SEM-III - Old - Mech - AM-III

12/5/16

QP Code : 28650

Duration: 3Hrs. **Total marks :100**

Note : **1. Question No.1 is compulsory.**
2. Answer any four from the remaining six questions.

1. a) Find $L\left[\frac{1}{\sqrt{\pi t}}\right]$ [5]

b) Find the orthogonal trajectory of the family of curves given by $e^x \cos y - xy = c$ [5]

c) Evaluate $\oint_C \log z \, dz$ where C is $|z| = 1$ [5]

d) Express the matrix A= $\begin{bmatrix} 3i & -1+i & 3-2i \\ 1+i & -i & 1+2i \\ -3-2i & -1+2i & 0 \end{bmatrix}$ as P + iQ where P is real Skew-symmetric matrix and Q is real Symmetric matrix. [5]

2. a) If $v = e^x \sin y$, prove that v is a harmonic function. Also find the corresponding harmonic conjugate function and analytic function. [6]

b) Evaluate $\int_C \frac{z^2}{(z^4-1)} dz$ where C is the circle $|z+i| = 1$. [6]

c) Reduce to normal form and find rank of the matrix : [8]

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

3. a) Solve the Differential Equations using Laplace Transformation

$$\frac{d^2y}{dt^2} + y = t \quad y(0) = 1, \quad y'(0) = 0$$ [6]

b) Find the sum of the residue at singular points of $f(z) = \frac{z}{(z-1)^2 (z^2-1)}$ [6]

c) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, then prove that 3 tan A= A tan 3 [8]

[P.T.O]

4. a) Find the analytic function whose real part is $\dfrac{\sin 2x}{\cosh 2y + \cos 2x}$ [6]

b) Examine whether the vectors $X_1 = [3 \quad 1 \quad 1]$ $X_2 = [2 \quad 0 \quad -1]$ $X_3 = [4 \quad 2 \quad 1]$ are linearly independent. [6]

c) Find Inverse Laplace Transform of (i) $\tan^{-1}\left[\dfrac{2}{s}\right]$

(ii) $\dfrac{1}{(s^2+4s+13)^2}$ using convolution theorem [8]

5. a) Consider the transformation $w = (1 + i)z + (2 - i)$ and determine the region in the w- plane into which the rectangular region bounded by $x = 0$. $y = 0$, $x = 1$, $y = 2$ in the z-plane is mapped under this transformation. Sketch the region. [6]

b) If $A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, find the characteristic roots and characteristic vectors of $A^3 + I$. [6]

c) Evaluate $\displaystyle\int_0^\infty e^{-\sqrt{2}\,t} \sin t \sinh t \, dt$ [8]

6. a) Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2}$ using residues [6]

b) State Cayley-Hamilton Theorem and verify for $A=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. [6]

c) Find Laplace Transform of (i) $\displaystyle\int_0^t u \cos^2 u \, du$

(ii) $t\sqrt{1+\sin t}$ [8]

7. a) Find Laplace Transform of the periodic functions :

$f(t) = t$ for $0 < t < 1$ $f(t) = 0$ for $1 < t < 2$ $f(t+2) = f(t)$ for $t > 0$ [6]

b) Find the Bilinear Transformation that maps the points $z = 1, i, -1$ into $w = i, 0, -i$. [6]

c) Find Laurent's series for $f(z) = \dfrac{2}{(z-1)(z-2)}$ when (i) $|z| < 1$

(ii) $1 < |z| < 2$ (iii) $|z| > 2$. [8]
