PROBABILITY & STATISTICS

(M.E CEM)

SEM-I

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Part -2

Theoretical Probability Distributions

Theoretical Probability Distributions

Discrete Continuous

1. Binomial
2. Poisson } Discrete
3. Normal
4. Standard
5. β } Continuous

1. Binomial distribution :

It is ideally used when there are only 2 possible outcomes of occurance for any event — yes / no, true / false, heads / tails, etc.

Let 'p' represent the probability of success of an event and q represent the probability of failure. Then, $p = 0.5$, $q = 0.5$ & $p + q = 1$

Let there be 'n' no. of such events occuring and let 'r' represent the no. of successes in 'n' events. Then, as per the binomial distribution, the probability of getting 'r' successes in 'n' events & is given by

$$P(r) = {}^{n}C_{r}\, p^{r} q^{(n-r)}$$

where ^nC_r represents the total no. of combinations of selecting 'r' objects at a time from 'n' object and is given by :

$$\frac{n!}{r!(n-r)!}$$

∴ Binomial Distribution

$$\boxed{P(r) = \frac{n!}{r!(n-r)!} \times p^r q^{(n-r)}}$$

Expected value of the distribution, i.e., the mean 'm'

$E(r) = m = np$

Std. deviation, $\sigma = \sqrt{npq}$

Variance, $\sigma^2 = npq$

Coeff. of variance $= \frac{\sigma}{m}$

Q: Which are the various scenarious in project management, which may fit into a binomial distribution?

A:
- Natural calamities
- Selection of equipments
- Occurrance of risks
- Presence / absence of human resources
- Availability of resources like electricity, water etc.
- tenderer getting and not getting projects.

If 'N' represents the no. of times, the ~~small~~ 'm' events are getting repeated, then the expected frequency of getting R successes

$$f(r) = N \times P(r)$$

eg: ① A tenderer is bidding for construction works, averagely 10 nos. in a year for the past 5 years. Find the probability that

(a) He gets atleast 7 works.
(b) He gets exactly 3 works
(c) He does not get any work
(d) He gets not more than 2 works
(e) He gets all the works.

Also determine the mean, standard deviation, variance and coeff. of variance, and frequency of 'r' successes for the above scenario.

Sol^n: $P(r) = {}^nC_r \, p^r \, q^{(n-r)}$

$= \dfrac{n!}{r!\,(n-r)!} \times 0.5^r \; 0.5^{(n-r)}$

(a) Probability of getting atleast 7 works

$= P(7) + P(8) + P(9) + P(10)$

$= {}^{10}C_7 (0.5)^7 (0.5)^3 + {}^{10}C_8 (0.5)^8 (0.5)^2 + {}^{10}C_9 (0.5)^9 (0.5)^1 + {}^{10}C_{10} (0.5)^{10} (0.5)^0$

$= 0.117 + 0.044 + 0.009 + 0.001$

$= \underline{0.171}$

(b) Exactly 3 works,

$P(3) = {}^{10}C_3 \; 0.5^3 \; 0.5^7$

$= \frac{10!}{3!\,(10-3)!} (0.5)^3 (0.5)^7$

$= \underline{\underline{0.117}}$

(c) Does not ~~gettin~~ get any work

$p(0) = \frac{10!}{0!\,(10-0)!} 0.5^0 \; 0.5^{10}$

$= {}^{10}C_0 \; 0.5^0 \times 0.5^{10}$

$= \underline{\underline{0.001}}$

d) Not more than 2

$= p(0) + p(1) + p(2)$

$= 0.001 + {}^{10}C_1 \; 0.5^1 \; 0.5^9 + {}^{10}C_2 \; 0.5^2 \; 0.5^8$

$= \underline{\underline{0.055}}$

) All work $= p(10) = \underline{\underline{0.001}}$

— The largest probability has been indicated for contractor getting minimum 7 works

$F(r) = N \times p(r)$

$= 5 \times 0.171$

$= 0.855$

$m = E(r) = np = 10 \times 0.5 = 5$

$\sigma = \sqrt{npq} = \sqrt{10 \times 0.5 \times 0.5}$

$= 1.58$

$\sigma^2 = 2.5$

Physically, it means, that in a period of 100 years of bidding, only in any 17 years, he ~~not~~ may get atleast 7 works.

2 2 material suppliers have been evaluated for delivering materials on time, based on which the following values are obtained

Supplier A $\rightarrow$ $\left.\begin{array}{l} p = 0.8 \\ q = 0.2 \end{array}\right\}$ reliable supplier

Supplier B $\rightarrow$ $\left.\begin{array}{l} p = 0.4 \\ q = 0.6 \end{array}\right\}$ unreliable supplier

Considering that both supplier are giving 25 deliveries, determine the probability of getting 22 deliveries on time in both the cases.

