

DSDT

(OLD COURSE) Q.P. Code : 4607

(3 Hours)

[Total Marks : 100

- N.B.:** (1) Solve any four questions out of remaining five questions.
 (2) All questions carry equal marks as indicated by figures to the right.
 (3) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 a) Use mathematical induction to show that

$$1+2+3+\dots+n = n(n+1)/2 \text{ for all natural number values of } n. \quad (05M)$$

b) Find the generating function for the following finite sequences

$$\text{i) } 2, 2, 2, 2, 2, 2 \quad \text{ii) } 1, 1, 1, 1, 1, 1 \quad (05M)$$

c) a) $A \oplus B = (A-B) \cup (B-A)$ b) $(A \cup B) = A^c \cap B^c$

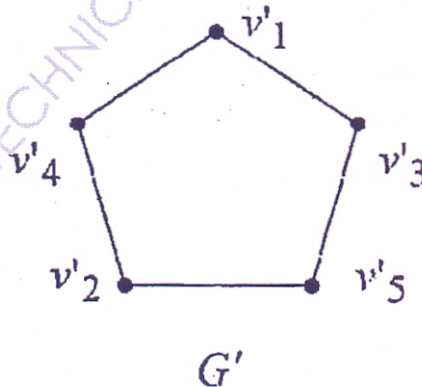
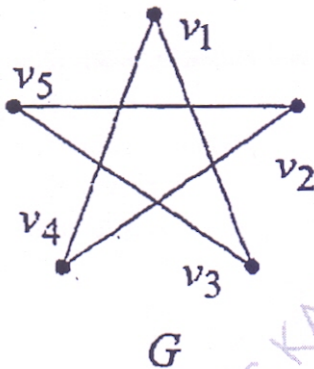
(05M)

d) How many vertices are necessary to construct a graph with exactly 6 edges in which each vertex is of degree 2.

(05M)

Q.2 a) Find if the following two graphs are isomorphic.

(04 M)

b) Let the functions $f, g,$ and h defined as follows:

(08 M)

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + 3$$

$$g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = 3x + 4$$

$$h: \mathbb{R} \rightarrow \mathbb{R}, h(x) = 4x$$

Find $\text{gof}, \text{fog}, \text{foh}, \text{gofoh}$ c) Consider set $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication module 7

(08 M)

I. Find the multiplication table of the above.

II. Find the inverse of 2, 3, and 5, 6

III. Prove that it is a cyclic group

IV. Find the orders and subgroups generated by $\{3, 4\}$ and $\{2, 3\}$

Q.3 a) How many friends must you have to guarantee that at least five of them will have birthdays in the same month.

(04 M)

b) Show that the $(3, 6)$ encoding function $e: \mathbb{B}^3 \rightarrow \mathbb{B}^6$ defined by

(08 M)

$e(000)=000000$ $e(001)=000110$
 $e(010)=010010$ $e(011)=010100$
 $e(100)=100101$ $e(101)=100011$
 $e(110)=110111$ $e(111)=110001$ is a group code.

c) Let R be a relation on set $S = \{a,b,c,d,e\}$, given as
 $R = \{(a,a), (a,d), (b,b), (c,d), (c,e), (d,a), (e,b), (e,e)\}$
 Find transitive closure using Warshall's Algorithm. (08 M)

Q.4 a) Draw Hasse Diagram for the poset $A = \{1, 2, 3, 6, 12, 24, 36, 72\}$ under the divisibility relation. Is this poset a lattice? Justify (04 M)

b) Consider Z together with binary operations of \oplus and \odot which are defined by (08 M)

$$x \oplus y = x + y - 1$$

$$x \odot y = x + y - xy$$

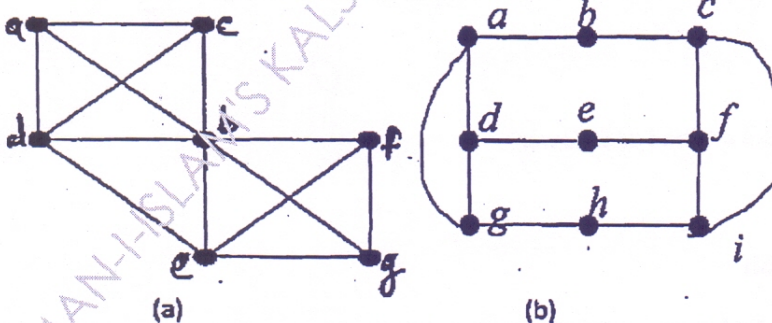
then prove that (Z, \oplus, \odot) is a ring.

c) Solve $a_r - a_{r-1} - 6a_r = -30$ given $a_0 = 20, a_1 = -5$ (08 M)

Q.5 a) Let R be the relation on the set of integers defined by $x R y$ if $x-y$ is divisible by 4. Show that R is an equivalence relation. (04 M)

b) i) Determine Hamiltonian Cycle and path in graph shown in (a)

ii) Determine Euler Cycle and path in graph shown in (b)



c) A survey of 500 television watchers produced the following information:

285 watch football games, 195 watch hockey games, 115 watch basket ball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch basketball and hockey games. 50 do not watch any three kinds of games. Find: (08 M)

- i) How many in the survey watch all 3 kinds of games?
- ii) How many watch exactly one of the sports languages?
- iii) Draw Venn Diagram showing results of the survey.

6. (a) Show that if every element in a group is its own inverse, then the group must be abelian. (04 M)

(b) Let $H =$ (08 M)

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Be a parity check matrix. Determine the group code $e_H : B^3 \rightarrow B^6$ (08 M)

(c) Solve the recurrence relation $a_r - a_{r-1} - a_{r-2} = -30$ given $a_0 = 20, a_1 = -5$ (08 M)

7. (a) If $f : A \rightarrow B$ be both one-to-one and onto then prove that $f^{-1} : B \rightarrow A$ is also both one-to-one and onto (04 M)

(b) Define the following with examples

- (i) Subgroup (08 M)
- (ii) Distributive Lattice
- (iii) Quantities
- (iv) Reflexive closure

(C) If addition and multiplication modulo is defined on set of integers $R = \{0, 2, 4, 6, 8\}$ then show that the system is an integral domain. (08 M)