

PROBABILITY & STATISTICS
(M.E CEM)
SEM-I

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Part -5

Simulation

Applications in

Civil

Engineering

SIMULATION

"The process of making uncertainties, certain to some extent is known as simulation."

It is the imitation of the operation of a real-world process or system over time.

§ Monte - Carlo Simulation:

This method is believed to be derived from gambling activities in a place called Monte - Carlo, in Monaco, and today has received wide application in various fields. It requires random nos. to be assigned to predict future performance of a system.

→ Numericals

① A contractor has kept the data as regards delay of his work and the penalties, which he had to pay, on 5 of his previously executed construction projects. The data is as follows.

Project	Delay time (months)	Penalty (₹)
1	30	20,00,000
2	2	20,000
3	18	15,00,000
4	4	1,00,000
5	13	7,00,000

Using Monte Carlo Simulation, simulate the mean delay and the mean penalty, which he should consider for his next 5 weeks.

Solⁿ:

Delay x	Penalty y	Deviation from Mean delay $x - \bar{x}$	Deviation from Mean penalty $y - \bar{y}$
30	29,00,000	16.6	11,36,000
2	20,000	-11.4	-8,44,000
18	15,00,000	4.6	6,36,000
4	1,00,000	-9.4	-7,64,000
13	7,00,000	-10.4	-1,64,000
$\bar{x} = 13.4$	$\bar{y} = 8,64,000$		

① Simulation Run for delay:

→ Assignment of random numbers

Random numbers should be so assigned so that maximum group (subrads) can be formed. For example, if a data set contains values between 0 & 100, and 90% of the values in such set lie between 0 to 20, there is no point grouping the data into 0-20, 20-40, 40-60, 60-80 & 80-100 sets, as maximum values would be concentrated in the set {0-20}. This would mean that 0 or 1 would have the same status as 19 or 20, which would lead to erroneous results.

In our case, an interval of '3' for delay would be ideal. (as lowest value is '2' & highest is '30') distributed