Sol^n : $n = 25$

$P(22) \rightarrow$ Supplier A $\rightarrow {}^{n}C_r \, p^r \, q^{n-r}$

$= {}^{25}C_{22} \, p^{22} \, q^{25-22}$

$= \dfrac{25!}{22! \times 3!} \, 0.8^{22} \, 0.2^{3}$

$= 0.136 = 13.6\%$

$P_{(22)} \rightarrow$ Supplier B $= \dfrac{25!}{22! \times 3!} \times 0.4^{22} \times 0.6^{3}$

$= 0.0000009$

$= 0\%$

Offer your comments on the above results.

- **Poisson's distribution** :

The limiting case of the binomial distribution is the Poisson's distribution. This distribution is used when the probability of success p is very close to 0 or 1.

$$P(r) = \frac{e^{-m} m^r}{r!}$$

where m = mean = np

std deviation, $\sigma = \sqrt{m}$

Variance, $\sigma^2 = m$

C.V $= \sigma/m$

$e = 2.7183$

① A company manufactures concrete blocks. During sampling, it was observed that 15 blocks are defective out of 1000. Find the probability that, in general while manufacturing 5000 blocks,

(a) 50 blocks are defective
(b) 100 blocks are defective
(c) 25 blocks are defective
(d) 800 blocks are defective
(e) 1000 blocks are defective

Soln : $p = \frac{15}{1000} = 0.015$

$m = np = 5000 \times 0.015 = 75$

$P(r) = e^{-m} m^r$

$\therefore$ (a) $P(50) = \dfrac{e^{-75}\, 75^{50}}{50!} = 4.9877 \times 10^{-4}$

$= 0.0005$

(b) $P(10) = 4.157 \times 10^{-21} \approx 0$

(c) $P(25) = 1.2996 \times 10^{-11} \approx 0$

(d) $P(80) = 0.0379$

(e) $P(100) = 9.2052 \times 10^{-4}$

② If 0.5% of doors manufactured in a factory are defective, find the probability that in a batch of 10,000 doors delivered to a mass housing ~~sample~~ construction site:

① No door is defective
② All doors are defective
③ 15 doors are defective
④ 20 doors are defective.

Comment on results obtained

Soln: $p = 0.005$

$m = np = 10000 \times 0.005 = 50$

(1) $P(0) = \dfrac{e^{-50}\, 50^{0}}{0!} = 1.929 \times 10^{-22}$

(2) $P(10000) = e^{-50}\, 50^{10000}$

(3) $p(15) = \frac{e^{-50}\, 50^{15}}{15!} = 4.50 \times 10^{-9}$

(4) $p(20) = \frac{e^{-50}\, 50^{20}}{20!} = 7.56 \times 10^{-7}$

③ In 25 years of a truck driver's career, it has been recorded that he has encountered 3 major and 25 minor accidents. His average journey per day is 100 kms. What is the probability that, if he has embarked on a journey to deliver precast units on an infrastructure site, 200 km from the manufacturing yard, he will be involved in an accident?

Sol^n: Total accidents $= 3 + 25 = 28$

Total driving exp $= 25 \times 100 \times 365$

$= 912500$ km

$\therefore p = \frac{28}{912500} = 3.068 \times 10^{-5}$

$m = np = 200 \times 3.068 \times 10^{-5}$

$= 6.137 \times 10^{-3}$

$\therefore$ Probability of him involved in an accident

$\therefore p(1) = \frac{e^{-6.137\times10^{-3}} \times (6.137\times10^{-3})^1}{1!}$

$= 6.099 \times 10^{-3}$

$= 0.006099$

$= \underline{0.6\%}$

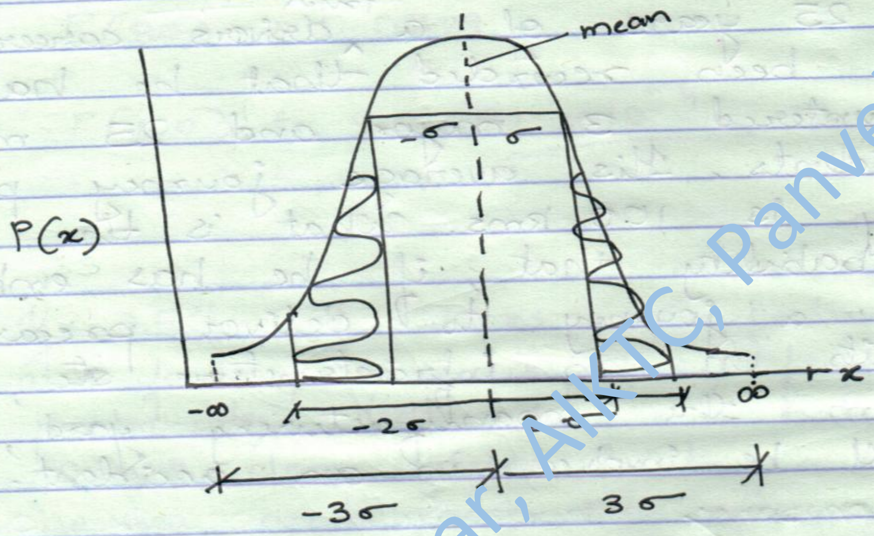
Probability of driver involved in 2 accidents

