Experiment 8
FS Expansion Method

Aim

To design FIR filter using Fourier series expansion method.

Theory

The FIR filter has the following properties.

1. It has $2Q + 1$ impulse response coefficients.
2. The impulse response is symmetric with respect to a_Q.
3. The duration of the response is $2 \times Q \times T$, where T is the sampling interval and is given by $1/F_S$.

The delay of Q samples introduces a phase change of $-\pi\upsilon Q$ for every υ, that is, a linear phase change with respect to normalized frequency. The group delay is a time delay introduced for each frequency. If the system introduces constant time delay for all frequencies, then all frequencies are delayed by equal amount when they pass via a filter and no phase distortion is introduced. The design of the filter will be done as per the specifications given by the user.

We have seen that the ideal brick-wall filter cannot be designed. The designer will try to design a filter that will closely approximate the desired specifications given by the user. The user will specify the desired response using the parameters as shown in Figure 1. Normally attenuation is plotted versus frequency rather than gain:

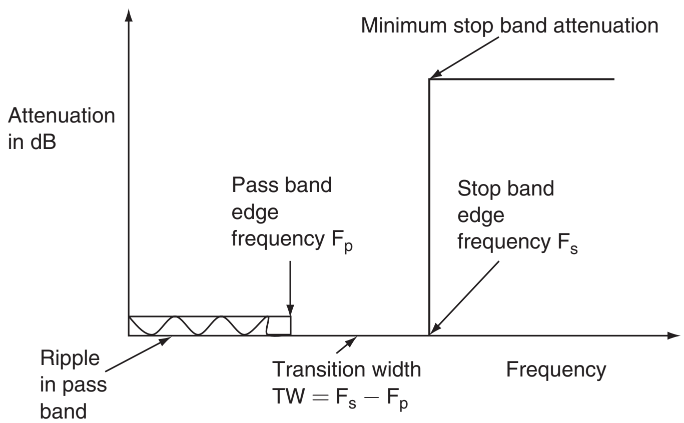

Figure 1 FIR filter specifications.

1. Pass band edge frequency, F_p.
2. Stop band edge frequency, F_s.
3. Transition width, $TW = F_s - F_p$.
4. Minimum stop band attenuation in dB.

The designer will trade-off the transition width for minimum stop band attenuation.

The reader is requested to refer to the Problem 1 of Chapter 7 for FIR filter design.

Experiment

Let us consider the filter specifications as follows: Let $F_S = 8000$ Hz. So $F_N = 4000$ Hz; $T = 1/8000$ s. Let the duration of the response be 0.0025 s. Consider LPF with cut-off frequency of 1000 Hz. We will use the following equation and find the value of Q:

$$\tau = 2QT$$
$$0.0025 = 2Q \times 1/8000$$
$$Q = 10$$

The resulting FIR filter has 21 coefficients.

The filter coefficients can be calculated using the following equation:

$$C_n = \frac{\sin(0.25n\pi)}{n\pi} \quad \text{for } 0 \le n \le 10$$

The symmetry in the impulse response means the coefficients $C_{-n} = C_n$.

Teaser *The reader is encouraged to write a MATLAB program to calculate the value of Q and the filter coefficients.*

The impulse response of the filter is obtained as shown in Figure 2. The MATLAB program is as follows.

```
%FIR filter design using rectangular window
clear all;
fs=8000;t=0.0025;
Q=t*fs/2;
Q=10;
disp(Q);
```

```
for i=1:Q,
    x(i)=(sin(0.25*pi*i))/(i*pi);
end
a(Q+1)=0.25;
for i=1:Q,
    a(i)=x(Q-(i-1));
end
for i=2:Q+1
a(i+Q)=x(i-1);
end
w1=window(@rectwin,21)
for i=1:21,
    a1(i)=a(i)*w1(i);
end
stem(a1);xlabel('coefficient number');
ylabel('amplitude');title('impluse response of the filter using
rectangular window');
for i=1:4000
sum(i)=0.0;
end
figure;
[h,w]=freqz(a1,1,256,8000);
plot(w/pi,20*log10(abs(h)));xlabel('normalized frequency');
ylabel('ampliudein dB');title('magnitude response plot-
rectangular window');
figure;
plot(w/pi,(atand(imag(h)/real(h))));xlabel('normalized
frequency');ylabel('angle in radians');title('Phase response
plot-kaise window');
freqz(a1,1,256,8000);title('magnitude and phase plot rectangular
window');
```

