

QP Code : 3110

Old course

(3 hours)

[ Total Marks: 100

1. Q 1 is compulsory.
2. Solve any four out of the remaining from Q. No. 2 to Q No 7.
3. Fig on right hand side indicate full marks.

Q. 1.

- a) Using Taylors series method solve  $\frac{dy}{dx} = x + y$  with  $x_0 = 1, y_0 = 0$  and carry to  $x = 1.1$  3
- b) Solve  $(D^4 - a^4)y = 0$  3
- c) Evaluate  $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}} dy dx$  3
- d) Evaluate  $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dx dy dz$  3
- e) Evaluate  $\int_0^4 \sqrt{x} (4 - x)^{\frac{3}{2}} dx$  4
- f) Using Euler's method, find the approximate value of  $y$  when  $\frac{dy}{dx} = xy$ , and  $y=2$  when  $x=0$  at  $x=1$  in five steps. 4

Q.2.

- a) Evaluate  $\int_0^{\infty} \frac{x^5(1+x^4)}{(1+x)^{16}} dx$  6
- b) Solve using Runge- Kutta method of fourth order  $\frac{dy}{dx} = x^2 + y^2$ , with the condition  $x=1$  at  $y=1.5$  in the interval (1.1.2) with  $h=0.1$ . 6
- c) Solve  $((1 + y^2)dx = (e^{\tan^{-1} y} - x)dy$  8

Q.3.

- a)  $(2xy \cos x^2 - 2xy + 1) dx + (\sin x^2 - x^2) dy = 0$  6
- b) Solve using method of variation of parameters,  $(D^2 + 1)y = \cot x$  6
- c) Show that  $\int_0^{\infty} \frac{\log(1+ax^2)}{x^2} dx = \pi\sqrt{a}, a \geq 0.$  8

[Turn over

Q.4.

- a) Solve  $y(x + y) dx - x(y - x) dy = 0$  6
- b) Solve  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 2 \log x$  6
- c) Solve  $(D^2 - 2D + 1)y = x^2 e^{3x}$  8

Q.5.

- a) In a electric circuit containing inductance  $L$ , resistance  $R$ , and voltage  $E$ , the current  $i$  is given by  $L \frac{di}{dt} + Ri = E$ . Find the current  $i$  at time  $t$ , if at  $t=0$  when  $i=0$  and  $L, R, E$  are constants. 6

- b) Change the order of integration.  $\int_0^a \int_x^{a^2/x} f(x, y) dx dy$  6

- c) Evaluate  $\iiint xyz(x^2 + y^2 + z^2) dx dy dz$  over the first octant of the sphere  $x^2 + y^2 + z^2 = a^2$  8

Q. 6.

- a) Find the total length of the loop of the curve  $9y^2 = (x + 7)(x + 4)^2$  6

- b) Change to polar coordinates and evaluate  $\int_0^1 \int_0^x (x + y) dx dy$  6

- c) Evaluate  $\iint_R \sqrt{xy - y^2} dx dy$  Over the region  $R$  of a triangle whose vertices are  $(0,0), (10,1)$  and  $(1,1)$ . 8

Q.7.

- a) Change the order of integration and evaluate  $\int_0^a \int_y^{\sqrt{ay}} \left(\frac{x}{x^2 + y^2}\right) dx dy$  6

- b) Find by double integration the area of the smaller region bounded by  $x^2 + y^2 = a^2$  and  $x + y = a$ . 6

- c) Find the volume of the tetrahedron bounded by planes,  $x + y + z = a$ ,  $x = 0$ ,  $y = 0$ ,  $z = 0$  8