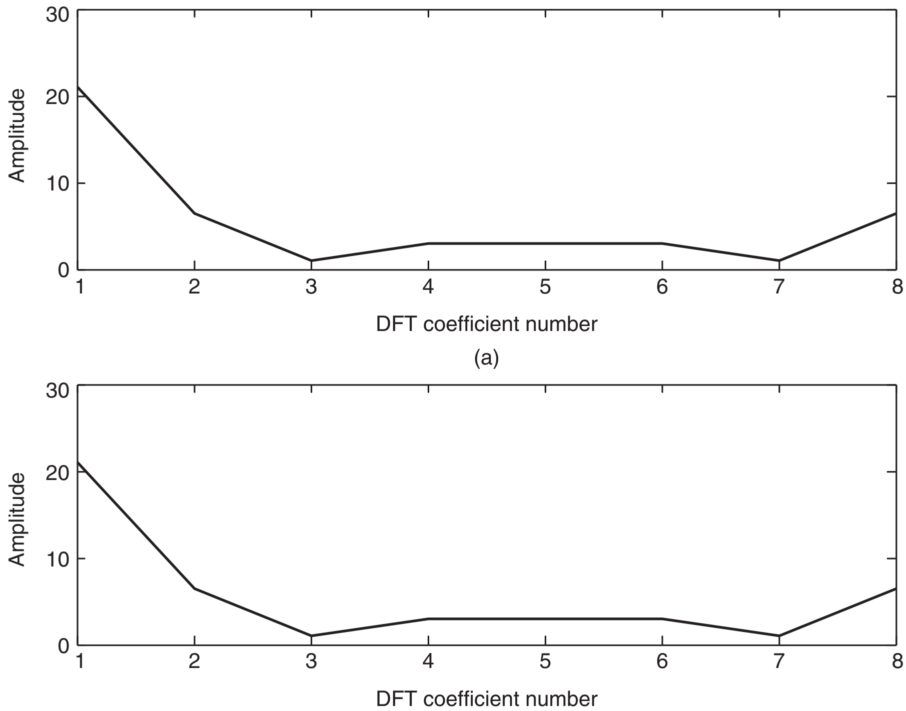


Figure 3 (Continued)



(b)

Figure 3 (a) Magnitude and (b) phase plot of modified signal in time domain.



(b)

Figure 4 Magnitude plot of (a) $x(n)$ and (b) $x(n-2)$. The plots are identical.

Experiment

We will calculate eight-point DFT of $x(n) = \{1\ 2\ 3\ 4\ 5\ 2\ 3\ 1\}$ and DFT of sequence shifted by two samples that is $x(n-2) = \{3\ 4\ 5\ 2\ 3\ 1\ 1\ 2\}$.

When a time shift of two samples occurs, we have to verify that there will be a phase shift in the kth DFT output by $2 \times \pi \times 2 \times k/N$. The reader has to write a MATLAB program to calculate the

DFT of sequence $x(n)$ and its shifted version $x(n - 2)$. The magnitude plot of the DFT outputs shows that the magnitude plot remains constant. Figure 4 shows the magnitude plot of the signal and shifted $x(n)$. The phase plots shown in Figure 5 indicate that the phase increases linearly for every DFT sample. The phase is warped in the sense that when the phase angle goes above 2π, it is reduced to the equivalent value below 2π. The MATLAB program is as follows.

```
%Time shifting property
clear all;
x=[1 2 3 4 5 2 3 1];
y=abs(fft(x));
y3=fft(x);
subplot(2,1,1);
plot(y);
title('DFT of x(n)');
xlabel('DFT coefficient number');
ylabel('Amplitude');
x1=[3 4 5 2 3 1 1 2]
y2=fft(x1);
y1=abs(fft(x1));
%disp(y1);
subplot(2,1,2);
plot(y1);
disp(y2)
disp(y3);
figure;
for i=1:8,
z(i)=atan(imag(y3(i))/real(y3(i)));
z1(i)=atan(imag(y2(i))/real(y2(i)));
end
%disp(z);
%disp(z1);
subplot(2,1,1);
plot(z);
title('DFT of x(n)');
xlabel('DFT coefficient number');
ylabel('angle');
subplot(2,1,2);
plot(z1)
title('DFT of x(n)');
xlabel('DFT coefficient number');
ylabel('Angle');
```