TABLE OF RANDOM NUMBERS

39 65 76 45 45	19 90 69 64 61	20 26 36 31 62	58 24 97 14 97	95 06 70 99 00
73 71 23 70 90	65 97 60 12 11	31 56 34 19 19	47 83 75 51 53	30 62 38 20 44
72 20 47 33 84	61 67 47 97 19	98 40 07 17 66	23 05 09 51 80	59 78 11 52 69
75 17 25 69 17	17 95 21 78 48	24 33 45 77 48	69 81 84 09 29	93 22 70 45 80
37 48 79 88 74	63 52 06 34 30	01 31 60 10 27	35 07 79 71 53	28 99 52 01 41
02 89 08 16 94	85 53 83 29 95	56 27 09 24 43	21 78 55 09 82	72 61 88 73 61
87 18 15 70 07	37 40 79 12 38	48 13 93 15 96	41 92 45 71 51	09 18 25 58 94
98 83 71 70 15	89 09 39 59 24	00 06 41 41 20	14 36 59 25 47	54 45 17 24 89
10 08 58 07 04	76 62 60 48 68	58 76 17 14 86	59 53 11 52 21	66 04 18 72 87
17 90 56 37 31	71 82 13 50 41	27 55 10 24 92	28 04 67 53 44	95 23 00 84 47
93 05 31 03 07	34 18 04 52 35	74 13 39 55 22	68 95 23 92 35	36 63 70 35 31
21 80 11 47 99	11 20 99 45 18	76 51 94 84 86	13 79 93 37 55	98 16 04 41 67
95 18 94 36 97	23 37 83 28 71	79 57 95 13 91	09 61 87 25 21	56 20 11 32 44
97 08 31 55 73	10 65 81 92 59	77 31 61 95 46	20 44 90 32 64	23 99 76 75 63
69 26 88 86 13	59 71 74 17 32	48 38 75 93 29	73 37 32 04 05	60 82 29 20 25
41 27 10 25 03	87 63 93 95 17	81 83 83 04 49	77 45 85 50 51	79 88 01 97 30
91 94 50 63 62	08 61 74 51 68	92 79 43 83 79	29 18 94 51 23	14 85 11 47 23
80 06 54 18 47	08 52 85 08 40	48 40 35 94 22	72 65 71 08 86	50 03 42 99 36
76 72 77 63 99	89 85 84 46 06	64 71 06 21 66	89 37 20 70 01	61 65 70 22 12
59 40 24 13 75	42 29 82 23 19	07 94 76 10 08	81 30 15 89 14	81 83 17 16 33
63 62 06 34 41	79 53 36 02 95	94 61 09 43 62	20 21 14 68 86	84 95 48 46 45
78 47 23 53 90	79 93 96 38 63	34 85 52 05 09	85 43 01 72 73	14 93 87 81 40
87 68 62 15 43	97 48 72 66 48	53 16 71 13 81	59 97 50 99 92	24 62 20 42 30
47 60 92 10 77	26 97 05 73 51	88 46 38 00 58	72 63 49 29 31	75 70 16 08 24
56 88 87 59 41	06 87 37 78 48	65 88 69 58 39	88 02 84 27 82	85 81 56 39 38
22 17 68 65 84	86 02 22 57 51	68 69 80 95 44	11 29 01 95 80	49 34 35 86 47
19 36 27 59 46	39 77 32 77 09	79 57 92 36 59	89 74 39 82 15	05 50 94 34 74
16 77 23 02 77	28 06 24 25 93	22 45 44 84 11	87 80 61 65 31	09 71 91 74 25
78 43 66 07 61	97 66 63 99 61	80 45 67 93 82	59 73 19 85 23	53 33 65 97 21
03 28 28 26 08	69 30 16 09 05	53 58 47 70 93	66 56 45 65 79	45 56 20 19 47
04 31 17 21 56	33 63 99 19 87	26 72 39 27 67	53 77 57 68 93	60 61 97 22 61
61 06 98 03 91	87 14 77 43 96	43 00 65 98 50	45 60 33 01 07	98 90 46 50 47
23 58 35 26 00	99 53 93 61 28	52 70 05 48 34	56 65 05 61 86	90 92 10 79 80
15 39 25 70 99	93 86 52 77 65	15 35 59 05 28	22 87 26 07 47	86 96 98 29 06
58 71 96 30 24	18 46 23 34 27	85 13 99 24 44	49 18 09 79 49	74 16 32 23 02
93 22 53 64 39	07 10 63 76 35	37 03 04 79 88	08 33 33 85 51	55 34 57 72 69
78 76 58 54 74	92 38 70 96 92	52 06 79 79 45	82 63 18 27 44	69 66 92 19 09
61 81 31 96 82	00 57 25 60 56	46 72 60 18 77	55 66 12 62 11	09 99 55 64 57
42 88 07 10 05	24 98 65 08 21	47 21 61 88 32	27 80 30 21 60	10 92 35 36 12
77 94 30 05 33	28 10 99 00 27	12 73 73 99 12	39 99 57 94 82	96 88 87 17 91

Random numbers should be assigned according to accuracy required. In the above case, 39 can be assigned as random no., as well as 396 can be assigned. If more accuracy is desired 3965 can be assigned as the 1st random number. The second random no. may be selected row-wise or column-wise & so on.

The random no. sel can be selected anywhere from the table (it is not necessary to select from 1st row, 1st column). The random nos. so selected is known as 'pseudo-random numbers', as the numbers are not purely "random".

Interval	Frequency	Probability	Cumulative Probability	Mean	Random nos. assigned
0-3	1	0.2	0.2	1.5	0 to 1999
3-6	1	0.2	0.4		
6-9	0	0	0.4	7.5	2000 to 3999
9-12	0	0	0.4		
12-15	1	0.2	0.6	13.5	4000 to 5999
15-18	1	0.2	0.8		
18-21	0	0	0.8	21	6000 to 7999
21-24	0	0	0.8		
24-27	0	0	0.8	28.5	8000 to 9999
27-30	1	0.2	1		
Σ	5				

→ For next 5 works,

Random no.	Simulated delay (months)
3965	7.5
7645	21
4519	13.5
9069	28.5
6461	21
$\text{Avg} = 18.3$	

Hence, the contractor should consider a mean delay of 18.3 months for his next 5 works.