Teaser *The reader is encouraged to use a MATLAB command to plot a magnitude and phase response (command freqz may be used).*

Magnitude and phase response is plotted in Figure 3.

Teaser *The reader is encouraged to try using different values of Q and study its effect on the response.*

When the value of *Q* increases, the order of the filter increases. The reader is advised to verify that increase in the value of *Q* decreases the width of the main lobe and hence the transition width. Figure 4 shows the magnitude and phase response that confirms the result.

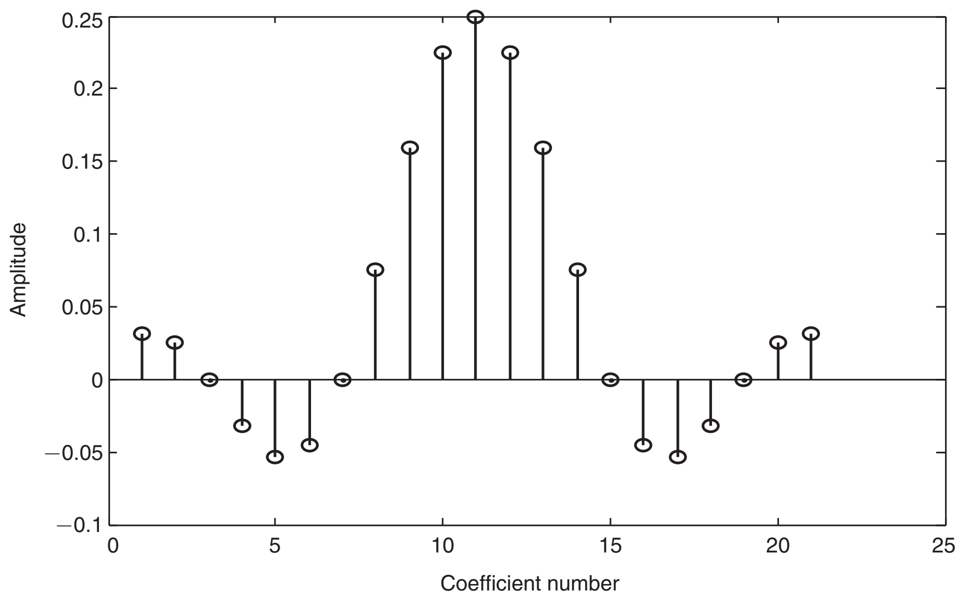

Figure 2 Impulse response of the designed filter.

