Experiment 13
Effect of Coefficient Quantization

Aim

To study the effect of coefficient quantization on the impulse response of the filter using direct form I realization and cascade realization.

Theory

We have studied the effect of coefficient quantization in Section 9.3 of Chapter 9 "*Quantization Effects in IIR Filters*". We have seen that owing to finite word length of quantized filter coefficients α_i and β_j, the following effects may occur.

1. The magnitude and phase response of $H^Q(Z)$ may differ appreciably from those of $H(Z)$.
2. If the poles of $H(Z)$ are closer to the unit circle in the Z domain, then the poles of $H^Q(Z)$ may lie just outside the unit circle resulting in an unstable system.

Kaiser had shown that such undesirable effects due to coefficient inaccuracy are far more pronounced when higher order systems are directly implemented using direct form I or direct form II realization than when they are implemented using cascade or parallel form realization.

Experiment

We will consider the example of the system transfer function for direct form I realization given as

$$H(Z) = \frac{1}{1 - 0.9Z^{-1} + 0.2Z^{-2}} \tag{1}$$

The same system, if implemented using cascade form realization will take the form

$$H(Z) = \frac{1}{(1 - 0.5Z^{-1})(1 - 0.4Z^{-1})} \tag{2}$$

We will now use "filter" command in MATLAB and observe the effect of coefficient quantization on the impulse response of the filter using direct form I and cascade realization using four bits with the first bit representing the sign bit. We have to enter coefficients of the numerator and denominator polynomial in the system transfer function. The quantized transfer functions for direct form I realization and cascade realization are represented in Eqs. (3) and (4), respectively. (Refer to Figure 1 in Chapter 9 to find the quantized nearest level.)

$$H'(Z) = \frac{1}{1 - 0.875Z^{-1} + 0.125Z^{-2}} \tag{3}$$

$$H''(Z) = \frac{1}{(1 - 0.5Z^{-1})(1 - 0.375Z^{-1})} \tag{4}$$

To observe the impulse response of the system for original transfer function, we will enter denominator coefficients as [1 0.9 0.2]. For direct form I and cascade realization after quantization, we will enter the denominator coefficients as [1 0.875 0.125] and [1 0.875 0.1875], respectively. After executing a MATLAB program, we get the impulse response for original, direct form I and cascade system as shown in Figures 1, 2 and 3, respectively.

The system impulse response for original, direct form I and cascade system goes to zero after 24, 40 and 23 samples, respectively. We find that the impulse response of the original system and the system with cascade realization closely match where as the response of the direct form I realization is significantly different. Thus we have verified the finding reported by Kaiser.

The MATLAB program is as follows.

```
%Quantization effect
num=input('Type in the numerator vector= ');
den=input('Type in the denominator vector= ');
N=max(length(num),length(den));
x=1;y0=0; S=0; zi=zeros(1,N-1);
for n=1:1000
    [y(n),zf]=filter(num,den,x,zi);
    if abs(y(n))<0.000001, break, end
    x=0;
    S=S+abs(y(n));
    y0=y; zi=zf;
end
disp(n);
plot(y);title('Impulse response of the system'); Xlabel('sample
number');
ylabel('Amplitude');
if n<1000
    disp('Stable Transfer Function');
else
    disp('Unstable Transfer Function');
end
```

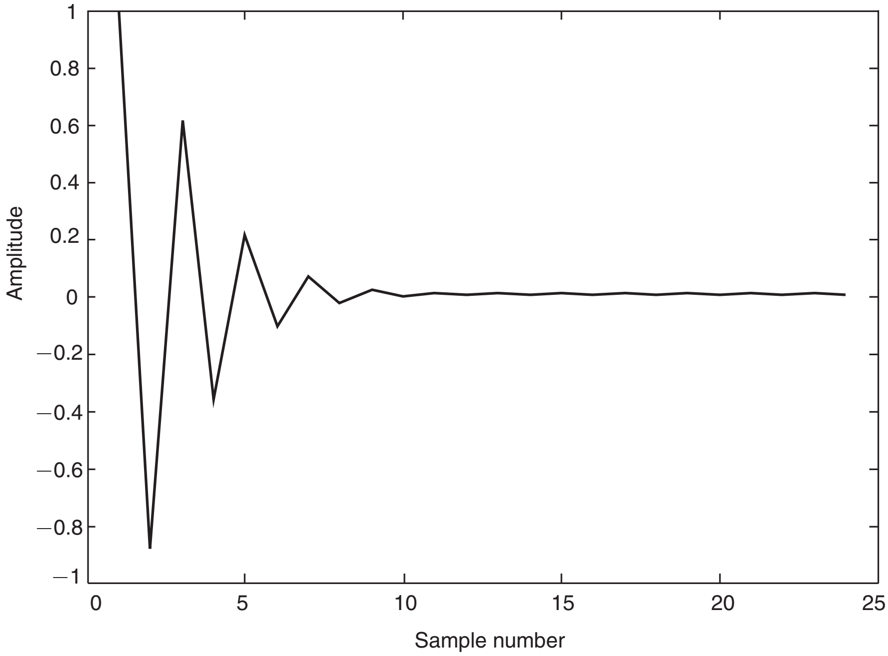

Figure 1 Impulse response for original system.