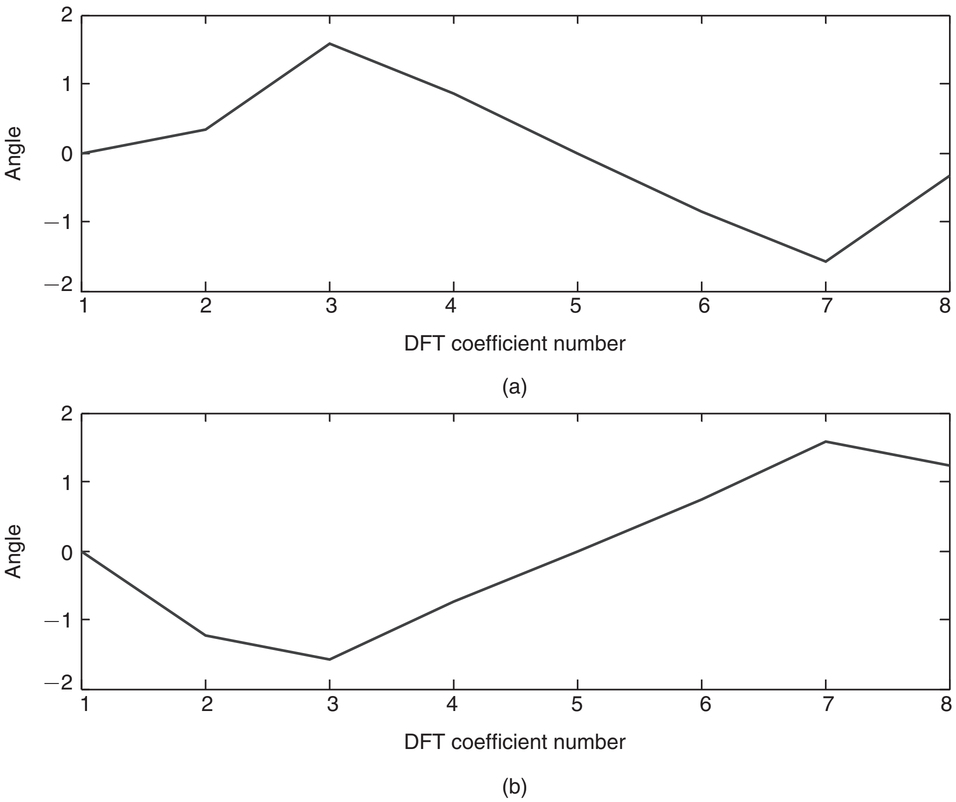


Figure 5 Phase plot for (a) $x(n)$ and (b) $x(n - 2)$. The phase angle is warped.

Property of Multiplication of Two DFTs and Circular Convolution

Theory

If

$$x_1(n) \xleftrightarrow{\text{DFT}} X_1(k)$$
$$x_2(n) \xleftrightarrow{\text{DFT}} X_2(k)$$

then the multiplication of the two DFTs is a DFT of a sequence which is a circular convolution of the two original sequences.

Experiment

Let us consider two sequences as $x(n) = \{1\ 2\ 2\ 1\}$ and $h(n) = \{1\ 2\ 1\ 0\}$. Let us calculate DFTs of the two sequences and multiply the two DFTs.

The MATLAB program is as follows.

```
clear all;
x=[1 2 2 1];
x1=fft(x);
disp(x1);
y=[1 2 1 0];
y1=fft(y);
for i=1:4,
z(i)=x1(i)*y1(i);
end
z=ifft(z1);
disp(z);
```

The reader is required to write a MATLAB program to calculate DFTs of two sequences using "fft" command in MATLAB. We will multiply the DFTs. We will calculate the IDFT of the multiplication and verify that the result is the circular convolution of the two sequences.

The result of execution of the program in MATLAB is found as follows:

Multiplication of two DFTs is = 24.0000, −2.0000 + 2.0000i, 0, −2.0000 − 2.0000i

The result of circular convolution is

5 5 7 7