SE- sem-III - 012 — COMPS - AM-III

12/5/16

QP Code : 28662

DATE: MAX. MARKS:100

TIME: DURATION: 3 HRS.

Instructions:

1) Question No. 1 is compulsory.
2) Attempt any **FOUR** of the remaining.
3) **Figures** to the **right** indicate **full** marks.

Q. 1) A) Find $L\{e^{3t}.\sin^2 t\}$ 05

B) Show that every square matrix can be uniquely expressed as the sum of a Symmetric and a skew–Symmetric matrix. 05

C) Find a Z – transform and the region of convergence of $f(k) = 2^k$, where $k \geq 0$. 05

D) Find the Fourier series expansion of $f(x) = x^2$, where $-\pi \leq x \leq \pi$. 05

Q. 2) A) Prove that the matrix, $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$ is orthogonal and hence find its inverse. 06

B) Find $L^{-1}\left\{\frac{s+2}{(s^2+4s+5)^2}\right\}$. 06

C) Obtain the Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $0 \leq x \leq 2\pi$ and $f(x + 2\pi) = f(x)$.

Also deduce that (i) $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

and (ii) $\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$ 08

Q. 3) A) Investigate for what values of λ and μ, the equations :

$x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have (i) no solution, (ii) a unique solution and (iii) infinite number of solutions. 06

B) Obtain complex form of Fourier series for $f(x) = e^{ax}$ in $(-\pi, \pi)$ where a is not an integer. 06

C) Solve $(D^2 - D - 2)y = 20.\sin(2t)$ with $y(0) = 1$ and $y'(0) = 2$. 08

Q. 4) A) Find the Laplace transform of $(t) = \begin{cases} asinpt & 0 < t < \frac{\pi}{p} \\ 0 & \frac{\pi}{p} < t < \frac{2\pi}{p} \end{cases}$.

and $f(t) = f\left(t + \frac{2\pi}{p}\right)$. 06

B) Find the inverse Z – transform of $F(z) = \frac{1}{(z-3)(z-2)}$, for $|z| > 3$. 06

C) Find the inverse Laplace transform of (i) $\frac{e^{4-3s}}{(s+4)^{\frac{5}{2}}}$ (ii) $\tan^{-1}\left(\frac{2}{s}\right)$ 08

Q. 5) A) Examine whether the following vectors are linearly independent or dependent :

(2, 1, 1), (1, 3, 1) and (1, 2, –1) 06

B) Using the convolution theorem, prove that $L^{-1}\left[\frac{1}{s}\ln\left(\frac{s+a}{s+b}\right)\right] = \int_0^t \frac{[e^{-bu} - e^{-au}]}{u} du$ 06

C) Express the function $f(x) = \begin{cases} sinx & |x| < \pi \\ 0 & |x| > \pi \end{cases}$ as Fourier Sine integral and

evaluate $\frac{1}{\pi}\int_0^{\infty} \frac{sinwx.sin\pi w}{1-w^2} dw$ 08

Q. 6) A) Find the Fourier transform of $f(x) = e^{-|x|}$. 06

B) Find Z{f(t)} where $f(t) = \sin\left(\frac{k\pi}{4} + a\right)$, where $k \geq 0$. 06

C) Find the Fourier expansion of $f(x) = 2x - x^2$, where $0 \leq x \leq 3$. Here f(x) is a periodic function having period 3. 08

Q. 7) A) Reduce the matrix, $A = \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$ to its normal form. Find its rank. 06

B) Evaluate $\int_0^{\infty} \frac{cos4t - cos3t}{t} dt$ using Laplace transform. 06

C) Show that the set of functions, $S = \left\{\sin\left(\frac{\pi x}{2L}\right), \sin\left(\frac{3\pi x}{2L}\right), \sin\left(\frac{5\pi x}{2L}\right), \ldots\right\}$ is an orthogonal set over (0, L). 08

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