Experiment 9
Effect of Different Windows

Aim

To study the effect of different windows on FIR filter response.

Theory

Use FIR filters design as done in Experiment 8 and apply different windows to it. The amplitude of the side lobes can be reduced by using a smooth window function rather than a rectangular window. Note that in frequency domain, Fourier transform of the window function gets convolved with the ideal filter response. The main lobe in the spectrum of the window leads to broadening of the transition band, and side lobes lead to Gibbs ringing. We would like to have a window function with thin main lobe and small side lobes. These window functions have broader main lobe but have much smaller side lobes. Therefore, they reduce the Gibbs ringing but lead to larger transition bands. Table 1 gives the equations for different window functions for ready reference.

Table 1 Different window function equations in time domain

Name of the Window	*Time Domain Sequence h(n),* $0 \leq n \leq M-1$
Blackman	$0.42 - 0.5\cos\left(\frac{2\pi n}{M-1}\right) + 0.08\cos\left(\frac{4\pi n}{M-1}\right)$
Hamming	$0.54 - 0.46\cos\left(\frac{2\pi n}{M-1}\right)$
Hanning	$\frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{M-1}\right)\right)$
Bertlett	$1 - \frac{2[n - \{(M-1)/2\}]}{M-1}$
Kaiser	$\frac{I_0\left[\beta\sqrt{\{(M-1)/2\}^2 - [n - \{(M-1)/2\}]^2}\right]}{I_0\left[\beta - \{(M-1)/2\}\right]}$
Rectangular	1

Experiment

Consider the use of Hamming window. Let us apply Hamming window to the filter coefficients obtained in Experiment 8. The window coefficients are calculated using the equation given for the Hamming window in Table 1. Consider $M = 21$.

Teaser *The reader is required to write a MATLAB program to calculate 21 window coefficients and multiply it with 21 filter coefficients obtained in Experiment 8.*

When the filter coefficients are multiplied by the Hamming window of length 21, we get the plot of modified filter coefficients as shown in Figure 1. The magnitude and phase response plot is shown in Figure 2 using freqz function in MATLAB. The MATLAB program is as follows.

```
%FIR filter design using Hamming window
clear all;
fs=8000;t=0.0025;
Q=t*fs/2;
disp(Q);
for i=1:Q,
    x(i)=(sin(0.25*pi*i))/(i*pi);
end
a(Q+1)=0.25;
for i=1:Q,
    a(i)=x(Q-(i-1));
end
disp(a);
for i=2:Q+1
a(i+Q)=x(i-1);
end
disp(a);
w1=window(@Hamming,21);
for i=1:21,
    a1(i)=a(i)*w1(i);
end
stem(a1);xlabel('coefficient number');
ylabel('amplitude');title('impluse response of the filter using
rectangular window');
for i=1:4000
sum(i)=0.0;
end
figure;
[h,w1]=freqz(a1,1,256,8000);
plot(w1/pi,20*log10(abs(h)));xlabel('normalized frequency');ylabel
('ampliudein dB');title('magnitude response plot-rectangular
window');
figure;
```

```
plot(w1/pi,(atand(imag(h)/real(h))));xlabel('normalized
frequency');ylabel('angle in radians');title('Phase response
plot-kaise window');
freqz(a1,1,256,8000);title('magnitude and phase plot-rectangular
window');
```

We can observe that the minimum stop band attenuation is −50 dB but the width of main lobe is increased and is equal to 1700 Hz. Compare the response with Figure 1 in Experiment 8, where attenuation is −20dB and width of main lobe is 1100 Hz.

We will use one more window, say the Blackman window, and calculate filter coefficients and write a program in MATLAB to calculate modified filter coefficients. The reader is then required to write a program to plot magnitude and phase response of the filter. The impulse response using Blackman window is shown in Figure 3. This is obtained by multiplying the filter coefficients by the Blackman window coefficients. The MATLAB program is as follows.

```
%FIR filter design using Blackman window, replace z by a to get
no window
clear all;
fs=8000;
t=0.0025;
Q=fs/2*t;
disp(Q);
for i=1:10,
    x(i)=(sin(0.25*pi*i))/(i*pi);
end
a(11)=0.25;
for i=1:10,
    a(i)=x(Q-(i-1));
end
for i=2:11
a(i+Q)=x(i-1);
end
w1=window(@Blackman,21);
for i=1:21,
   z(i)=a(i)*w1(i);
end
stem(a);xlabel('coefficient number'); ylabel('amplitude');title
('impluse response of the filter using Blackman window');
for i=1:4000
sum(i)=0.0;
end
figure;
[h,w]=freqz(z,1,256,8000);
plot(w/pi,20*log10(abs(h)));xlabel('normalized frequency');
ylabel('ampliudein dB');title('magnitude response plot');
figure;
```

```
plot(w/pi,(atand(imag(h)/real(h))));xlabel('normalized frequency');
ylabel('angle in radians');title('Phase response plot');
freqz(z,1,256,8000);title('magnitude and phase plot-using Blackman
window');
```

The magnitude and phase response using a MATLAB program is plotted in Figure 4. We can observe that the minimum stop band attenuation is −85 dB but the width of main lobe is increased and is equal to 2400 Hz. Compare the response with Figure 2 using Hamming window.

