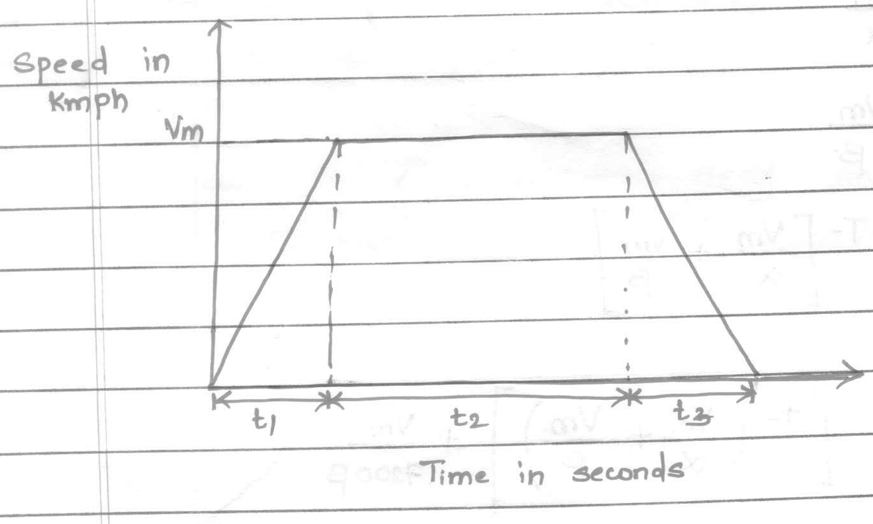
SYSTEM OF TRACTION

Trapeziodal Speed time Curve.



Let α = Acceleration in kmphps

β = Retardation in kmphps

V_m = Crest Speed in kmph

T = total time of run in sec.

Time for acceleration in seconds $t_1 = \frac{V_m}{\alpha}$

Time for retardation in seconds $t_3 = \frac{V_m}{\beta}$

Time for free running in seconds $t_2 = T - (t_1 + t_3)$

$$= T - \left[\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right]$$

Total distance of run in km

S = Distance travelled during acceleration

+ Distance travelled during free run

+ Distance travelled during braking

$$S = \frac{1}{2} \frac{V_m t_1}{3600} + \frac{V_m t_2}{3600} + \frac{1}{2} \frac{V_m t_3}{3600}$$

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Substituting $t_1 = \frac{V_m}{\alpha}$

$t_3 = \frac{V_m}{\beta}$

and $t_2 = T - \left[\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right]$

$$S = \frac{V_m^2}{7200\alpha} + \frac{V_m}{3600}\left[T - \left(\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right)\right] + \frac{V_m^2}{7200\beta}$$

$$S = \frac{V_m^2}{7200\alpha} + \frac{V_m}{3600}T - \frac{V_m^2}{3600\alpha} - \frac{V_m^2}{3600\beta} + \frac{V_m^2}{7200\beta}$$

$$S = \frac{V_m T}{3600} - \frac{V_m^2}{7200\alpha} - \frac{V_m^2}{7200\beta}$$

$$\frac{V_m^2}{3600}\left[\frac{1}{2\alpha} + \frac{1}{2\beta}\right] - \frac{V_m T}{3600} + S = 0$$

$$V_m^2\left[\frac{1}{2\alpha} + \frac{1}{2\beta}\right] - V_m T + 3600S = 0$$

Substituting $K = \left[\frac{1}{2\alpha} + \frac{1}{2\beta}\right]$ in above equation

