

(OLD COURSE) Q.P. Code : 4515

(3 Hours)

[Total Marks : 100

- N.B.:** (1) Question 1 is compulsory.
 (2) Answer any four questions from remaining.
 (3) Figures to the right indicate marks.

1. (a) Show that the set of functions $\left\{1, x, \frac{3x^2-1}{2}\right\}$ is orthogonal over $(-1,1)$ 5

(b) Find the value of K such that $W = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \left(\frac{kx}{y} \right)$ 5

is analytic.

(c) Find the laplace transform of $e^{4t} \sin^3 t$ 5

(d) Check whether the matrix A is unitary or not? 5

$$A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$$

2. (a) Find complex form of fourier series for $f(x) = e^{2x}$ in $(0, 2)$ 6

(b) Using convoluion theorem Find iaiace transform of $\frac{S^2}{(S^2+a^2)^2}$ 6

(c) Find Fourier series of $f(x) = x^2$ in $(0, 2\pi)$ 8

3. (a) Find bilinear transformation which maps the points $z = 1, i, -1$ on to the points $w = i, 0, -i$ 6

(b) Find eigen value and eigen vectors of $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ 6

(c) Solve by using laplace transform $(D^2 - 3D + 2)y = 4e^{2t}$ with $y(0) = -3$ and $y'(0) = 5$ 8

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4. (a) Find half range sine series for $f(x) = lx - x^2$, $0 < x < l$. 6

(b) Find the image of $|z - 2| = 3$ under the transformation $w = \frac{1}{z}$ 6

(c) Evaluate (i) $L^{-1} [\text{Cot}^{-1}(S+1)]$ 4

(ii) $L^{-1} \log \left(\frac{S+a}{S+b} \right)$ 4

5. (a) Verify Cayley Hamilton Theorem and hence find A^{-1} for A 6

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) Find $L \left[\text{Sin h} \left(\frac{t}{2} \right) \text{Sin} \left(\frac{\sqrt{3}}{2} t \right) \right]$ 6

(c) Investigate for what values of λ, μ the following linear equations. 8

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu \quad \text{have}$$

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

6. (a) Evaluate $\int_0^{\infty} \frac{\text{Cos} at - \text{Cos} bt}{t} dt$ 6

(b) Construct an analytic function $f(z) = u + iv$ where $u + v = e^x (\cos y + \sin y)$ 6

(c) Reduce matrix A to normal form and hence find its rank 8

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

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7. (a) Find the orthogonal trajectory of family of curves $3x^2y - y^3 = \text{Constant}$
(b) Determine l, m, n & find A^{-1}

$$\text{if } A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$$

is orthogonal.

- (c) Find the laplace transform of

$$f(t) = a \sin pt, \quad 0 < t < \frac{\pi}{p}$$

$$f(t) = 0, \quad \frac{\pi}{p} < t < \frac{2\pi}{p}, \quad \text{and } f(t) = f\left(t + \frac{2\pi}{p}\right)$$

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