(REVISED COURSE)   QP Code : **1037**

(3 Hours)   [Total Marks : 80]

N.B. (1) Question no. **1** is **compulsory.**
    (2) Solve any **three** questions from the remaining **six** questions
    (3) **Each** questions carry **equal** marks.

1. (a) Evaluate $\int_0^\infty \dfrac{x^4}{4^x}dx$ 3

   (b) Find P.I. of $(D^2 - 4D + 4)y = e^x + \cos 2x$ 3

   (c) Show that $\nabla = 1 - E^{-1}$ 3

   (d) Evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1+x^2}} \dfrac{dy\,dx}{1+x^2+y^2}$ 3

   (e) Solve $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\dfrac{x}{y}\right)dy = 0$ 4

   (f) Evaluate $\displaystyle\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dx\,dy$ by changing to polar co-ordinates   4

2. (a) Solve $y^4\,dx = \left(x^{-3/4} - y^3 x\right)dy$ 6

   (b) Change the order of integration and evaluate   6

   $\displaystyle\int_0^1 \int_x^{1/x} \dfrac{y}{(1+xy)^2(1+y^2)}dy\,dx$

   (c) (1) P.T. $\displaystyle\int_0^\infty \dfrac{x^{m-1}}{(a+bx)^{m+n}}dx = \dfrac{1}{a^n b^m}\beta(m,n)$ 4

   (2) P.T. $\displaystyle\int_0^\infty \dfrac{\log(1+ax^2)}{x^2}dx = \pi\sqrt{a}$, where $a > 0$ 4

3. (a) Evaluate $\displaystyle\int_0^{\log 2}\int_0^x\int_0^{x+\log y} e^{x+y+z}dz\,dy\,dx$ 6

   (b) Find the area bounded between the parabola $x^2=4ay$ and $x^2 = -4a\,(y-2a)$ 6

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(c)   Solve by the method of variation of parameters   8

$$\frac{d^2y}{dx^2} + y = \sec x \tan x$$

4. (a)   Find the length of the cardioid $r = a(1 - \cos\theta)$ lying outside the circle $r = a\cos\theta$ 6

(b)   Solve $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 2xe^{3x} + 3e^x \cos 2x$ 6

(c)   Using R.K. Method of fourth order, solve.

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \text{ given } y(0) = 1 \text{ at } x = 0.2, 0.4$$ 8

5. (a)   Solve $x \sin x\, dy + (xy\cos x - y\sin x - 2)dx = 0$

(b)   Solve $\dfrac{dy}{dx} = 2 + \sqrt{xy}$ with $x_0 = 1.2$, $y_0 = 1.6403$ by modified Euler's method, for   6

    $x = 1.4$ correct to 4-decimal places, (taking $h = 0.2$)   6

(c)   Evaluate $\displaystyle\int_0^6 x f(x)dx$ by

        (a)   Trapezoidal rule   8
        (b)   Simpson's 1/3rd rule
    using the following table

x	0	1	2	3	4	5	6
f(x)	0.146	0.161	0.176	0.190	0.204	0.217	0.230

6. (a)   The charge Q on the plate of a condensor of Capacity C charged through a
    resistance R by a steady voltage V satisfies the differential equation   6

    $R\dfrac{dQ}{dt} + \dfrac{Q}{c} = V$, If $Q = 0$ at $t = 0$, show that $i = \dfrac{V}{R}e^{-t/RC} \quad \because \quad i = \dfrac{dQ}{dt}$

(b)   Evaluate $\displaystyle\iint_A x^2 dx dy$ where A is the region in the first qadrant bounded by the

    hyperbola $xy = 16$ and the lines $y = x$, $y = 0$ and $x = 8$.   6

(c)   Find the volume of the tetrahedron bounded by the planes, $x = 0, y = 0, z = 0$ and
    $x + y + z = a$ 8

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