SEM III - CO - AM - III - CBGS

14/05/15

Q.P. Code : 4827

(3 Hours) [ Total Marks : 80

N.B.: (1) Question **No.1** is **compulsory**.
(2) Attempt any **three** from the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

Q1 a Find Laplace Transform of $\frac{\sin t}{t}$ [20]

b Prove that $f(z)=\sinh z$ is analytic and find its derivative

c Find Fourier Series for $f(x)=9-x^2$ over $(-3,3)$

d Find $Z\{f(k)*g(k)\}$ if $f(k)=\frac{1}{3^k}, g(k)=\frac{1}{5^k}$

Q2 a Prove that $\overline{F}=ye^{xy}\cos z\, i+xe^{xy}\cos z\, j-e^{xy}\sin z\, k$ is Irrotational. Find Scalar Potential for $\overline{F}$

Hence evaluate $\int_C \overline{F}\bullet d\overline{r}$ along the curve C joining the points $(0,0,0)$ and $(-1,2,\pi)$ [6]

b Find the Fourier series for $f(x)=\frac{\pi-x}{2};\ 0\le x\le 2\pi$. [6]

c Find Inverse Laplace Transform of i) $\frac{s+29}{(s+4)(s^2+9)}$ ii) $\frac{e^{-2s}}{s^2+8s+25}$ [8]

Q3 a Find the Analytic function $f(z)=u+iv$ if $u+v=\frac{x}{x^2+y^2}$ [6]

b Find Inverse Z transform of $\frac{1}{(z-1/2)(z-1/3)}$, $1/3<|z|<1/2$ [6]

c Solve the Differential Equation $\frac{d^2y}{dt^2}+y=t$, $y(0)=1$, $y'(0)=0$, using Laplace Transform [8]

Q4 a Find the Orthogonal Trajectory of $3x^2y-y^3=k$ [6]

b Using Greens theorem evaluate $\int_C (xy+y^2)dx+x^2dy$, C is closed path formed by $y=x, y=x^2$ [6]

c Find Fourier Integral of $f(x)=\begin{cases}\sin x & 0\le x\le\pi\\ 0 & x>\pi\end{cases}$. Hence show that $\int_0^\infty \frac{\cos(\lambda\pi/2)}{1-\lambda^2}d\lambda=\frac{\pi}{2}$ [8]

Q5 a Find Inverse Laplace Transform using Convolution theorem $\frac{s}{\left(s^4+8s^2+16\right)}$ [6]

b Find the Bilinear Transformation that maps the points $z=1,i,-1$ into $w=i,0,-i$ [6]

c Evaluate $\int_C \overline{F}\bullet d\overline{r}$ where C is the boundary of the plane $2x+y+z=2$ cut off by co-ordinate planes and $\overline{F}=(x+y)i+(y+z)j-xk$. [8]

Q6 a Find the Directional derivative of $\phi=x^2+y^2+z^2$ in the direction of the line $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$ at ( 1,2,3 ) [6]

b Find Complex Form of Fourier Series for $e^{2x}$ ; $0<x<2$ [6]

c Find Half Range Cosine Series for $f(x)=\begin{cases} kx ; & 0\le x\le l/2 \\ k(l-x) ; & l/2\le x\le l \end{cases}$ ,hence find $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\dots$ [8]

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