② Simulation Run for penalty:

As the minimum penalty is ₹20,000/-, and max^m penalty is ₹20,00,000/- we can consider a set of 50,000 for assignment of random nos.

Interval (in 1000s)	Frequency	Prob.	Cum. Prob.	Mean	Assignment of random nos.
0 - 50	1	0.2	0.2	25,000	0 to 1999
50 - 100	1 ✓	0.2	0.4	3,50,000	
100 - 150	1 ✗	0	0.4		
150 - 200	—	0	"		
200 - 250	—	0	"		
250 - 300	—	0	"		
300 - 350	—	0	"		
350 - 400	—	0	"		
400 - 450	—	0	"		
450 - 500	—	0	"		
500 - 550	—	0	"		
550 - 600	—	0	"	2000 to 3999	
600 - 650	—	0	"	10,50,000	
650 - 700	1	0.2	0.6		
700 - 750	—	0	"		
750 - 800	—	0	"		
800 - 850	—	0	"		
850 - 900	—	0	"		
900 - 950	—	0	"		
950 - 1000	—	0	"		
1000 - 1050	—	0	"		
1050 - 1100	—	0	"		
1100 - 1150	—	0	"		
1150 - 1200	—	0	"		
1200 - 1250	—	0	"		
1250 - 1300	—	0	"		

once a value is considered in 1 set, it must not be considered again. 1 lakh can either be included in 50-100 range or 100-150 range, but only once

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Interval (in 1000s)	Frequency	Prob.	Cumm. Prob.	Mean	Assigned random nos.
1350-1400	—	0	"	17,00,000	6000 to 7999
1400-1450	—	0	"		
1450-1500	1	0.2	0.8		
1500-1550	—	0	0.8		
1550-1600	—	0	"		
1600-1650	—	0	"		
1650-1700	—	0	"		
1700-1750	—	0	"		
1750-1800	—	0	"		
1800-1850	—	0	"		
1850-1900	—	0	"		
1900-1950	—	0	"		
1950-2000	1	0.2	"	19,75,000	8000 to 9999
$\Sigma = 5$					

→ For next 5 works.

Random no.	Penalty (₹)
0289	25,000
8718	19,75,000
9883	19,75,000
1008	25,000
1190	25,000
Arg	8,05,000

Hence the contractor should consider a mean penalty of ₹ 8,05,000/- for his next 5 works

③ Simulation for deviation from mean delay:-

Interval	Frequency	Prob.	Cumu. Prob	Mean	Assignment of random nos.
-12 to -8	2	0.4	0.4	-8	0 to 3999
-8 to -4	—	—	0.4		
-4 to 0	1	0.2	0.6	0	4000 to 5999
0 to 4	—	—	0.6		
4 to 8	1	0.2	0.8	8	6000 to 7999
8 to 12	—	—	0.8		
12 to 16	—	—	0.8	16	8000 to 9999
16 to 20	1	0.2	1		
	$\Sigma = 5$			18	

→ For next 5 works

Random no.	Simulated deviation from mean delay (months)
6362	10
0634	-8
4179	0
5336	0
0295	-8
	Avg = -1.2

∴ For his next 5 works, contractor should consider a delay of $(13.4 - 1.2)$ months, i.e. 12.2 months

→ Significance of simulating from the mean for deviations from the mean:
It is more "precise" likely to be

④ Simulation for deviation from mean penalty:

Interval (in lakhs ₹)	Frequency	Probability Cum.	Cumm. Prob.	Means	Assignment of random nos.
-10 to -5	2	0.4	0.4	-7.5	0 to 3999
-5 to 0	1	0.2	0.6	0	4000 to 5999
0 to 5	0	0	0.6		
5 to 10	1	0.2	0.8	7.5	6000 to 7999
10 to 15	1	0.2	1	12.5	8000 to 9999
$\Sigma = 5$					

→ For next 5 works

Random no.	Simulated deviation from mean (in lakh ₹)
2021	-7.5
8543	12.5
5997	0
7263	7.5
8802	12.5
Avg = 5	

Hence the contractor should consider a mean penalty of $(8.64 + 5)$ lakh, i.e ₹ 13,60,000/- for his next 5 works.

The results of simulation would be more accurate if large data set is available.

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