Experiment 12
Bilinear Transformation Mapping

Aim

To study and plot the mapping function used in bilinear transformation method of IIR filter design.

Theory

We have to study the mapping between S domain and Z domain when we are transforming analog filter into a DT filter using bilinear transformation method. This method uses a mapping function which is a bilinear function relating S to Z given by

$$S = \frac{Z-1}{Z+1} \quad \text{or} \quad Z = \frac{1+S}{1-S} \tag{1}$$

Consider the mapping of points from S plane to Z plane.

1. Let $S = 0$, that is $\omega = 0$. Using Eq. (1), we get $Z = 1$.
2. Consider ω varying from 0 to ∞ in the S domain. When $S = j\infty$, $Z = -1$. So as ω varies from 0 to ∞, it traverses from $Z = 1$ to $Z = -1$ on unit circle in anticlockwise direction.
3. Consider ω varying from 0 to $-\infty$ in the S domain. When $S = -j\infty$, $Z = -1$. So as ω varies from 0 to $-\infty$, it traverses from $Z = 1$ to $Z = -1$ on unit circle in clockwise direction.
4. When S is negative, using Eq. (1) we get $|Z| < 1$. So points on the left-hand side of imaginary axis map onto the area inside the unit circle in the Z domain.
5. When S is positive, using Eq. (1) we get $|Z| > 1$. So points on the right-hand side of imaginary axis map onto the area outside the unit circle in the Z domain.

Experiment

Let us do this as a pen and paper exercise. The reader is encouraged to take different complex values of S and map them onto complex Z domain. The reader is encouraged to verify the mapping shown in Figure 1.

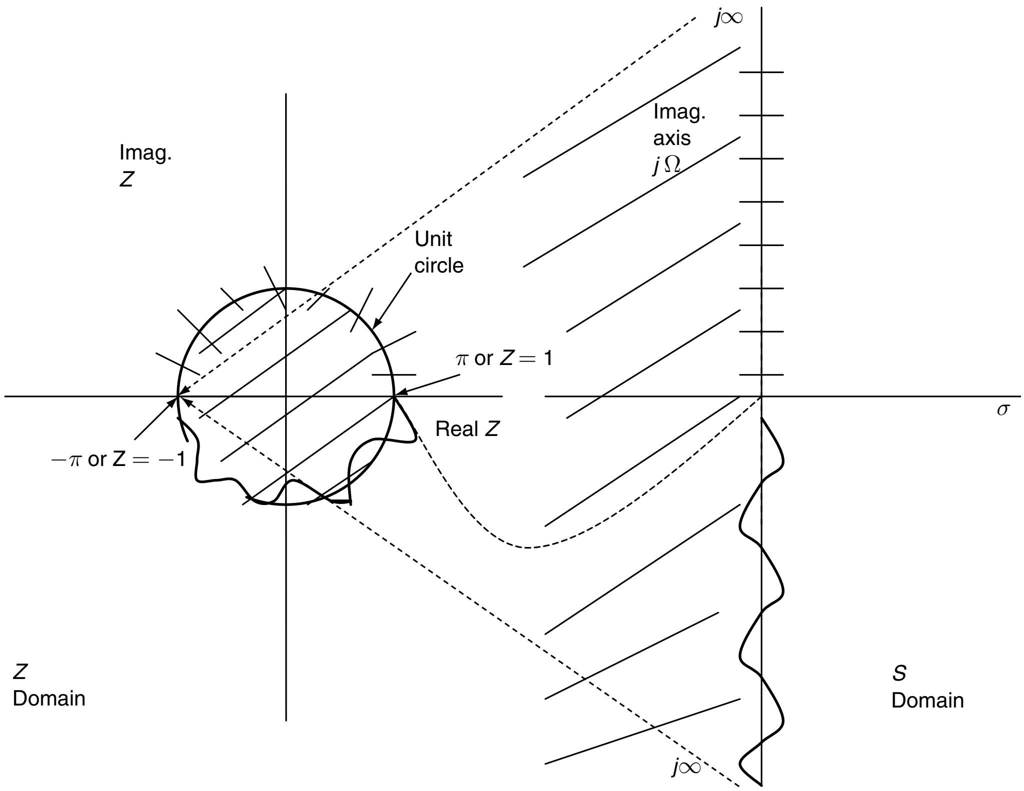

Figure 1 Mapping between *S* domain and *Z* domain for bilinear transformation method.