SE – sem-III – old – Mechanical – AM-III

12/5/17

QP Code : 553702

(OLD COURSE)

(3 Hours) [Total Marks : 100

N.B :

1. Q1 is compulsory.

2. Solve any four out of the remaining from Q.2 to Q. 7.

3. Figures on the right hand side indicate marks.

Total Marks: 100 Time: Three hours

Q.1.a. Find the Laplace transform of $\frac{\cos 2t \sin t}{e^t}$ 5

b. Prove that $u = \cos x \cosh y$, is a harmonic function. Find its harmonic conjugate and hence the analytic function. 5

c. Evaluate $\int_c \log z dz$, where c is the unit circle in the z plane. 5

d. If $A=\begin{bmatrix}3 & 2 & 2\\ 1 & 3 & 1\\ 5 & 3 & 4\end{bmatrix}$, find B such that $AB=\begin{bmatrix}3 & 4 & 2\\ 1 & 6 & 1\\ 5 & 6 & 4\end{bmatrix}$ 5

Q.2.a. Reduce the Matrix to normal form and hence find the Rank of A.

$$\begin{bmatrix}1 & 2 & -2 & 3\\ 2 & 5 & -4 & 6\\ 1 & -3 & 2 & -2\\ 2 & 4 & -1 & 6\end{bmatrix}$$ 6

b. If f (z) u + iv, is analytic and $u + v = \frac{2\sin 2x}{(e^{2y}+e^{-2y}-2\cos 2x)}$ find f (z). 6

c. Evaluate $\int_0^\pi \left(\frac{\cos at - \cos bt}{t}\right) dt$ 8

Q.3.a. Find the bilinear transformation which maps the points z=1,i,-1 on to the points w=i,0,-i. 6

b. . Find the image of the region bounded by x=0,x=2,y=0,y=2, in z plane under the transformation w=(1+i)z . 6

c. Find, 1. $L[\int_0^t \frac{e^{-u}\sin u}{u} du]$, 2. $L^{-1}[\frac{4s+12}{s^2+8s+12}]$ 8

Q.4.a. Evaluate $\int_c \frac{e^{3z}}{(z-i)} dz$ where c is the curve $|z-2|+|z+2|=6$ 6

[TURN OVER]

b. Find the non singular matrices P and Q such that PAQ is in Normal form, hence find Rank of A. Where 6

$$A=\begin{bmatrix}1 & 2 & -1 & 2\\ 2 & 5 & -2 & 3\\ 1 & 2 & 1 & 2\end{bmatrix}$$

c. Find $L^{-1}\left(tan^{-1}\frac{2}{s^2}\right)$ 8

Q.5.a. Verify Cayley Hamilton Theorem for $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and find A^{-1}. 6

b. Evaluate $\int_C \frac{\cos\pi z^2+\sin\pi z^2}{(z-z^2)}dz$ where c is the circle $|z-2|=4$, Using residue theorem. 6

c. Solve using Laplace transform.

$(D^2+2D+5)\,y=e^{-t}\sin t,\quad given\ that,\ y(0)=0, y'(0)=1$. 8

Q.6.a. Find the Laurent's series expansion which represents the function

$$f(z)=\frac{1}{(z-1)(z-2)}\ in\ the\ regions\ 1<|z-1|<2\ and\ \ |z|<1$$ 6

b. Find the analytic function $f(z)=u+jv$, where $v=e^{-x}(ysiny+xcosy)$. 6

c. Find the Eigen values and Eigen vectors of the matrix A and A^2, if $A=\begin{bmatrix}3 & -1 & 1\\ -1 & 3 & -1\\ 1 & -1 & 3\end{bmatrix}$ 8

Q.7.a. Solve the following system of equations, if consistent.

4x-2y+6z=8, x-y-3z=-1, 15x-3y+9z=21 6

b. Find the inverse Laplace transform using convolution theorem.

$$f(s)=\frac{s}{(s^2+4)(s^2+2)}$$ 6

c. Evaluate $\int_0^{2\pi}\frac{\cos 3\theta}{5-4\cos\theta}d\theta$ 8
