S.E. (SEM-IV) old

ME - AM-IV

13/05/16

QP Code : 28871

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question no. **1** is **compulsory**
(2) Attempt **any four** questions out of the remaining **six** questions.
(3) Figures to the **right** indicate **full** marks.

1. (a) Obtain the complex form of Fourier feries for $f(x) = e^{ax}$ in $(0, a)$. 5

(b) If the p.d.f. of a random variable x is $f(x) = \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3....$, find E (x). 5

(c) Solve the partial differential equation 5

$$x^2 \frac{\partial^2 u}{\partial x \partial y} + 3y^2 u = 0$$

(d) Find the probability of getting a total 7 at least once in three tosses of a pair of fair dice. 5

2. (a) Find the Fourier series for $f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$ in $(0, 2\pi)$. 6

(b) A continuous random variable has the p.d.f. $f(x) = kx^2e^{-x}$, $x \geq 0$. Find K, mean and variance. 6

(c) How many dice must be thrown so that the probability of getting 6 is more than $\frac{1}{2}$? 8

3. (a) Using a normal distribution find a probability that in a group of 100 persons there will be 55 males assuming that the probability of a person being male is $\frac{1}{2}$. 6

(b) Two regression lines are given by
$20x - 9y - 107 = 0$ and $4x - 5y + 33 = 0$.
Obtain (i) Coefficient of correlation between x and y 6

(c) Find a Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$ and gence deduce that 8

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ...$$

4. (a) Expand $f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ 6

into a Fourier series where $f(x+2) = f(x)$.

[TURN OVER]

(b) Calculate the correlation coefficient for the following data : 6

X:	23	27	28	29	30	31	33	35	36	39
Y:	18	22	23	24	25	26	28	29	30	32

(c) A dice is tossed 180 times, using normal distribution find the probability that total 4 will turn up at least 35 times. 8

5. (a) Find the Fourier expansion of $f(x) = 4-x^2$ in $(0, 2)$. 6

(b) If the probability that an individual suffers a bad reaction from injection of a medicine is 0.001, determine a probability that out of 2000 individuals exactly 3 suffer a bad reaction. 6

(c) A rod of length ℓ has its ends A and B kept at 0^0C and 100^0C respectively until steady state condition prevails. Temperature of the end A is raised to 25^0C and that of the end B is reduced to 75^0C and kept so, find the temperature distribution u (x, t). 8

6. (a) Find the Fourier expansion of $f(x) = \begin{cases} 0, & -2 < x < -1 \\ 1+x, & -1 < x < 0 \\ 1-x, & 0 < x < 1 \\ 0, & 1 < x < 2. \end{cases}$ 6

(b) The mean height and S.D.height of 8 randomiy chosen soldiers are 166.9 and 8.29 cms. respectively. The corresponding values of 6 randomly chosen sailors are 170.3 and 8.5 cms respectively. Based on this data, can we conclude that soldiers are in general shorter than sailors. 6

(c) Two regression lines are given by : $20x - 9y - 107 = 0$ and $4x - 5y + 33 = 0$. Obtain 8

(i) Coefficeint of correlation between x & y.

(ii) Mean values of x and y.

7. (a) Show that the functions $f_1(x) = 1$, $f_2(x) = x$ are orthogonal on (-1, 1). Determine the constants a and b such that the function $f_3(x) = -1 + ax + bx^2$ is orthogonal to both f_1 and f_2 on that interval. 6

(b) Find the Fourier integral representation for $f(x) = \begin{cases} 1-x^2 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1. \end{cases}$ 6

(c) Fit a second degree parabolic curve to the following data and estimate the production in 1982. 8

Year (x)	1974	1975	1976	1977	1978	1979	1980	1981
Production (y): (in tons)	12	14	26	42	40	50	52	53

——— X X X ———