

QP Code : 545802

( 3 Hours )

[ Total Marks: 80

- N.B.:** (1) Question No.1 is compulsory.  
 (2) Attempt any Three from the remaining.

1. (a) Find the extremal of the functional 5  

$$\int_0^1 [y'^2 + 12xy] dx$$
 subject to  $y(0) = 0$  and  $y(1) = 1$ .
- (b) Verify Cauchy - Schwartz inequality for  $u = (1, 2, 1)$  and  $v = (3, 0, 4)$  also find the angle between  $u$  &  $v$ . 5
- (c) If  $\lambda$  &  $X$  are eigen values and eigen vectors of  $A$  then prove that  $\frac{1}{\lambda}$  and  $X$  are eigen values and eigen vectors of  $A^{-1}$ , provided  $A$  is non singular matrix. 5
- (d) Evaluate  $\int_C \frac{e^{2z}}{(z+1)^4} dz$  where  $C: |z| = 2$  5
2. (a) Find the extremal that minimises the integral 6  

$$\int_{x_0}^{x_1} (16y^2 - y''^2) dx$$
- (b) Find eigen values and eigen vectors of  $A^{-1}$  6  
 where  $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$
- (c) Obtain Taylor's and two distinct Laurent's expansion of  $f(z) = \frac{z-1}{z^2-2z-3}$  8  
 indicating the region of convergence.

[ TURN OVER

3. (a) Verify Cayley-Hamilton Theorem for 6

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \text{ and hence find } A^{-1}$$

- (b) Using Cauchy Residue Theorem, evaluate 6

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$$

- (c) Show that a closed curve 'C' of given fixed length (perimeter) which encloses maximum area is a circle. 8

4. (a) Find an orthonormal basis for the subspace of  $\mathbb{R}^3$  by applying Gram-Schmidt process where  $S = \{(1,1,1), (0,1,1), (0,0,1)\}$ . 6

- (b) Find  $A^{50}$ , where 6

$$A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$$

- (c) Reduce the following Quadratic form into canonical form & hence find its rank, index, signature and value class where. 8

$$Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$$

5. (a) Using the Rayleigh-Ritz method, find an approximate solution for the 6

extremal of the functional  $\int_0^1 \left\{ xy + \frac{1}{2} y'^2 \right\} dx$  subject to  $y(0) = y(1) = 0$ .

- (b) Prove that  $W = \{(x,y) | x = 3y\}$  subspace of  $\mathbb{R}^2$ . Is  $W_1 = \{(a,1,1) | a \in \mathbb{R}\}$  subspace of  $\mathbb{R}^3$ ? 6

[ TURN OVER

- (c) Prove that A is diagonalizable matrix. Also find diagonal form and 8

transforming matrix where  $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

6. (a) By using Cauchy Residue Theorem, evaluate  $\int_0^{2\pi} \frac{\cos^2 \theta}{5+4\cos\theta} d\theta$ . 6

- (b) Evaluate  $\int_C \frac{z+4}{z^2+2z+5} dz$  where  $C : |z+1+i|=2$ . 6

- (c) (i) Determine the function that gives shortest distance between two given points. 5

- (ii) Express any vector (a,b,c) in  $\mathbb{R}^3$  as a linear combination of  $v_1, v_2, v_3$  where  $v_1, v_2, v_3$  are in  $\mathbb{R}^3$ . 3