SE - sem-III - OLD - Civil
AM-III 20/11/15

(OLD COURSE) **QP Code : 1051**

(3 Hours) **[Total Marks : 100**

1. Q1 is compulsory
2. Solve any four out of the remaining from Q.2 to Q. 7.
3. Figures on the right hand side indicate marks.

Total Marks: 100 Time: Three hours

Q.1.

a. Find the Laplace transform of $\frac{\cos 2t \sin t}{e^t}$ 5

b. Prove that $u = \cos x \cos hy$, is a harmonic function. Find its harmonic conjugate and hence the analytic function. 5

c. Prove that the set of functions cos nx is orthogonal, for n=1,2,3,….,on $(0,2\pi)$ 5

d. If A=$\begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix}$, find B such that AB=$\begin{bmatrix} 6 & 6 & 2 \\ 2 & 9 & 1 \\ 10 & 9 & 4 \end{bmatrix}$ 5

Q.2. a. Reduce the Matrix to normal form and hence find the Rank of A.

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ 1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$ 6

b. Find the inverse Laplace transform using convolution theorem,

$f(s) = \frac{s}{(s^2+4)(s^2+1)}$ 6

c. Find the Fourier series for $f(x) = x \ in(0,2\pi)$. 8

Q.3. a. Find the bilinear transformation which maps the points z=1,i,-1 on to the points w=i,0,-i. 6

b. find the complex form of Fourier series $f(x) = e^{-x}$, in(-1,1). 6

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c. Find the Eigen values and Eigen vectors of A and , $if \quad A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ 8

Q.4. a. Verify Cayley Hamilton Theorem for A= $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1}. 6

b. Find the image of the region bounded by x=0,x=2,y=0,y=2, in z plane under the transformation w=(1+i)z . 6

c. Find, 1. $L[\int_0^t \frac{e^{-u}\sin u}{u} du]$, 2. $L^{-1}[\frac{4s+12}{s^2+8s+12}]$ 8

Q.5. a. Find the non singular matrices P and Q such that PAQ is in Normal form, hence find Rank of A. Where 6

$$A=\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -2 & 3 \\ 1 & 2 & 1 & 2 \end{bmatrix}$$

b. Find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$. 6

c. Find the analytic function f (z), where u + v = $e^x(\cos y + \sin y)$ 8

Q.6. a. Prove that every square matrix can be uniquely expressed as sum hermitian and skew hermitian matrices. 6

b. Find the half range sine series for f(x) = x $(\pi - x)$ in $(0, \pi)$. 6

c. Solve using Laplace transform,

(D^2+2D+5) y = $e^{-t}\sin t$, $given\ that, y(0) = 0, y'(0) = 1$. 8

Q.7.a. Prove that A is unitary. A= $\begin{bmatrix} \frac{2+i}{3} & \frac{2i}{3} \\ \frac{2i}{3} & \frac{2-i}{3} \end{bmatrix}$ 6

b. Find the Fourier series for f(x) = x^2, in the interval $(-l, l)$. 6

c. Solve the following system of equations, if consistent.

4x-2y+6z=8, x+y-3z=-1,15x-3y+9z=21 8