Experiment 7
Gibbs Phenomenon

Aim

To study Gibbs phenomenon.

Theory

Consider a DT signal $x(n)$ containing 32 samples. Pass it via a rectangular window of size 32. We see that in the time domain, signal $x(n)$ gets multiplied by a rectangular window. We know that multiplication in the time domain is equivalent to a convolution in the Fourier transform domain. This convolution results in the ripple in the stop band of the signal. This phenomenon of oscillations in the stop band is named as Gibbs phenomenon.

Experiment

Consider the following simple signal

$$x(n) = \{1\ 1\ 1\ 1\ 1\ \ldots\ 1\ 1\}$$

with 32 samples each of value 1. Let us consider a rectangular window $w(n)$ of size 32 such that

$$w(n) = \{0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\}$$

It passes only 16 samples of data sequence.

We will write a MATLAB program to multiply the two signals. The output is as shown in Figure 1. We will calculate FFT of both $x(n)$ and $w(n)$ and will convolve the result and plot it or we may calculate FFT of $x(n) \times w(n)$ and plot it. The result is shown in Figure 2. We see that FFT output is symmetrical around sample number 16 and if we concentrate on sample number 0 to 15, we find that there is a main lobe and there are side lobes. The side lobes correspond to the stop band frequencies for the windowed signal. These ripples in stop band or oscillations in stop band are called Gibbs phenomenon.

If we now take 128-point FFT of the windowed signal, we get better resolution in the FFT output as shown in Figure 3.

The MATLAB program is as follows.

```
%windowing and Gibbs phenomenon
clear all;
a=[0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0
0 0];
b=fft(a,64);
y=abs(b);
%axis(-pi pi 0 4);
for i=1:31,
    z(i)=y(32-i);
end
z(32)=16;
for i=1:31,
    z(i+32)=y(i);
end
plot(z);
```

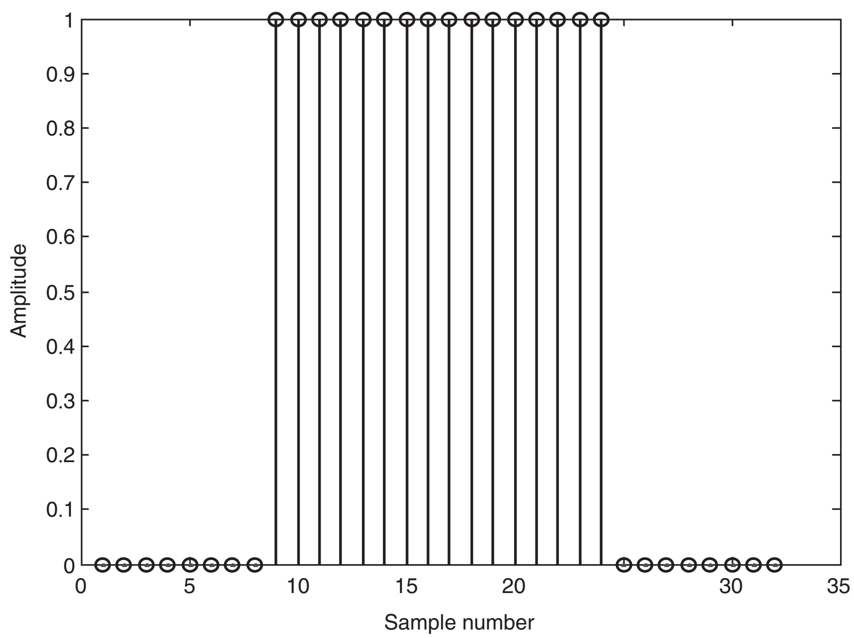

Figure 1 Multiplication of $x(n)$ and $w(n)$.

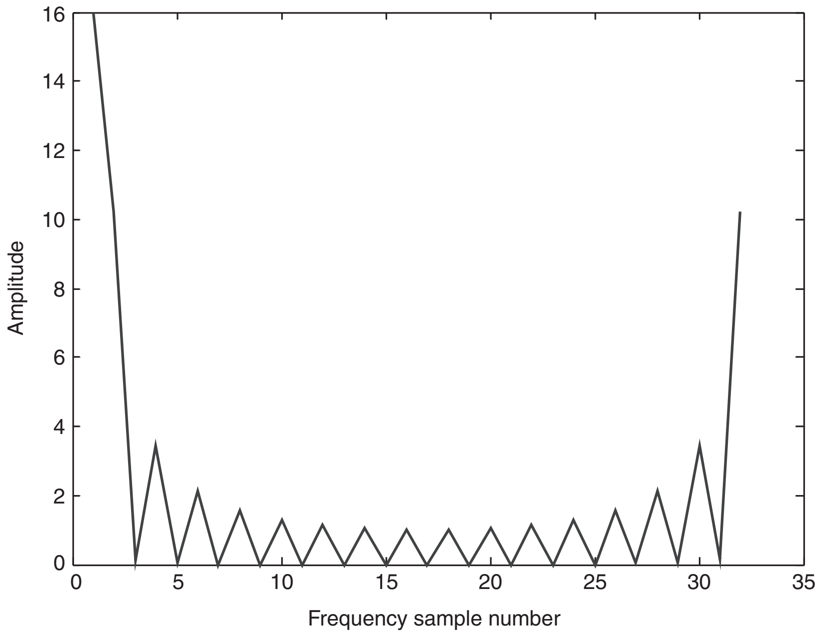

Figure 2 Plot of FFT (32 point) of $x(n) \times w(n)$.

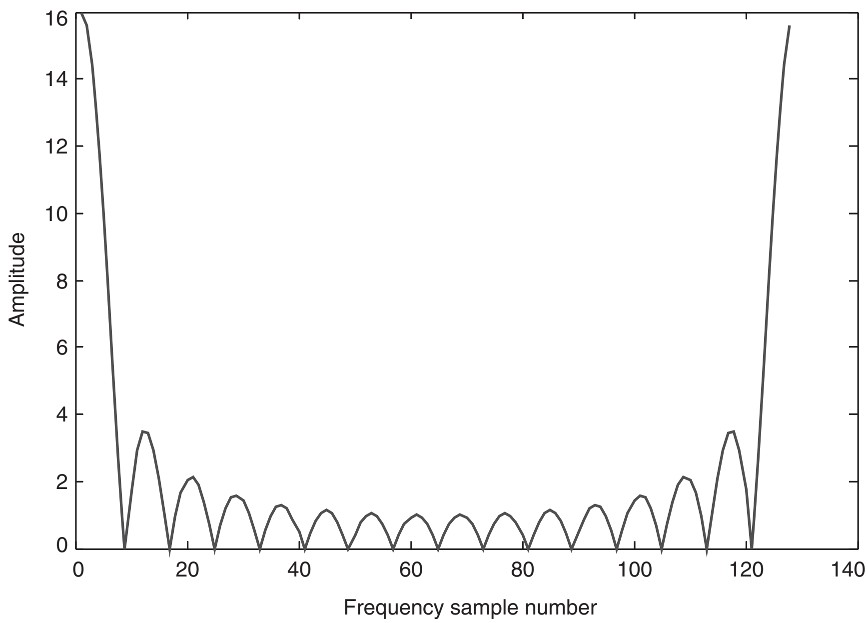

Figure 3 Plot of FFT (128 point) of $x(n) \times w(n)$.