We get

$$V_m^2 K - V_m T + 3600S = 0$$

$$V_m = \frac{T \pm \sqrt{T^2 - 4K \times 3600S}}{2K}$$

$$= \frac{T}{2K} \pm \sqrt{\frac{T^2}{4K^2} - \frac{3600S}{K}}$$

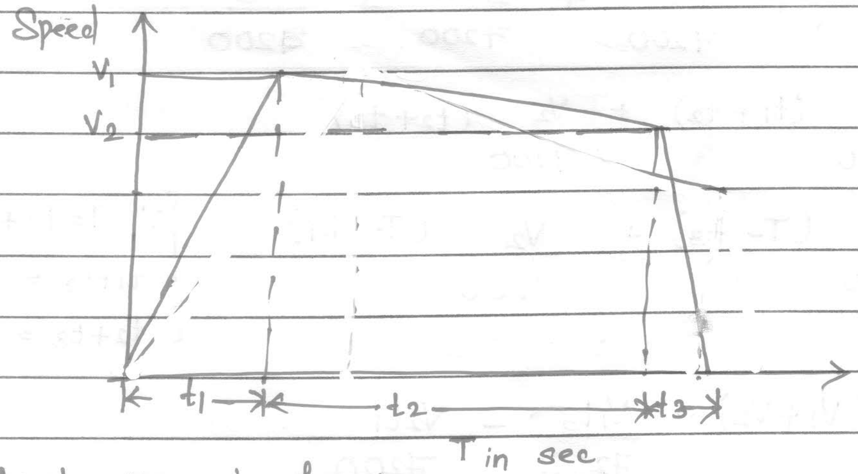
- +ve sign cannot be adopted as value of Vm obtained by using +ve sign will be much highes than that is possible in practice

- Hence -ve sign will be used and therefore

We have

$$\boxed{V_m = \frac{T}{2K} - \sqrt{\frac{T^2}{4K^2} - \frac{3600S}{K}}}$$

Quadrilateral Speed Time Curve



α = Acceleration in kmphps

βc = Coasting retardation in kmphps

β = Braking retardation in kmphps

V_1 = Maximum speed at the end of acceleration in kmph

V_2 = Speed at the end of coasting in Kmph

T = Total time of run in second

Time of acceleration in seconds $t_1 = \dfrac{V_1}{\alpha}$

Time of coasting in seconds $t_2 = \dfrac{V_1 - V_2}{\beta c}$

Time of braking in seconds $t_3 = \dfrac{V_2}{\beta}$

Total distance travelled in km

S = Distance travelled during acceleration
+ Distance travelled during coasting
+ Distance travelled during retardation

$$S = \frac{1}{2}\frac{V_1 \times t_1}{3600} + \frac{V_1+V_2}{2} \times \frac{t_2}{3600} + \frac{1}{2}\frac{V_2 \times t_3}{3600}$$

$$S = \frac{V_1 t_1}{7200} + \frac{V_1 t_2}{7200} + \frac{V_2 t_2}{7200} + \frac{V_2 t_3}{7200}$$

$$S = \frac{V_1}{7200}(t_1+t_2) + \frac{V_2}{7200}(t_2+t_3)$$

$$S = \frac{V_1}{7200}(T-t_3) + \frac{V_2}{7200}(T-t_1)$$

$\left[\because T = t_1+t_2+t_3,\quad t_1+t_2 = (T-t_3) \ \& \ t_2+t_3 = (T-t_1)\right]$

$$S = \frac{T}{7200}(V_1+V_2) - \frac{V_1 t_3}{7200} - \frac{V_2 t_1}{7200}$$

Substitute $t_3 = \frac{V_2}{\beta}$ and $t_1 = \frac{V_1}{\alpha}$

$$S = \frac{T}{7200}(V_1+V_2) - \frac{V_1}{7200} \times \frac{V_2}{\beta} - \frac{V_2}{7200} \times \frac{V_1}{\alpha}$$

$$S = \frac{T}{7200}(V_1+V_2) - \frac{V_1 V_2}{7200\beta} - \frac{V_1 V_2}{7200\alpha}$$

$$7200\,S = T(V_1+V_2) - V_1 V_2\left[\frac{1}{\alpha} + \frac{1}{\beta}\right]$$

Now we have

$$V_2 = V_1 - \beta c t_2 \qquad \left[\because \; t_2 = \frac{V_1 - V_2}{\beta} \right]$$

and $T = t_1 + t_2 + t_3$

$$t_2 = T - t_1 - t_3$$

$$V_2 = V_1 - \beta c \left[T - t_1 - t_3 \right]$$

$$V_2 = V_1 - \beta c \left[T - \frac{V_1}{\alpha} - \frac{V_2}{\beta} \right]$$

$$\left[V_2 - \frac{\beta c}{\beta} V_2 \right] = V_1 - \beta c \left[T - \frac{V_1}{\alpha} \right]$$

$$V_2 = \left[1 - \frac{\beta c}{\beta} \right] = V_1 - \beta c \left[T - \frac{V_1}{\alpha} \right]$$

$$V_2 = \frac{V_1 - \beta c T + \frac{\beta c}{\alpha} V_1}{\left[1 - \frac{\beta c}{\beta} \right]}$$

