

Q. P. Code : 610802

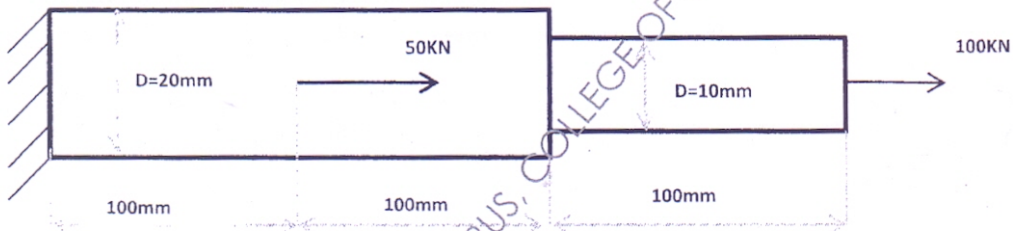
(Old Course) R-2007

TIME: 4 HRS

MAXIMUM MARKS: 100

- Question No. 1 is compulsory.
- Attempt any four questions from the remaining.

- Q1. (a) Explain different types of finite elements used in 1D and 2D analysis. 5
- (b) With the help of typical spring mass system, explain principle of Stationary Total Potential. 5
- (c) Find the displacement, stresses and strain in the elements of step bar shown. 10
Take $E = 210 \text{ GPa}$.



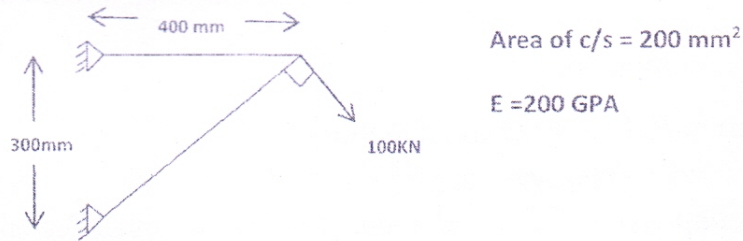
- Q2. (a) Solve the following differential equation using Galerkin method and compare the result at $x=0.5$ using Exact Method. 10

$$\frac{d^2y}{dx^2} + \sin(\pi x) = 0 \quad ; 0 \leq x \leq 1 ; y(0) = 0, y(1) = 0$$

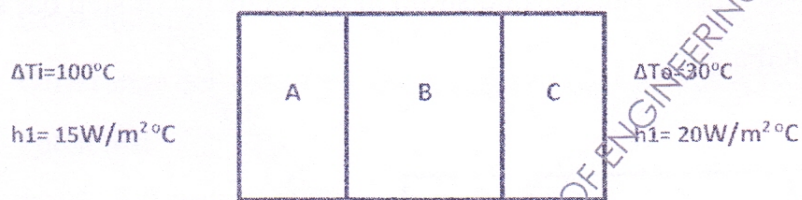
- (b) Explain with neat sketch the properties of Shape Function and Derive quadratic shape functions for 1D element. 10

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Q.3 (a) Evaluate the following truss completely for reactions, stresses and strains.



(b) Find the temperature at two surfaces and at interfaces for a composite wall shown.



$K_A = 50 \text{ W/m}^\circ\text{C}$, $K_B = 30 \text{ W/m}^\circ\text{C}$, $K_C = 60 \text{ W/m}^\circ\text{C}$, $L_A = 50 \text{ mm}$, $L_B = 60 \text{ mm}$,
 $L_C = 60 \text{ mm}$.

Q.4 (a) Using R-R Method mapped over general element solve,

$$\frac{d}{dx} \left(a \frac{du}{dx} \right) + bu = 0; \quad 0 \leq x \leq L$$

Global boundary conditions are, $u(0) = u_0$ and $a \frac{du}{dx}(L) = 0$

Use Lagrange's Linear shape Functions. Write global matrix equation using 3 elements and constant $a = 10$, $b = 5$.

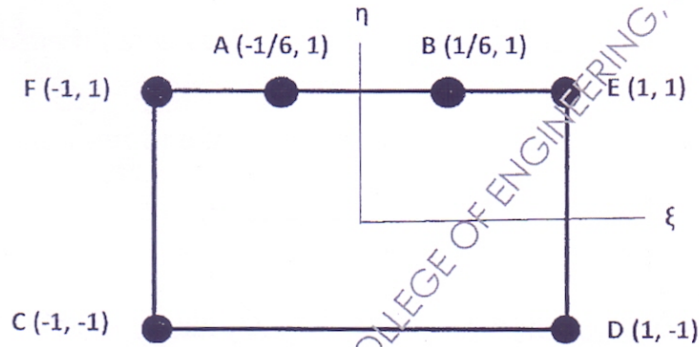
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- (b) In pure bending of a prismatic beam having bending rigidity EI , the functional for the bending energy can be written as 8

$$I(x, u, u'') = \int_0^L \left[\frac{EI}{2} (u'')^2 - qu \right] dx$$

Where, u - The Lateral Displacement, q -The Distributed Load. Find the Differential Equation and Boundary Conditions.

- Q.5 (a) Using Serendipity concept, find the shape functions for following 6 noded rectangular element in natural coordinate system 6



- (b) Derive shape functions for CST element. 6
- (c) Evaluate the following integral using Newton Cotes Method and compare answer with exact. 8

$$I = \int_0^h x \cdot \frac{d\phi_1}{dx} \cdot \frac{d\phi_3}{dx} dx$$

Where

$$\phi_2 = \frac{4x}{h} \left(1 - \frac{x}{h}\right),$$

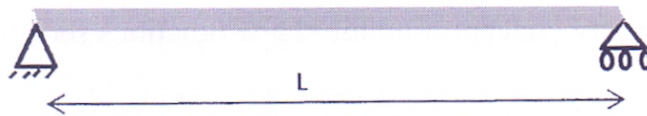
$$\phi_3 = -\frac{x}{h} \left(1 - \frac{2x}{h}\right)$$

r	W1	W2	W3	W4	W5
1	1				
2	1/2	1/2			
3	1/6	4/6	1/6		
4	1/8	3/8	3/8	1/8	
5	7/90	32/90	12/90	32/90	7/90

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Q.6 (a) Show that Consistent mass matrix overestimate and Lumped mass matrix underestimate the natural frequency of a bar element. 10

(b) Find natural frequency of the simply supported beam shown using one element and consistent mass matrix.(assume different parameter)



Q.7 (a) Write notes on following. 10

1. Compatibility,
2. Difference between Lagrange's element and serendipity elements
3. Patch test
4. Jacobian Matrix
5. Sources of Error in a typical FEM solution.

(b) The Four noded rectangular element is shown in figure. Calculate the temperature at point P. 10

Nodal Temperatures $T_1 = 55^\circ\text{C}$, $T_2 = 65^\circ\text{C}$, $T_3 = 75^\circ\text{C}$, $T_4 = 85^\circ\text{C}$

