

28

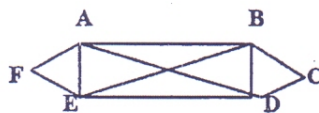
QP Code : 30745

(3 Hours)

[Total Marks :80

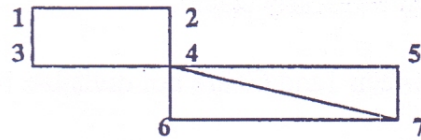
- N.B. :** (1) Question no. 1 is compulsory.
(2) Attempt any **three** questions from the remaining **five** questions.
(3) **All** questions carry equal marks as indicated by **figures** to the **right**.
(4) Assumptions made should be clearly stated.

1. (a) Find how many integers between 1 and 60 are not divisible by 2 nor by 3 and nor by 5 respectively. 6
(b) By using mathematical induction prove that $1 + a + a^2 + \dots + a^n = \frac{1 - a^{n+1}}{1 - a}$, where $n \geq 0$ 6
(c) Let $A = \{1, 2, 3, 4, 5\}$ and R be the relation defined by $a R b$ if and only if $a < b$. Compute R, R^2 and R^3 . Draw digraph of R, R^2 and R^3 . 8
2. (a) Show that a group G is Abelian, if and only if $(ab)^2 = a^2 b^2$ for all elements a and b in G . 6
(b) Let $A = \{1, 2, 3, 4, 6\} = B$, $a R b$ if and only if a is multiple of b . Find R . Find each of the following (i) $R(4)$ (ii) $R(G)$ (iii) $R(\{2, 4, 6\})$. 6
(c) Show that the (2,5) encoding function $e: B^2 \rightarrow B^5$ defined by $e(00) = 00000$, $e(01) = 01110$, $e(10) = 10101$, $e(11) = 11011$ is a group code. How many errors will it detect and correct? 8
3. (a) State pigeon hole and extended pigeon hole principle. Show that 7 colors are used to paint 50 bicycles, at least 8 bicycle will be of same color. 6
(b) Define distributive lattice. Show that in a bounded distributive lattice, if a complement exists, its unique. 6
(c) Functions f, g, h are defined on a set, $X = \{1, 2, 3\}$ as $f = \{(1, 2) (2, 3) (3, 1)\}$, $g = \{(1, 2) (2, 1) (3, 3)\}$, $h = \{(1, 1) (2, 2) (3, 1)\}$ (i) Find $f \circ g, g \circ f$ are they equal? (ii) Find $f \circ g \circ h$ and $f \circ h \circ g$ 8
4. (a) Define Euler path and Euler circuit, determine whether the given graph has Euler path and Euler circuit. 6

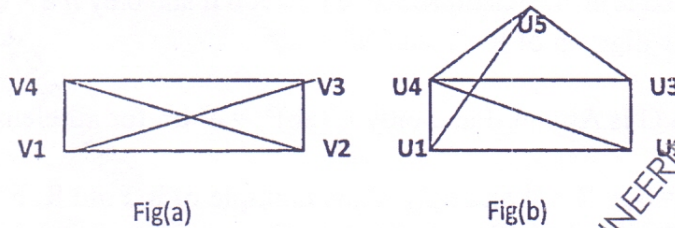


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- (b) Define Hamiltonian path and Hamiltonian circuit, determine whether the given graph has Hamiltonian path and Hamiltonian circuit. 6



- (c) Define isomorphic graphs. Show that the following two graphs are isomorphic. 8



5. (a) What is an Universal and existential quantifiers? Prove the distribution law. 6
 $(p \vee q) \wedge r \equiv (p \vee q) \wedge (p \vee r)$
- (b) Let $A = \{1, 2, 3, 4\}$ and let $R = \{(1, 2), (2, 3), (3, 4), (4, 1)\}$ Find transitive closure of R by using Warshall's algorithm. 6
- (c) Prove that the set $A = \{0, 1, 2, 3, 4, 5\}$ is a finite Abelian group under addition modulo 6. 8
6. (a) Find the ordinary generating functions for the given sequences: 6
 (i) $\{1, 2, 3, 4, 5, \dots\}$ (ii) $\{2, 2, 2, 2, \dots\}$ (iii) $\{1, 1, 1, 1, \dots\}$
- (b) Define group, monoid, semigroup. 6
- (c) Solve the following recurrence relation: $a_n - 7a_{n-1} + 10a_{n-2} = 0$ with initial condition $a_0 = 1, a_2 = 6$ 8