$p(2) = \frac{e^{-6.137\times10^{-3}} \times (6.137\times10^{-3})^2}{2!}$

- Normal Distribution

$$P(x) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right) e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

where, m = mean deviation
σ = std deviation



Data within

$m \pm 3\sigma = 99.99\%$

$m \pm 2\sigma = 95\%$

$m \pm \sigma = 70\%$

$$T = \frac{t_o + 4t_m + t_p}{6}$$

Three point time estimate — PERT
" " cost " — Cost anal

Cost control estimate are also based o
normal distribution

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2}$$

eg: ① In the gradation process of construction contractors adopted by Construction Industry Development Council (CIDC). Based on budget & schedule achievement contractor's organisations were rated on a scale of [illegible] to 5, based on their performance. ~~(on~~ 1 being very poor & 5 excellent)

A reputed construction firm based on execution of 50 of its similar industrial projects has got the following ratings

① Project no	② Ratings for performance (maintaining budget)	③ Ratings for performance (maintaining schedule)
1	5	5
2	1	5
3	3	2
4	4	1
5	4	4
6	1	2
7	2	4
8	3	4
9	4	3
10	3	5
11	2	5
12	4	3
13	5	3
14	2	2
15	4	5
16	1	2
17	5	4
18	5	3
19	5	4
20	5	5
21	4	5

①	②	③
22	2	1
23	5	3
24	5	5
25	4	5
26	3	1
27	2	4
28	3	3
29	2	3
30	2	1
31	4	1
32	2	5
33	4	3
34	4	4
35	1	2
36	2	2
37	2	3
38	4	5
39	4	5
40	2	1
41	3	3
42	1	1
43	1	2
44	5	2
45	4	4
46	3	2
47	3	2
48	1	2
49	3	1
50	1	1

Determine the mean and standard deviation values for ratings given for budget achievements and schedule achievement. In case they don't fit assuming the normal distribution determine the probability of construction company getting grades between 3–5 for its future projects.

Draw the graphs for above distribution in excel.

Soln :

Project nos	$(x-\bar{x})^2$	$(y-\bar{y})^2$	Project nos.	$(x-\bar{x})^2$	$(y-\bar{y})^2$
1	3.686	3.764	26	0.006	4.244
2	4.326	3.764	27	1.166	0.884
3	0.006	1.124	28	0.006	0.004
4	0.846	4.244	29	1.166	0.004
5	0.846	0.884	30	1.166	4.244
6	4.326	1.124	31	0.846	4.244
7	1.166	0.884	32	1.166	3.764
8	0.006	0.884	33	0.846	0.004
9	0.846	0.004	34	0.846	0.884
10	0.006	3.764	35	4.326	1.124
11	1.166	3.764	36	1.166	1.124
12	0.846	0.004	37	1.166	0.004
13	3.686	0.004	38	0.846	3.764
14	1.166	1.124	39	0.846	3.764
15	0.846	3.764	40	1.166	4.244
16	4.326	1.124	41	0.006	0.004
17	3.686	0.884	42	4.326	4.244
18	3.686	0.004	43	4.326	1.124
19	3.686	0.884	44	3.686	1.124
20	3.686	3.764	45	0.846	0.884
21	0.846	3.764	46	0.006	1.124
22	1.166	4.244	47	0.006	1.124
23	3.686	0.004	48	4.326	1.124
24	3.686	3.764	49	0.006	4.244
25	0.846	3.764	50	4.326	4.244

$\bar{x} = 3.08 \quad \bar{y} = 3.06$

$$\therefore \quad \sigma_x = \sqrt{\frac{1}{n} \Sigma (x - \bar{x})^2} = \sqrt{\frac{1}{50} (91.66)} = 1.352$$

$$\sigma_y = \sqrt{\frac{1}{n} \Sigma (y - \bar{y})^2} = 1.434$$