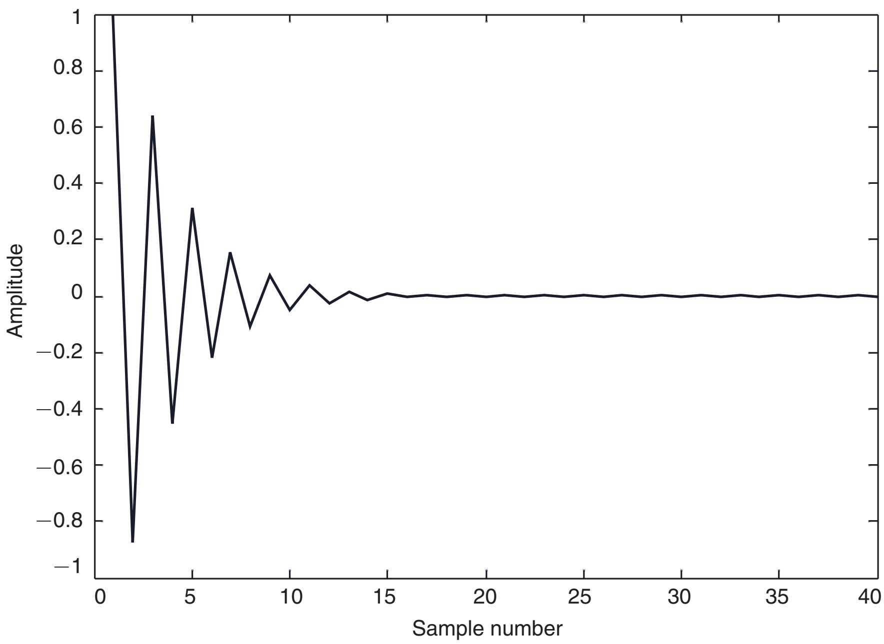

Figure 2 Impulse response for direct form I implementation.

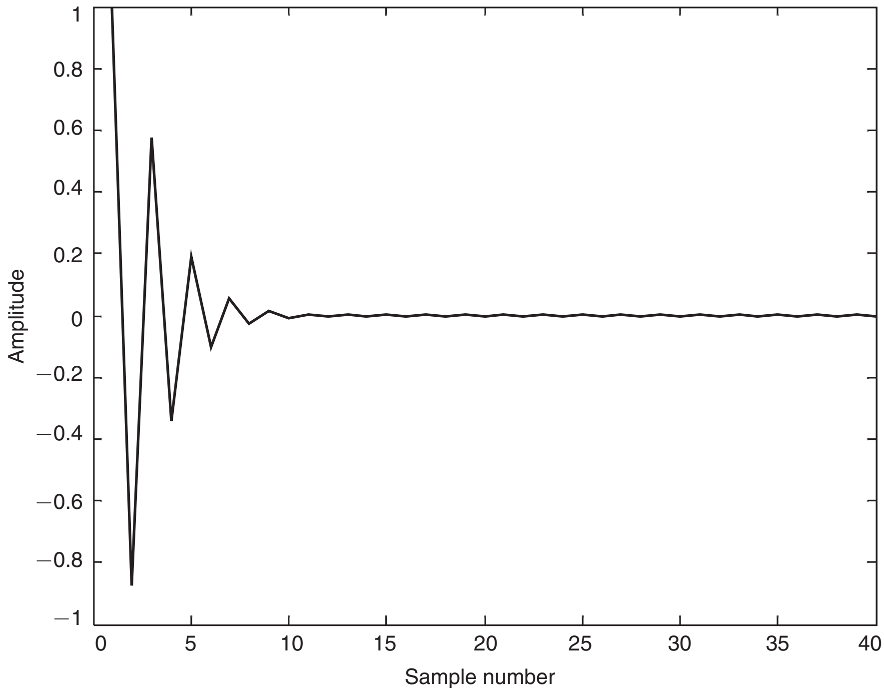

Figure 3 Impulse response for cascade realization.