(OLD COURSE)   QP Code : 4533

(3 Hours)   [Total Marks : 100

Note :   1. Question No.1 is compulsory.
          2. Answer any four from the remaining six questions.

1. a) If $L\{f(t)\} = \dfrac{s}{s^2 + s + 4}$, find $L\{e^{-2t} f(2t)\}$ [5]

    b) Find the orthogonal trajectory of the family of curves given by
       $2x - x^3 + 3xy^2 = a.$ [5]

    c) Evaluate $\oint\limits_{C} \log z \; dz$ where C is $|z| = 1$ [5]

    d) Express the matrix $A = \begin{bmatrix} 2i & 2+i & 1-i \\ -2+i & -i & 3i \\ -1-i & 3i & 0 \end{bmatrix}$ as P + iQ where P is real Skew-

    symmetric matrix and Q is real Symmetric matrix.   [5]

2. a) Determine the analytic function whose imaginary part is
       $\left(x^4 - 6x^2y^2 + y^4\right) + \left(x^2 - y^2\right) + 2xy$ [6]

    b) Evaluate $\int\limits_{C} \dfrac{4z-1}{z^2 - 3z - 4} dz$ where C is the ellipse $x^2 + 4y^2 = 4$.   [6]

    c) Reduce to normal form and find rank of the following matrix :   [8]
       $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

3. a) Solve the Differential Equations using Laplace Transformation

       $\dfrac{d^2y}{dt^2} - 3\dfrac{dy}{dt} + 2y = 4e^{2t}$ $y(0) = -3, \; y'(0) = 5$ [6]

    b) Find the sum of the residue at singular points of $f(z) = \dfrac{z}{(z-1)^2(z^2-1)}$ [6]

    c) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, then prove that $3\tan A = A\tan 3$ [8]

                                        [TURN OVER

RJ-Con. 8865-15.

4. a) Prove that $u = \log\sqrt{x^2 + y^2}$ is harmonic and find its harmonic conjugate [6]

b) Examine whether the vectors $X_1 = [3 \quad 1 \quad 1]$ $X_2 = [2 \quad 0 \quad -1]$ $X_3 = [4 \quad 2 \quad 1]$
are linearly independent or dependent. [6]

c) Find Inverse Laplace Transform of (i) $\cot^{-1}(as)$

(ii) $\dfrac{(s+3)^2}{(s^2 + 6s + 5)^2}$ using convolution theorem [8]

5. a) Find the image of the rectangle bounded by $x = 0, y = 0, x = 1, y = 2$ under the transformation $w = (1 + i)z + (2 - i)$. Sketch the region. [6]

b) Find the Eigen value and Eigen vector of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ [6]

c) Evaluate the Integral $\displaystyle\int_0^\infty \dfrac{e^{-\sqrt{2}t} \sin t \sinh t}{t} \, dt$ [8]

6. a) Evaluate $\displaystyle\int_0^{2\pi} \dfrac{d\theta}{25 - 16\cos^2 \theta}$ [6]

b) Verify Cayley-Hamilton Theorem for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and hence find A^{-1}, $A^3 - 5A^2$. [6]

c) Find Laplace Transform of (i) $\displaystyle\int_t^\infty \dfrac{\cos u}{u} \, du$

(ii) $t\left(\dfrac{\sin t}{e^t}\right)^2$ [8]

7. a) Find Laplace Transform of the following periodic functions :

$f(t) = K\dfrac{t}{T}$ for $0 \le t \le T$, $f(t+T) = f(t)$ [6]

b) Find the Bilinear Transformation that maps the points $z = 1, i, -1$ into $w = i, 0, -i$. [6]

c) Obtain Laurent and Taylor's series for $\dfrac{z-1}{z^2 - 2z - 3}$ indicating region of convergence [8]