

REVISED COURSE

(3 Hours)

[Total Marks : 100]

- N.B. : (1) Question No. 1 is compulsory.
 (2) Attempt any three questions from question No. 2 to question no. 6.
 (3) Figures to the right indicate full marks.

1. (a) Solve the equation $7\cosh x + 8\sinh x = 1$ for real values of x 3
 - (b) If $z(x+y) = (x-y)$ find $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} \right)^2$ 3
 - (c) If $u = r^2 \cos 2\theta$, $v = r^2 \sin 2\theta$ find $\frac{\partial(u, v)}{\partial(r, \theta)}$ 3
 - (d) Prove that $\sec^2 x = 1 + x^2 + \frac{2x^4}{3} + \dots$ 3
 - (e) Find the rank of the Matrix by reducing it to normal form. 4

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$
 - (f) Find n^{th} derivatives of $\frac{x}{(x-1)(x-2)(x-3)}$ 4
2. (a) If α, β are the roots of the equation $x^2 - 2\sqrt{3}x + 4 = 0$ find the value of $\alpha^3 + \beta^3$ 6
 - (b) Examine whether the vectors 6
 $X_1 = [3 \ 1 \ 1]$, $X_2 = [2 \ 0 \ -1]$
 $X_3 = [4 \ 2 \ 1]$
 are linearly independent.
 - (c) (i) State and prove Euler's theorem for a Homogeneous function in two variables. 4
 (ii) If $y = x \cos u$ find the value of $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$ 4

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3. (a) Is the following system has trivial or non trivial solution ? Obtain the non trivial solution if exist. 6

$$\begin{aligned}3x_1 + 4x_2 - x_3 - 9x_4 &= 0 \\2x_1 + 3x_2 + 2x_3 - 3x_4 &= 0 \\2x_1 + x_2 - 14x_3 - 12x_4 &= 0 \\x_1 + 3x_2 + 13x_3 + 3x_4 &= 0\end{aligned}$$

- (b) Discuss the stationary points for Maxima and Mininima of $x^3 + xy^2 - 12x^2 - 2y^2 + 21x + 10$ 6

- (c) (i) If $\tan(x+iy) = a + ib$ prove that $\tanh 2y = \frac{2b}{1+a^2+b^2}$ 4
 (ii) Separate into real and imaginary parts of $\log(3+4i)$ 4

4. (a) If $x = u \cos v$, $y = u \sin v$ 6

$$\text{Prove that } \frac{\partial(u,v)}{\partial(x,y)}, \frac{\partial(x,y)}{\partial(u,v)} = 1$$

- (b) Show that $\log[e^{i\alpha} + e^{i\beta}] = \log\left[2\cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right)$ 6

- (c) (i) Solve the system of equation by Gauss Jordan Method
 $x + 2y + 6z = 22$, $3x + 4y + z = 26$, $6x - y - z = 19$ 4

- (ii) Solve the system of equation by Gauss Siedel Method.
 Correct upto three decimal.

$$\begin{aligned}2x - 4y + 49z &= 49 \\43x + 2y + 25z &= 23 \\3x + 53y + 3z &= 91\end{aligned}$$

5. (a) Prove that $\cos^6\theta + \sin^6\theta = \frac{1}{8}[3\cos 4\theta + 5]$ 6

- (b) Find the value of a and b 6

$$\text{if } \lim_{x \rightarrow 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$$

- (c) (i) If $y = e^x \cos 2x \cos x$ find y_n 4
 (ii) If $y = e^{\tan^{-1}x}$ prove that $(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$ 4

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6. (a) Find non-Singular Matrices P & Q such that,

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

is reduced to normal form. Also find rank.

(b) If $u = f(e^{y-z}, e^{z-x}, e^{x-y})$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

- (c) (i) Fit a straight line to the following data :

| | | | | | |
|----------------|------|------|------|------|------|
| Year x : | 1951 | 1961 | 1971 | 1981 | 1991 |
| Production y : | 10 | 12 | 8 | 10 | 15 |

- (ii) Fit a second degree parabolic curve to the following data :

| | | | | | | | | | |
|-----|---|---|---|---|----|----|----|----|---|
| x : | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| y : | 2 | 6 | 7 | 8 | 10 | 11 | 11 | 10 | 9 |

Course: F.E. (OLD) & (CBSGS)(ALL BRANCHES)

QP Code: 28579 (2nd query)

Correction:

Q1 (e) an element in position (3, 1) in matrix is 3

Q4 (a) left hand side of the result to prove is a product of two jacobians

(i.e instead of “ , ” read as “ . ”)

Date and Time 10/05/2016 11:30 AM

Hemant . sahani 12114503 Hemant . sahani

Block No:-6

Shawkr Arora 12114551 Arora

Block No:-8

Snehal Singh 12114641 S.Singh

Block No:-9

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Question paper is of 80 marks instead of 100 marks

Date and Time 10/05/2016 10:15 AM