SE - sem - III - OLD - Electrical

EM - III

20/11/15

QP Code : 1057

(3 Hours) **[Total Marks : 100**

N.B. 1. Question No.1 is compulsory and attempts any four questions from Q.No.2 to 7

2. Figures to the right indicate full marks.

Q.1 a) Determine value of k such that $w = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{kx}{y}$ is analytic. 05

b) Find the Laplace transform of $e^{4t}Sin^3 t$. 05

c) Show that the set of functions $\{1, x, \frac{3x^2 - 1}{2}\}$ is orthogonal over (-1,1). 05

d) Find the image of $|z-2| = 3$ under the transformation $w = \frac{1}{z}$. 05

Q.2 a) Find the bilinear transformation which maps the points 1, i ,-1 in Z-planeonto the points 0,1, ∞ in W-plane. 06

b) Using Laplace transforms evaluate $\int_0^\infty e^{-t}\frac{Sin^2 t}{t}dt$ 06

c) Find half range sine series for the function $f(x) = \frac{\pi}{4}$;0< x <π. 08

Hence deduce that $\frac{\pi}{4}(\frac{\pi}{2} - x) = \frac{Cosx}{1^2} + \frac{Cos3x}{3^2} + \frac{Cos5x}{5^2} + ...$

Q.3 a) Find orthogonal trajectory of family of curves $3x^2y - y^3 = const.$ 06

b) Using convolution theorem find inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)^2}$ 06

c) Find Fourier series for the function $f(x) = x^2$ over (0,2π). 08

Q.4 a) State true or false with proper justification. 'There does not exist an analytic function whose real part is $x^3 - 3x^2y - y^3$ 06

b) Obtain complex form of fourierseries for $f(x) = Coshax$ in (-π,π). 06

c) Using Laplace transforms Solve $\frac{d^2y}{dt^2} + y = t$ with y(0)=1 and y'(0)=0. 08

Q.5 a) Find inverse Laplace transform of i) $\cot^{-1}(s+1)$ ii) $\log(\frac{s+a}{s+b})$ 06

b) Express $f(x) = 1; -1 \prec x \prec 1$ and is zero otherwise as a Fourier integral. 06

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c) Obtain all possible Laurent's series expansions of $f(z)=\dfrac{z-1}{z^2-2z-3}$ about z=0.

Q.6 a) Construct analytic function $f(z)=u+iv$ where $u+v=e^x(\cos y+\sin y)$ 06

b) Find Laplace transform of $(1+2t-3t^2+4t^3)H(t-2)$. 06

c) State Dirichlet's conditions for a function f(x) to have fourier series over (a,b).Obtain fourier series for $f(x)=e^{-x}$ over $0<x<2\pi$. $f(x+2\pi)=f(x)$. 08

Q.7 a) Find fourier series for $f(x)=x$ $\quad 0<x\le\pi$ 06

$= 2\pi - x \quad \pi\le x<2\pi$

b) State and prove Cauchy's integral formula for $f(z)=u+iv$ 06

c) Using residue theorem evaluate i) $\displaystyle\int_0^{2\pi}\frac{d\theta}{5+3Sin\theta}$ 08

ii) $\displaystyle\int_{-\infty}^{\infty}\frac{x^2+x+2}{x^4+10x^2+9}dx$

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