Teaser *The reader is encouraged to write a MATLAB program to use other window functions also and check the results.*

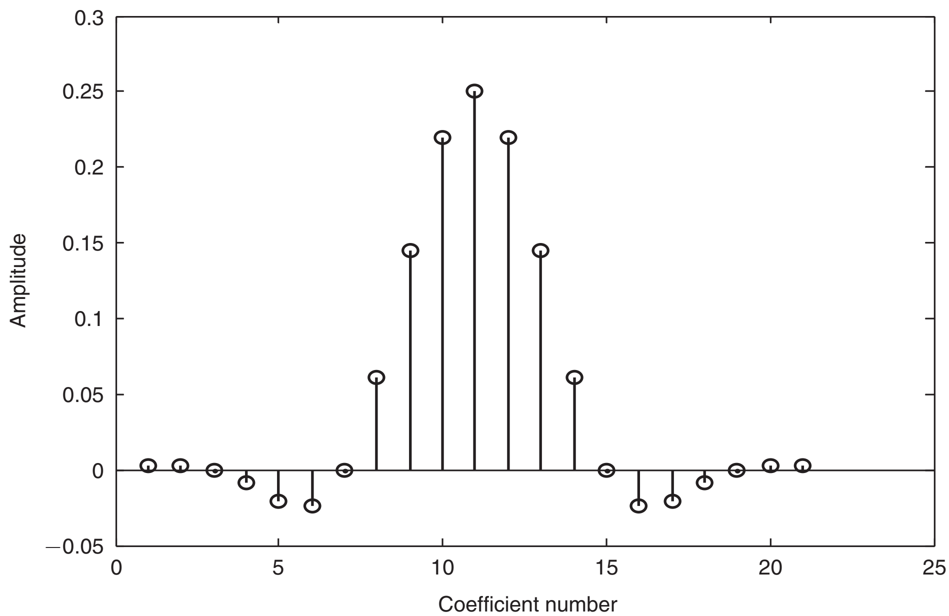
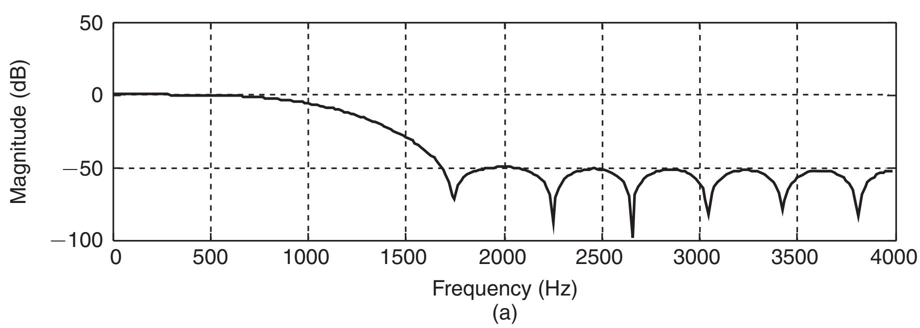


Figure 1 Plot of windowed filter coefficients.



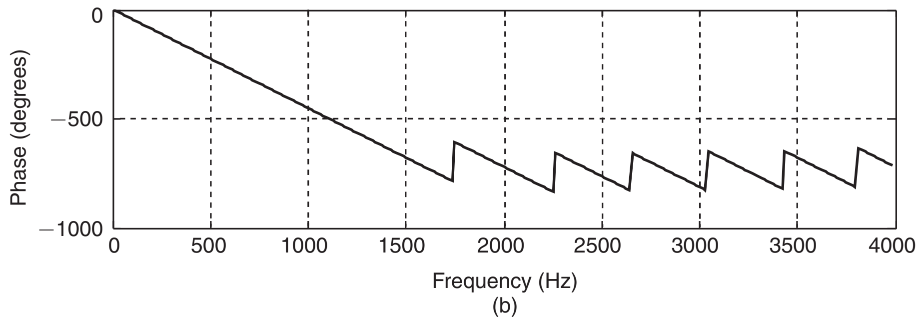


Figure 2 Plot of (a) magnitude and (b) phase response.

