Experiment 11
Butterworth Filter Design

Aim

To design Butterworth IIR filters and study the response.

Theory

Butterworth filters of order n are characterized by the magnitude response given by

$$|P_n(\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}, \quad n \geq 1$$

We will use bilinear transformation (BLT) method to design IIR low-pass filter. The steps involved in filter design using BLT method are as follows.

Step 1: Pre-warp the critical frequency, that is, convert the digital cut-off frequency into the corresponding analog frequency using Eq. (1).

$$\omega_A = \tan(\omega_D T / 2) \tag{1}$$

Step 2: Perform frequency scaling. The analog filter given has cut-off frequency of 1 rad. We have to scale the transfer function so that it will have the required cut-off frequency. We have to substitute $S = S/\omega_A$ in transfer function of the filter.

Step 3: Use Eq. (2) to convert the transfer function from S domain to Z domain.

$$S = \frac{Z-1}{Z+1} \quad \text{or} \quad Z = \frac{1+S}{1-S} \tag{2}$$

Experiment

Design a second-order DT Butterworth filter whose cut-off frequency is 1 kHz at a sampling frequency of 10,000 Hz using BLT method. Plot the corresponding magnitude and phase response.

Let us use a MATLAB command (Butter) for design of IIR low-pass filter with given specifications. The specifications are cut-off frequency is 0.1 of the Nyquist frequency. We will then use freqz function to plot magnitude and phase response. The designed filter coefficients obtained are

Numerator coefficients: 0.0018   0.0141   0.0494   0.0987   0.1234   0.0987
                               0.0494   0.0141   0.0018

Denominator coefficients: 1.0000   −6.3904   18.0003   −29.1711   29.7314

                                 −19.5056   8.0410   −1.9037   0.1981

Magnitude and phase response is shown in Figure 1. The MATLAB program is as follows.

```
%Butterworth LPF design using BLT for problem 3
clear all;
B=[0.0018    0.0141    0.0494    0.0987    0.1234    0.0987
   0.0494    0.0141    0.0018];
A=[1.0000    -6.3904    18.0003   -29.1711    29.7314   -19.5056
8.0410    -1.9037    0.1981];
freqz(B,A,256,10000);title('magnitude and phase plot for LPF DT
butterworth filter -using BLT method ')
```
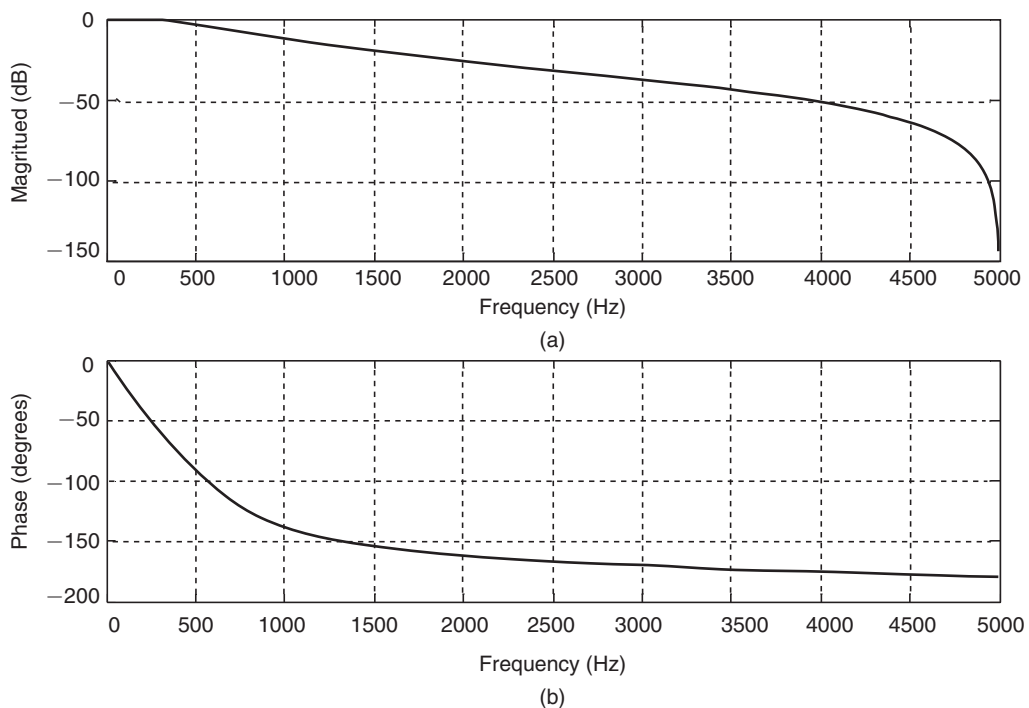

Figure 1 (a) Magnitude and (b) phase response of the filter.