F.E. - Sem-II - CBSGS - AM-II - 18/11/2015

QP Code : 5801

(3 Hours) **[Total Marks : 80**

N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

1. (a) Evaluate $\int_0^2 x^2 (2-x)^3 dx$ [3]

(b) Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$ [3]

(c) Prove that $E = 1 + \Delta$ [3]

(d) Solve $\left[y\left(1+\frac{1}{x}\right) + \cos y\right] dx + (x + \log x - x\sin y)\, dy = 0$ [3]

(e) Change to polar coordinates and evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2)\, dy\, dx$ [4]

(f) Evaluate $\int_0^1 \int_0^x xy\, dy\, dx$ [4]

2. (a) Solve $\frac{dy}{dx} + \frac{4x}{x^2+1} y = \frac{1}{(x^2+1)^3}$ [6]

(b) Change the order of integration and evaluate

$$\int_0^2 \int_{\sqrt{2x}}^2 \frac{y^2\, dx\, dy}{\sqrt{y^4 - 4x^2}}$$ [6]

(c) Prove that $\int_0^{\pi/2} \frac{\log(1 + a\sin^2 x)}{\sin^2 x} dx = \pi\left[\sqrt{a+1} - 1\right] \quad a > -1$ [8]

3. (a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{1}{(x+y+z+1)^3} dz\, dy\, dx$ [6]

(b) Find by double integration the area enclosed by the curve $9xy = 4$ *and the line* $2x + y = 2$ [6]

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QP-Con. 7157-15.

(c) Using method of Variation of Parameter solve $\frac{d^2y}{dx^2} + a^2 y = \sec ax$ [8]

4. (a) Find the perimeter of the cardioide $r = a(1+\cos\theta)$. [6]

(b) Solve $(D^2+4)y = \cos 2x$ [6]

(c) Apply Runge-kutta Method of fourth order to find an approximate value of y for $\frac{dy}{dx} = \frac{1}{x+y}$ with $x_0 = 0, y_0 = 1$ at $x = 1$ taking $h = 0.5$ [8]

5. (a) Solve $(y - xy^2)dx - (x + x^2 y)dy = 0$ [6]

(b) Using Taylor Series Method obtain the solution of following differential equation $\frac{dy}{dx} = 1 + y^2$ with $y_0 = 0$ when $x_0 = 0$ for $x = 0.2$ [6]

(c) Find the approximate value of $\int_0^6 e^x \, dx$

by i) Trapezoidal Rule , ii) Simpson's 1/3rd Rule, iii) Simpson's 3/8th Rule [8]

6. (a) A resistance of 100 ohms and inductance of 0.5 henries are connected in series with a battery of 20 volts. Find the current at any instant if the relation between L, R, E is $L\frac{di}{dt} + Ri = E$. [6]

(b) $\iint y\,dx\,dy$ over the area bounded by the $x = 0, y = x^2, x + y = 2$ [6]

(c) Find the volume bounded by the paraboloid $x^2 + y^2 = az$ and the cylinder $x^2 + y^2 = a^2$ [8]