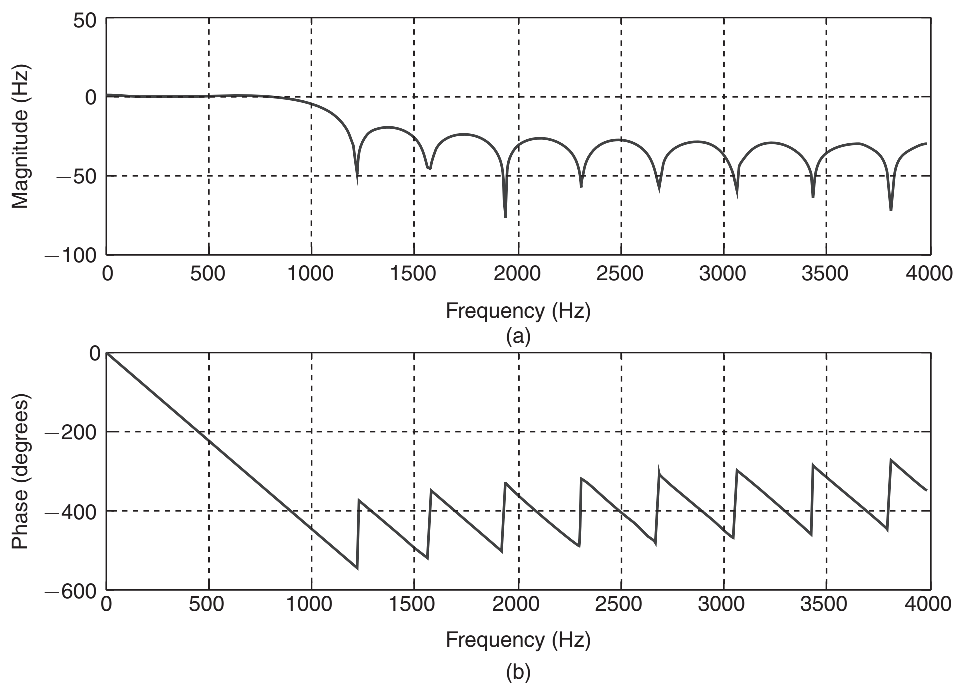

Figure 3 (a) Magnitude and (b) phase response of the designed filter with $Q = 21$.

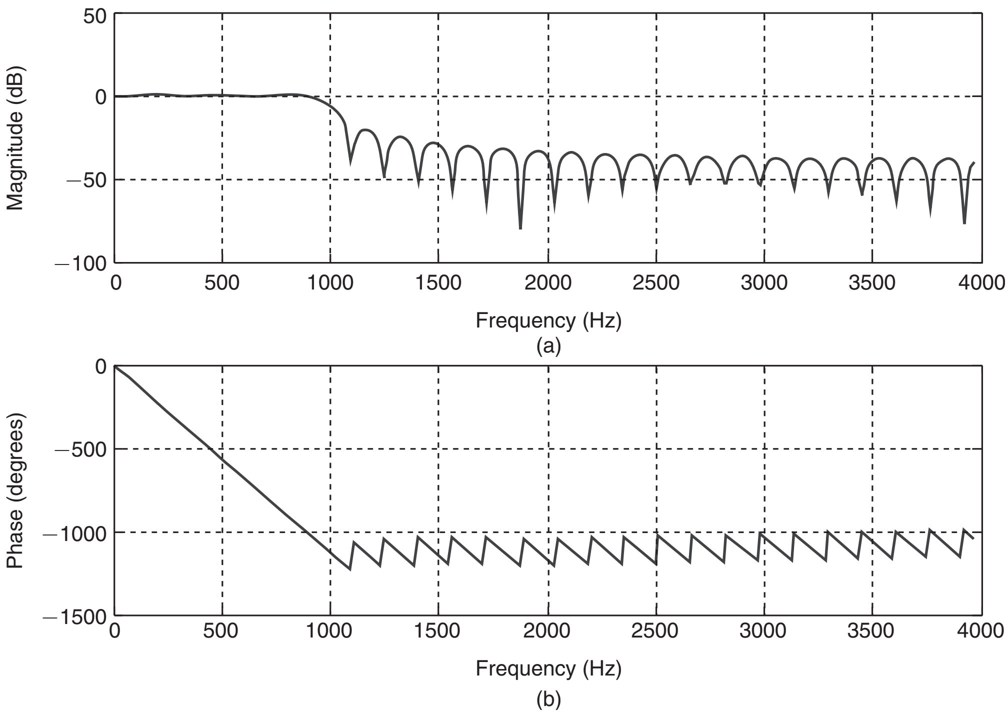

Figure 4 (a) Magnitude and (b) phase response of the designed filter with $Q = 51$.