Experiment 4
Circular Convolution

Aim

To study the circular convolution for calculation of linear convolution and aliasing effect.

Theory

Convolution of the two sequences in the time domain is equivalent to the multiplication of their DFTs in the frequency domain. For detailed theory refer to Section 5.5 on properties of DFT, namely, *property of multiplication of two DFTs and circular convolution*. Refer to Section 5.1 for aliasing effect in time domain.

Experiment

Consider the following sequences:

$$x(n) = \{1\ 2\ 2\ 1\}$$
$$h(n) = \{1\ 2\ 1\}$$

Here $x(n)$ is the data sequence and $h(n)$ is the impulse response of the filter. When we want to filter the data sequence, we have to convolve the two sequences linearly. If we want to make use of the FFT for reducing the number of computations, then we have to use the property of DFT, namely, circular convolution property. Circular convolution property states that the multiplication of the two DFTs will result in a circular convolution of the two sequences when reverted to time domain. The two sequences considered above are having four and three samples each. We will append extra zero to $h(n)$ so we can treat it as 4-periodic, that is, $h(n) = \{1\ 2\ 1\ 0\}$. We want to convolve the two sequences linearly for, say, calculation of output of a system with impulse response $h(n)$ and input $x(n)$. The linearly convolved sequence will contain $L + M - 1 = 4 + 3 - 1 = 6$ samples, where L denotes number of points in data sequence and M denotes the number of points in the other sequence, namely, the impulse response of the filter. To obtain linear convolution using circular convolution, append extra zeros to both the sequences until the length of the each sequence is $L + M - 1$.

If we make use of circular convolution property of DFT, we see that when we multiply the DFTs of the two sequences, we obtain circular convolution of the two sequences in time domain. Let us take four-point DFT of both the sequences and multiply. We will then revert back in time domain by taking IDFT of the multiplication result. Repeat the procedure for eight-point DFT. The method of circular convolution using FFT is shown in Figure 1. The figure indicates that the

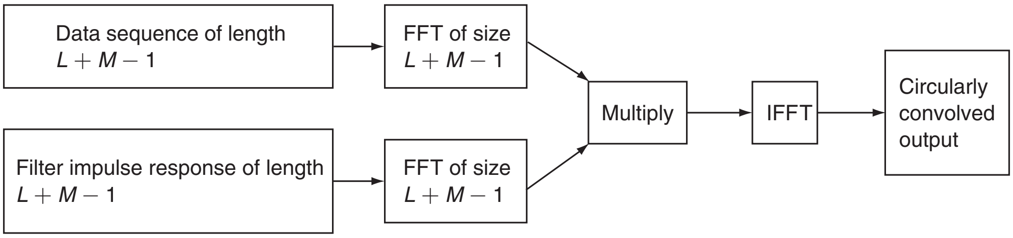

Figure 1 Method of circular convolution using FFT.

length of the data sequence must be $L + M - 1$. When we use eight-point FFT, the result of circular convolution is the same as that of linear convolution.

According to the sampling theorem in the frequency domain, we must take the number of DFT point to be greater than $L + M - 1$, that is $4 + 3 - 1 = 6$. When we take the eight-point DFT, this condition is satisfied. There is no aliasing. When we take a four-point DFT ($4 < 6$) there will be an overlapping of the samples in the time domain and these overlapping samples will add up. The result using a MATLAB program is shown in Table 1.

Table 1 Convolution result using eight-point DFT and four-point DFT

Sample No.	*1*	*2*	*3*	*4*	*5*	*6*
Output using eight-point DFT	1.0	4.0	7.0	7.0	4.0	1.0
Output using four-point DFT	5	5	7	7	–	–

We see that sample 1 and sample 5 overlap to give the output as $1 + 4 = 5$ and sample 2 and sample 6 overlap to give the output as $4 + 1 = 5$. This is exactly the aliasing effect in time domain.

Teaser *The reader can verify that the result of eight-point IFFT is the linear convolution of two sequences.*

The MATLAB program is as follows.

```
%sampling theorem in Frequency domain
clear all;
x=[1 2 2 1 ];
h=[ 1 2 1 0 ]
x1=fft(x);
h1=fft(h);
for i=1:4,
z(i)=x1(i)*h1(i);
end
disp(z);
```

```
w=ifft(z);
disp(abs(w));
x=[1 2 2 1 0 0 0 0 ];
h=[1 2 1 0 0 0 0 0];
x1=fft(x,8);
h1=fft(h,8);
for i=1:8,
z(i)=x1(i)*h1(i);
end
disp(z);
w=ifft(z);
disp(abs(w));
```

The result of circular convolution using 4-point FFT is

5 5 7 7

The result of circular convolution using 8-point FFT is

1 4 7 7 4 1