[OLD COURSE]   **QP Code : 28647**

Time: 3 hours   Total Marks: 100

N.B. (i) Question No.1 is compulsory.

(ii) Attempt any FOUR questions from remaining six questions.

(iii) Figures to the right indicate full marks.

Q. 1 (a) Find k such that   5

$$f(z) = \frac{1}{2} \log\left(x^2 + y^2\right) + i \tan^{-1} \frac{kx}{y} \quad \text{is analytic}$$

(b) Prove that   5

$$f_1(x) = 1, \ f_2(x) = x, \ f_3(x) = \frac{3x^2 - 1}{2} \quad \text{are orthogonal over } (-1, 1)$$

(c) Find the Laplace Transform of $t\, e^{3t} \sin t$ 5

(d) Prove that the matrix A is orthogonal   5

$$A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix} \text{ and hence find } A^{-1}$$

Q.2 (a) Evaluate   6

$$\int_0^\infty \frac{\cos at - \cos bt}{t}\, dt$$

(b) Reduce matrix A to normal form and find its rank   6

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & 1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

(c) Find the bilinear transformation under which $1, i, -1$ from the z-plane   8
are mapped onto $0, 1, \infty$ of w-plane

Q.3 (a) Find Inverse Laplace Transform of   6

$$\frac{2s^2 - 4}{(s + 1)(s - 2)(s - 3)}$$

(b) Obtain complex form of Fourier series for $f(x) = e^{ax}$ in $(-\pi, \pi)$ where a is not an   6
integer

(c) If $v = e^x \sin y$, prove that v is harmonic function. Also find the corresponding   8
harmonic conjugate function and analytic function

GE-Con. 9453-16.

[TURN OVER

Q.4 (a) Examine whether the vectors $X_1 = [3, 1, 1]$, $X_2 = [2, 0, -1]$, $X_3 = [4, 2, 1]$ are linearly   6
dependent or independent

(b) Construct an analytic function whose real part is $e^x \cos y$ 6

(c) Solve   8

$$\frac{dy}{dt} + 3y = 2 + e^{-t}, \text{ if } y = 1 \text{ at } t = 0$$

Q.5 (a) Find the image of the circle   6

$$(x - 3)^2 + y^2 = 2 \text{ under the transformation } w = 1/z$$

(b) Find non-singular matrices P and Q such that PAQ is in normal form and hence find   6
the rank of A

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

(c) Find a Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$ and hence deduce that   8

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots$$

Q.6 (a) Find a cosine series of period 2π to represent $\sin x$ in $0 \le x \le \pi$ 6

(b) Using Convolution Theorem find the inverse Laplace Transform of   6

$$\frac{(s+3)^2}{\left(s^2 + 6s + 5\right)^2}$$

(c) Determine the Eigen Values and the associated Eigen Vectors for the   8

$$\text{matrix } A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

[TURN OVER

GE-Con. 9453-16.

Q.7 (a) Find the Laplace Transform of $t^2 H(t-3)$ 6

(b) Express the function 6

$$f(x) = -e^{kx} \text{ for } x < 0$$
$$= e^{-kx} \text{ for } x > 0$$

as Fourier Integral and hence prove that

$$\int_0^\infty \frac{\omega \sin \omega x}{\omega^2 + k^2} d\omega = \frac{\pi}{2} e^{-kx} \text{ if } x > 0, k > 0$$

(c) Find the characteristic equation of the matrix 8

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ and verify that it is satisfied by A and hence obtain A^{-1}

GE-Con. 9453-16.