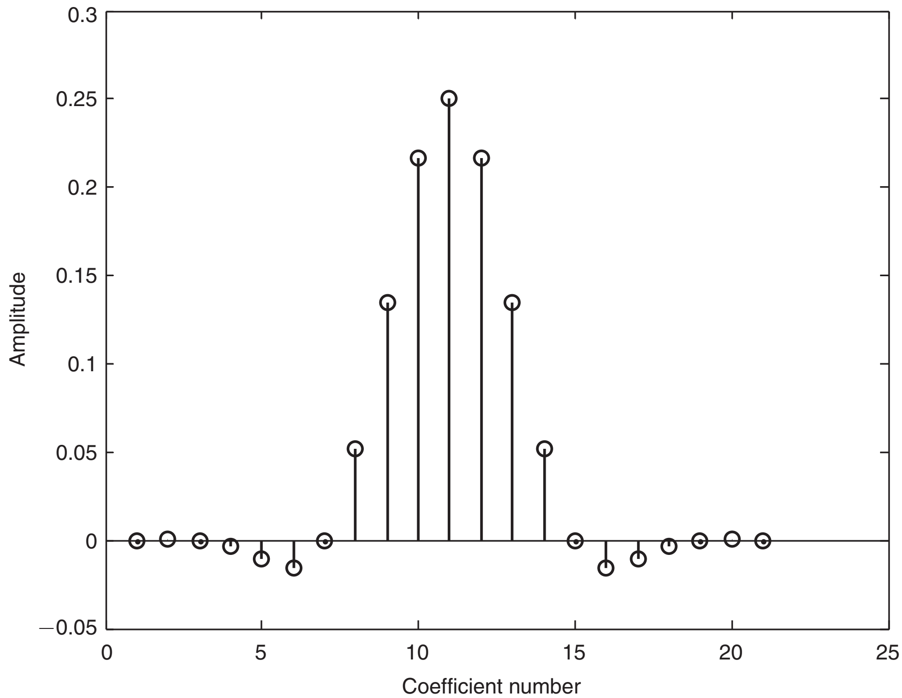


Figure 3 Impulse response using Blackman window function.

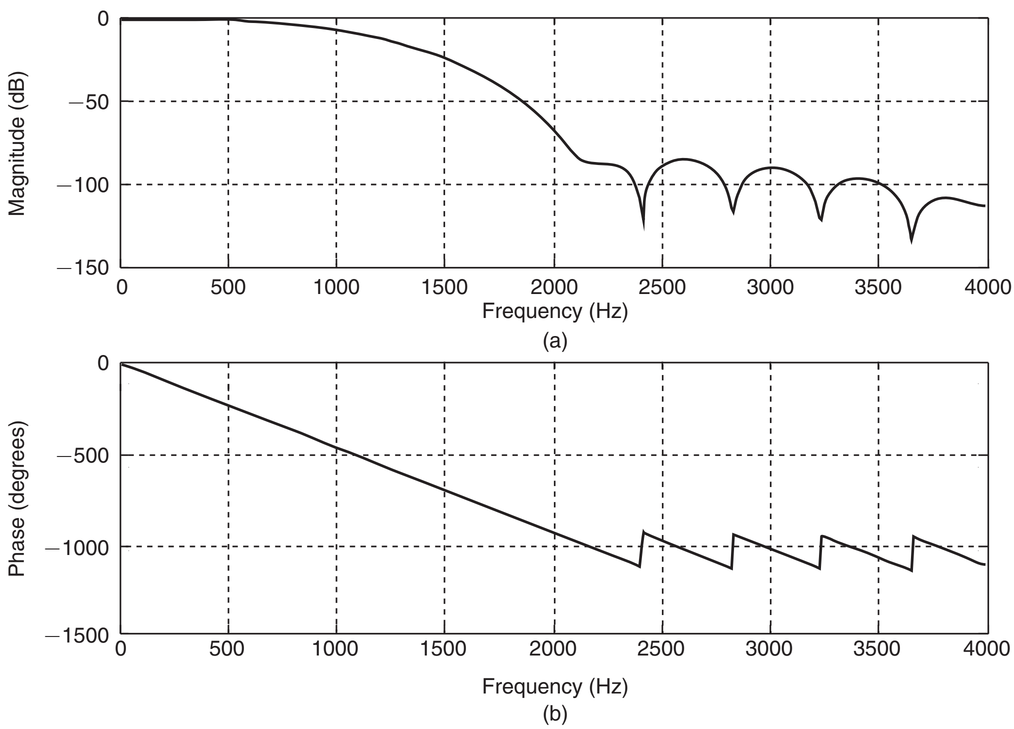


Figure 4 (a) Magnitude and (b) phase response using Blackman window function.