

QP CODE : 537700

(Old Course)

(3 Hours)

[Total Marks:100]

Instructions:-

1. Question one is compulsory.
2. Attempt any four questions from remaining six questions.
3. Statistical table is permitted.

1.(a) A vector field \vec{F} is given by $\vec{F} = (y \sin z - \sin x)\vec{i} + (x \sin z + 2yz)\vec{j} + (xy \cos z + y^2)\vec{k}$. Prove that it is irrotational and hence find its scalar potential. 5

(b) Find the value of k such that the following will be the probability density function. Find $P(x \leq 1.5)$

$$f(x) = \begin{cases} kx & , 0 \leq x \leq 1 \\ k & , 1 \leq x \leq 2 \\ k(3-x) & , 2 \leq x \leq 3 \end{cases} \quad 5$$

(c) A sample size of 100 is drawn from a population of standard deviation 10. If the sample mean is 62.15, Find 99% confidence limits for the mean of the population. 5

(d) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$. 5

2(a) Evaluate $\iint_S \vec{F} \cdot d\vec{s}$ where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the region bounded by $y^2 = 4x, x = 1, z = 0, z = 3$. 6

(b) The probability that an electronic component will fail in less than 1200 hrs. of continuous use is 0.25. Using normal approximation to Binomial distribution, find the probability that among 200 such components fewer than 45 will fail in less than 1200 hrs of continuous use. 6

(c) Evaluate $\int_0^\pi \frac{d\theta}{3 + 2 \cos \theta}$. 8

3(a) Theory predicts that the proportion of beans in the four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans the number in the four groups were 882, 313, 287 and 118 does the experiment result support the theory. 6

(b) Find all possible Laurent's series expansion of the function $f(z) = \frac{7z-2}{z(z-2)(z+1)}$ about $z = -1$. 6

(c) Calculate the coefficients of correlation between X and Y from the following data 8

X:	3	5	4	6	2
Y:	3	4	5	2	6

[TURN OVER :

4.(a) The probability that a man aged 60 will live upto 70 is 0.65. What is the probability that out of 10 such men now at 60 at least 7 will live upto 70. 6

(b) Evaluate $\int_C \frac{dz}{z^3(z+4)}$ where C is the circle $|z| = 2$. 6

(c) Verify Green's Theorem for $\int_C \left(\frac{1}{y} dx + \frac{1}{x} dy \right)$ where C is the boundary of the region defined by

$x = 1, x = 4, y = 1$ & $y = \sqrt{x}$. 8

5(a) A continuous random variable has probability density function $f(x) = 6(x - x^2), 0 \leq x \leq 1$.

Find (i) Mean (ii) Variance (iii) median 6

(b) Seven dice are thrown 729 times. How many times do you expect at least 4 dice to show three or five. 6

(c) The regression lines of a sample are $x+6y=6$ and $3x+2y=10$. Find (i) sample mean \bar{x} and \bar{y} (ii) Coefficient of correlation between x and y. Also estimate y when $x=12$. 8

6.(a) If $\vec{F} = (2xyz^2)\mathbf{i} + (x^2z^2 + z \cos yz)\mathbf{j} + (2x^2yz + y \cos yz)\mathbf{k}$, find its scalar potential and work done in moving a particle from A (0,0,1) to B(1, $\frac{\pi}{4}$, 2) along the straight line AB. 6

(b) If the variance of a poisson distribution is 2, find the probabilities of $r=1,2,3,4$ from the recurrence relation of poisson distribution. 6

(c) Calculate the first four moments about the mean for the following data. 8

x:	1	2	3	4	5	6	7	8	9
f:	1	6	13	25	30	22	9	5	2

7.(a) Using Gauss's Divergence theorem evaluate $\iiint_S (ax^2 + by^2 + cz^2) ds$ over the sphere

$x^2 + y^2 + z^2 = 1$. 6

(b) Find mean and variance of Poisson distribution. 6

(c) The height of six randomly chosen sailors are in inches: 63,65,68,69,71 and 72. The height of 10 randomly chosen soldiers are 61,62,65,66,69,70,71,72 and 73. Discuss in the light that these data throw on the suggestion that soldiers on an average are taller than sailors. 8