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Tractive Effort (Important)

- It is the effective force on the wheel rims of locomotive which is required for its propulsion
- The tractive effort is a vector quantity which is tangential to the wheel. It is measured in newtons.
- The notation used for tractive effort is F_t
- In order to propel the train forward the train, the tractive effort or force should be equal to the some of the following forces:

1] Tractive effort required for linear & angular acceleration of train (F_a)
2] Tractive effort for overcoming the effect of gravity (F_g)
3] Tractive effort for overcoming train resistance (F_r)

1] Tractive effort required for linear & angular acceleration of train (F_a)

According to laws of dyamamic force it is required to accelerate the motion of the body and is given by an expression as

Force = Mass × Acceleration

$$F = m \times a$$

Consider a train of weight W tonnes being accelerated at α kmphps

The weight of train = 1000 W kgf

Mass of train = 1000 W kg

Acceleration = α kmphps

$$\alpha \text{ in } m/s^2 = \alpha \times \frac{1000}{3600} \; m/s^2$$

$$= 0.2778 \, \alpha \; m/s^2$$

Tractive effort required for linear acceleration

$$F_a = m \times \alpha$$

$$= 1000 \times 0.2778$$

$$\boxed{F_a = 277.8 \, W \alpha} \text{ newtons}$$

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$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

$$F = m \times a$$

Consider a train of weight W tonnes being accelerated at α Kmphps

$$\text{The weight of train} = 1000\, W \text{ kgf}$$

$$\text{Mass of train} = 1000\, W \text{ kg}$$

$$\text{Acceleration} = \alpha \text{ Kmphps}$$

$$\alpha \text{ in } m/s^2 = \alpha \times \frac{1000}{3600} \; m/s^2$$

$$= 0.2778\, \alpha \; m/s^2$$

Tractive effort required for linear acceleration

$$F_a = m \times \alpha$$

$$= 1000 \times 0.2778$$

$$\boxed{F_a = 277.8\, W \alpha} \text{ newtons}$$

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The tractive effort required for the angular acceleration depends upon the individual weight, radius of gyration (It refers to the distribution of the component of an object around an axis) etc.

Hence the equivalent or accelerating weight of train is taken as We which is higher then the dead weight (W) [Weight of train] requiring for linear acceleration to be considered for the tractive effort of angular acceleration

So in practice We is higher than W

The normal value lies between 10 to 12%.

Hence the tractive effort required for acceleration

$$\boxed{F_a = 277.8\, W_e\, \alpha \text{ newtons}}$$

2] Tractive effort for overcoming the effect of Gravity (F_g)

When a train is on slope, a force of gravity equal to the component of the dead weight along the slope acts on the train and tends to cause its motion down the gradient or slope

When train moves on an up gradient the gravity component of dead weight parallel to the track

ie. $W \sin\theta$ will cause the train to come down

Hence force due to gradient

$$F_g = 1000\, W \sin\theta$$

Gradient is expressed in %

$$G = \sin\theta \times 100 \qquad \sin\theta = \frac{G}{100}$$

W = dead weight of train in tonne

θ = Angle of slope

G = Gradient which is represents in % G

$$F_g = 1000\, W \times \frac{G}{100}$$

$$= 10\, W G \times 9.81$$

$$= 98.1\, W G$$

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- When the train is going up a gradient, the tractive effort will be required to balance this force due to gradient
- But while going down the gradient, the force will add to the tractive effort

3] Tractive Effort for Overcoming Train resistance (F_r)

Train resistance consists of all the forces resisting the motion of a train when it is running at uniform speed on straight and level track.

Train resistance is due to

(i) the friction (It is a force that opposes the motion of an object) at various parts of the rolling stock.

