SEM-III - Old - Electrical - EM-III

12/5/16

QP Code : **28653**

(3 Hours) [Total Marks : 100

(Old course)

INSTRUCTIONS:

- Question number 1 is COMPULSORY.
- Answer any FOUR questions from the remaining six questions.

1. a) Find $L\{(t+e^{-t}+\sin t)\}$ (5)

b) Find the constant a in the analytic function $\frac{1}{2}\log(x^2+y^2)+i\tan^{-1}\frac{ay}{x}$ (5)

c) Find Fourier Series of the function $f(x)=\frac{x(\pi^2-x^2)}{12}$; $-\pi<x<\pi$ (5)

d) Evaluate the complex line Integral $\int_0^{1+i}(x-y+ix^2)dz$ along the straight line from z=0 to z=1+I (5)

2. a) Evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)}dz$ where C is (a) $|z|=3$ (b) $|z-i|=2$ (6)

b) Find the Fourier series of $f(x)=2x-x^2$ in $[0,2]$ Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{16}$ (8)

c) Using Laplace transform show that $\int_0^{\infty}\frac{e^{-\sqrt{2}t}\sin t\sinh t}{t}dt=\frac{\pi}{8}$ (6)

3. a) Using Counter Integration and residue theorem evaluate

$$\int_0^{2\pi}\frac{1}{5+4\cos\theta}d\theta$$ (6)

b) Find the half range Fourier sine series for $f(x)=\begin{cases} x; & 0<x<\frac{\pi}{2} \\ \pi-x; & \frac{\pi}{2}<x<\pi \end{cases}$ (8)

and hence deduce that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4}=\frac{\pi^4}{96}$

c) Find $L^{-1}\left\{\frac{s^2+5}{(s^2+4s+13)^2}\right\}$ (6)

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4. a) Using Laplace Transform solve the differential equation (8)

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t; y(0)=0, y'(0)=1$$

b) Show that $f(z)=(x^3-3xy^2+2xy)+i(3x^2y-x^2+y^2-y^3)$ is analytic and hence find $f'(z)$ (6)

c) Find the Fourier integral representation of $f(x)=\begin{cases} e^{ax} & ; x\le 0 \\ e^{-ax} & ; x\ge 0 \end{cases}$ (6)

and hence show that $\int_0^\infty \frac{\cos sx}{s^2+a^2}ds=\frac{\pi}{2a}e^{-ax}$ for x>0

5. a) Find i) $L\left\{\frac{\cos at-\cos bt}{t}\right\}$ ii) $L\{t\sin t.\cos 3t\}$ (6)

b) Find a bilinear transformation which maps z=1, i,-1 into w=i, 0, -i (6)

c) Find Laurent series for $\frac{1}{z^2-5z+6}$ in $2<|z|<3$ (8)

6. a) Find complex Fourier series of f(x)=sinh2x+cosh2x ;$-\pi<x<\pi$ (6)

b) $w=z^2+z$ maps the circle $|z|=1$ in the z-plane into the car diode $\rho=2(1+\cos\phi)$ in the w-plane (6)

c) If $L\{F(t)\}=\frac{1}{s\sqrt{s+1}}$ find $L\{tF(2\sqrt{t})\}$ (8)

7. a) Find i) $L^{-1}\left\{\frac{(s+1)e^{-2s}}{s^2+2s+2}\right\}$ ii) $L^{-1}\left\{\frac{1}{s}\log\frac{s^2+a^2}{(s+b)^2}\right\}$ (6)

b) Find the constants a & b if $1, x, -1+ax+bx^2$ are orthogonal over(-1,1) (6)

c) Find α using Laplace Transform if $\int_0^\infty e^{-2t}\sin(t+\alpha)\cos(t-\alpha)dt=\frac{3}{8}$ (8)