SE-Sem-IV - Computers Old - AM-IV

15/5/17

Q. P. Code : 09881

(3 Hours) [Total marks : 100

OLD COURSE

N.B. : -
1) Question No. **1** is **compulsory.**
2) Attempt any **four** questions from the remaining **six** questions.
3) **Figures** to the **right** indicate **full** marks.

Q.1 a) If $A = \begin{pmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{pmatrix}$, find the eigen values of $A^3 + 5A + 8I$. 05

b) Obtain the dual of the following L.P.P.: 05

$$\begin{aligned} &Minimize\ Z = x_1 - 3\,x_2 - 2x_3 \\ &subject\ to \\ &3x_1 - x_2 + 2x_3 \le 7 \\ &2x_1 - 4\,x_2 \ge 12 \\ &-4x_1 + 3x_2 + 8x_3 = 10 \\ &x_1,\ x_2 \ge 0, \quad x_3\ is\ unrestricted. \end{aligned}$$

c) Prove that $f(z) = (x^3 - 3xy^2 + 2xy) + i(3x^2y - x^2 + y^2 - y^3)$ is analytic and find $f'(z)$ and $f(z)$ in terms of z. 05

d) Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}\,dz,$ where C is the circle $|z| = 3$. 05

Q.2 a) Find the characteristic equation of the matrix A given below and hence, the matrix represented by 06

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ where

$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$.

b) Solve the following L.P.P. by simplex method: 06

$$\begin{aligned} &Maximize\ Z = 3x_1 + 2\,x_2 \\ &subject\ to \\ &x_1 + x_2 \le 4 \\ &x_1 - x_2 \le 2 \\ &x_1,\ x_2 \ge 0 \end{aligned}$$

c) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$. 08

TURN OVER

Q. 3 a) Show that the following function 06

$$f(z) = \begin{cases} \dfrac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0 \\ 0 & z = 0 \end{cases}$$

is not analytic at the origin although Cauchy-Riemann equations are satisfied.

b) If $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$, find the characteristic roots and characteristic vectors of $A^3 + I$. 06

c) Using Penalty (Big M or Charne's) solve the following L.P.P.: 08

$Maximize\ Z = 3x_1 - x_2$
$subject\ to$
$2x_1 + x_2 \leq 2$
$x_1 + 3x_2 \geq 3$
$x_2 \leq 4$
$x_1, x_2 \geq 0$

Q. 4 a) Find p if $f(z) = r^2 \cos 2\theta + i\, r^2 \sin p\theta$ is analytic. 06

b) If $A = \begin{pmatrix} \pi & \pi/4 \\ 0 & \pi/2 \end{pmatrix}$, find $\cos A$. 06

c) Use dual simplex method to solve the following L.P.P.: 08

$Maximize\ Z = -3x_1 - 2x_2$
$subject\ to$
$x_1 + x_2 \geq 1$
$x_1 + x_2 \leq 7$
$x_1 + 2x_2 \leq 10$
$x_2 \leq 3$
$x_1, x_2 \geq 0$

Q. 5 a) Find Laurent's series which represents the function 06

$f(z) = \frac{2}{(z-1)(z-2)}$ when $(i)\ |z| < 1,\ (ii)\ 1 < |z| < 2.$

b) Find an analytic function whose imaginary part is $e^{-x}(y \cos y - x\sin y)$. 06

c) If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 0 \\ 1/2 & 2 \end{pmatrix}$, prove that both A and B are not diagonalizable but AB is diagonalizable. 08

TURN OVER

Q. 6 a) Find the residues of $f(z) = \frac{\sin \pi z}{(z-1)^2(z-2)}$ at its poles. 06

b) Find the image of the rectangular region bounded by $x = 0,\ x = 3,\ y = 0,\ y = 2$ under the transformation $w = z + (1 + i)$. 06

c) Using the method of Lagrangian multipliers solve the following non-linear programming problem: 08

$$Maximize\ Z = {x_1}^2 + {x_2}^2 + {x_3}^2$$
$$subject\ to$$
$$x_1 + x_2 + 3x_3 = 2$$
$$5x_1 + 2x_2 + x_3 = 5$$
$$x_1, x_2, x_3 \geq 0$$

Q. 7 a) Find the bilinear transformation which maps the points $2, i, -2$ onto the points $1, i, -1$. 06

b) Use Kuhn-Tucker method to solve the following N.L.P.P.: 06

$$Maximize\ Z = 10x_1 + 4x_2 - 2{x_1}^2 - {x_2}^2$$
$$subject\ to$$
$$2x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

c) Evaluate $\int_C \frac{z+6}{z^2-4} dz$ where C is the circle 08

$(i)\ |z| = 1, \quad (ii)\ |z-2| = 1, \quad (iii)\ |z+2| = 1.$