(ii) friction at the track

(iii) Air resistance

Train resistance depends upon various factors such as shape, size and condition of track etc.. and is expressed in newtons per tonne of the dead weight

The general equation for train resistance is given as

$$R = K_1 + K_2 V + K_3 V^2$$

where K_1, K_2 and K_3 are constants depending upon the train and the track

R is the resistance

V is speed in Kmph

The first two terms represents mechanical resistance last term represents air resistance

Tractive effort required to overcome the train resistance

$$F_r = \underline{Wr}$$

Total Tractive Effort (F_t)

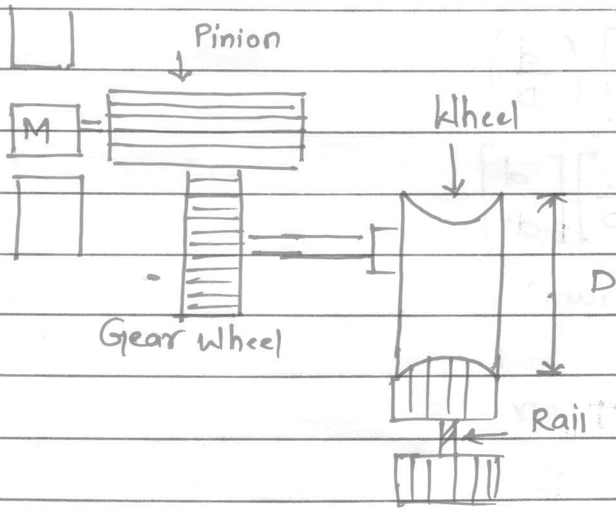
$$F_t = F_a \pm F_g + F_r$$

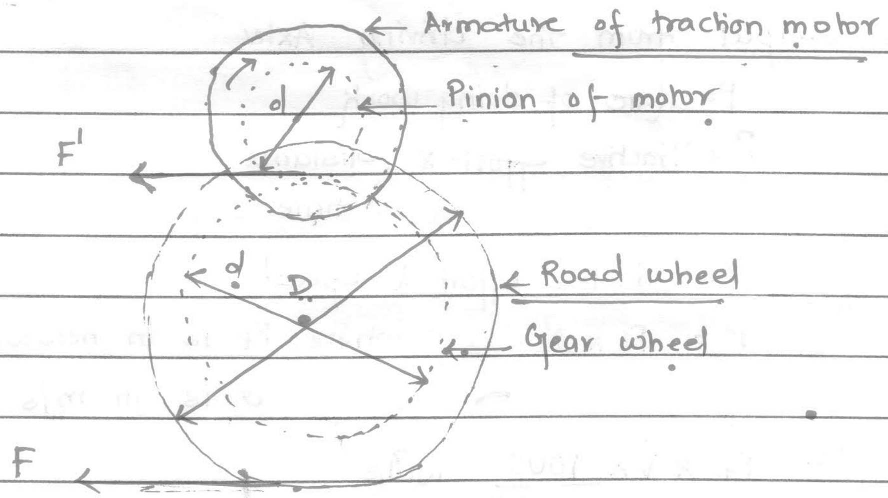
$$F_t = \underline{277.8\, W_e \alpha \pm 98.1 WG + Wr}$$

+ve sign for the motion up gradient

-ve sign for the motion down gradient

Mechanics of Train Movement





- Referring to the fig the motor armature is connected to the pinion whose diameter is d'
- The pinion transfer its motion to a general wheel with diameter d
- Let the torque developed by the motor is T Nm and tractive effort at the edge of pinion is F'

The relation between these two is given by

$$F = \eta F' \left(\frac{d}{D}\right)$$

η = Efficiency of transmission gear
d = Gear wheel diameter
D = Wheel diameter

Substituting F' we get

$$F = \eta \left[\frac{2T}{d'}\right]\left(\frac{d}{D}\right)$$

$$= \eta\, T \left[\frac{2}{D}\right]\left[\frac{d}{d'}\right]$$

$$\frac{d}{d'} = \gamma \text{ (Gear ratio)}$$

$$F = \eta\, T\, \frac{2\gamma}{D}$$

Power output from the driving Axles

P = Rate of doing work

P = Tractive effort x $\frac{\text{distance}}{\text{time}}$

P = Tractive effort x Speed.

$P = F_t \times v$. where F_t is in newtons

v is in m/s

$$P = F_t \times V \times \frac{1000}{3600} \text{ watts.}$$

$$= \frac{F_t \times V}{3600} \text{ KW}$$

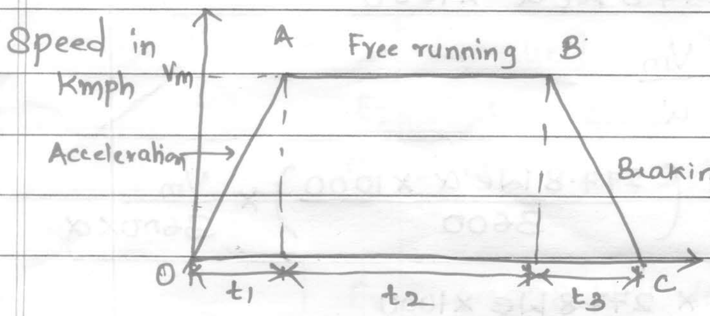
Energy Output from Driving Axles

Consider the train is running as per the trapeziodal speed time curve

S_1 = Distance travelled in km during acceleration

S_2 = Distance travelled in km during free running

S_3 = Distance travelled in km during braking



The total energy required for the train to run will be

E = [Energy required by the train for acceleration] + [Energy required by the train for overcoming gravity] + [Energy required by the train for overcoming train resistance]

$$E = E_a + E_g + E_r$$

① **Energy required by the train for acceleration (E_a)**

We know that

$$F_a = 277.8\, W_e\, \alpha$$

Where W_e is the effective weight in tonne

α is acceleration in kmphps

$$F_a = 277.8\, W_e\, \alpha \times 1000$$

Energy required during acceleration

E_a = (Avg power during acceleration) $\times$ (Time of acceleration)

$$Ea = \left(\frac{1}{2} Fa \times \frac{Vm}{3600} \right) \times \frac{t_1}{3600}$$

$$= \frac{Vm}{2} \times \frac{Fa}{3600} \times \frac{t_1}{3600}$$

Substituting the value of Fa

$$Fa = 277.8\, We\, \alpha \times 1000$$

and $t_1 = \frac{Vm}{\alpha}$

$$Ea = \frac{Vm}{2} \left(\frac{277.8\, We\, \alpha \times 1000}{3600} \right) \times \frac{Vm}{3600 \times \alpha}$$

$$= \frac{V_m^2 \times 277.8\, We \times 1000}{2 \times 3600 \times 3600}$$

$$\boxed{Ea = 0.01072\, V_m^2\, We} \quad -(1)$$

② Energy required by train to overcome gravity component (Eg)

We know that

$$Fg = 98.1\, WG$$

From which we can write

$$Eg = 98.1\, WG\, (S_1 + S_2)$$

where $S_1 + S_2$ is distance travelled during acceleration and free running with gradient % G

It is assumed that the train has to overcome gradient during acceleration and free running also therefore $S_1 + S_2$ is taken as

$$Eg = 98.1\, WG\, (S_1 + S_2) \times 1000$$

$$= 98.1\, WG\, (S_1 + S_2) \times \frac{1000}{3600} \text{ watt hr}$$

$$\boxed{Eg = 98.1\, WG\, (S_1 + S_2) \times 0.2778 \text{ watt hr}} \quad -(2)$$

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③ Energy required by train to overcome train resistance (E_r)

We know that expression

$$F_r = W r$$

from which we may write

$$E_r = W \cdot r\,(S_1 + S_2) \quad Nwkm$$

W = Dead weight of train in tonne

r = Train resistance in Newton/tonne

S_1 = Distance travelled during acceleration

S_2 = Distance travelled during free running.

$$E_r = W \cdot r\,(S_1 + S_2) \times 1000 \quad Nw \cdot km$$

$$E_r = W \cdot r\,(S_1 + S_2) \times \frac{1000}{3600}$$

$$\boxed{E_r = 0.2778\, W r\,(S_1 + S_2)} \quad \text{— (3)}$$

Total Energy output of driving axle

$$E = E_a + E_g + E_r$$

$$= [0.00172\, V_m^2 W_e] + [98.1 \times 0.2778\, W G\,(S_1 + S_2)] + [0.2778\, W r\,(S_1 + S_2)]$$

$$\boxed{E = 0.00172\, V_m^2 W_e + 0.2778\,[(98.1\, W G + W r)(S_1 + S_2)]}$$

Course Owner Name

Rokhaiya Banu

Banu