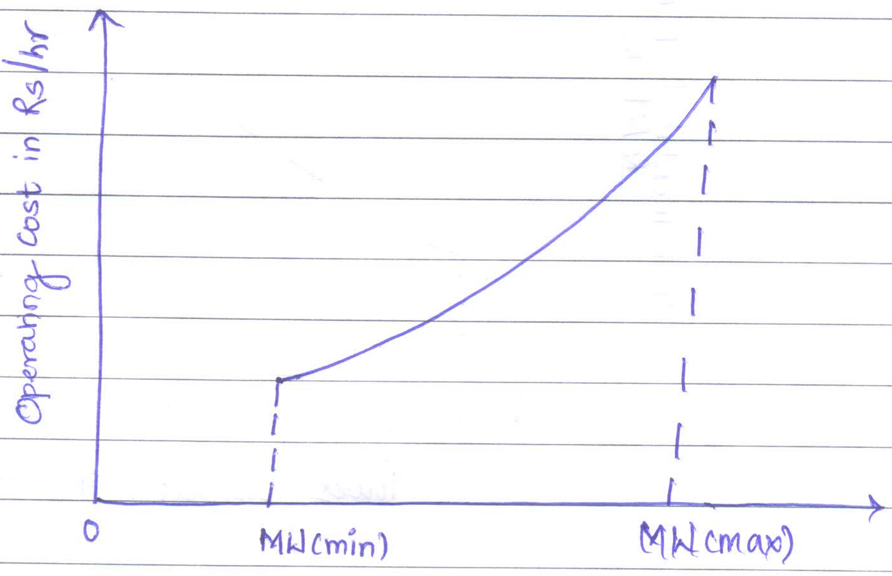
MODULE 2 : ECONOMIC SYSTEM OPERATION

- Generator operating cost : input - output
- Heat rate and IFC curve
- Constraints in operation
- Coordinate equation
- Exact coordinate equation
- Bmn coefficient Transmission loss formular
- Economic exchange of power between the areas
- Economic operation with limited fuel supply and shared generators.
- Optimal unit commitment
- Reliability considerations.

Generator operating cost : input - output.

- The major components of generating operating cost is the fuel input /hour, while maintenance contributes only to a small extent.
- The fuel cost plays an important role in case of thermal and nuclear stations
- The input - output curve of a unit (A unit consists of a boiler, turbine and generator) that can be expressed in a million kilocalories per hour.



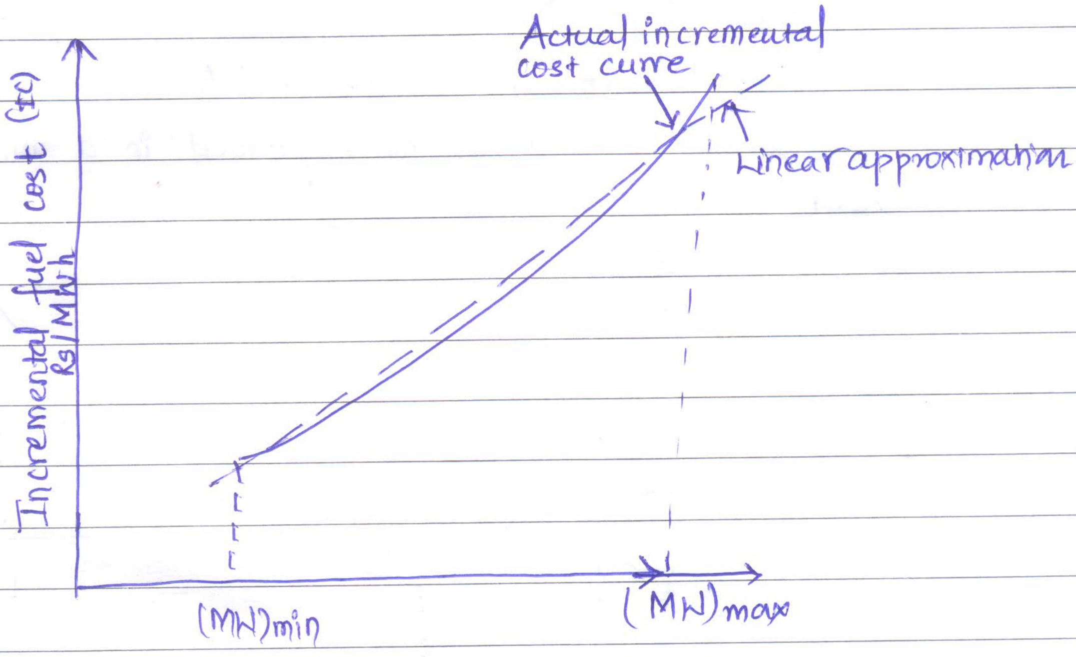
- A typical curve is as shown in figure: where (MW)min is the minimum loading limit below which it is uneconomical (or may be technically infeasible) to operate the unit and (MW) max is the minimum output limit
- The input-output curve has discontinuities at a steam value openings
- By fitting a suitable degree polynomial, an analytical expression for operating cost can be written as follows.

$C_i(P_{Gi})$ Rs/hour at output P_{Gi}

where i = number of units

$$C_i = \frac{1}{2} a_i P_{Gi}^2 + b_i P_{Gi} + d_i \text{ Rs/hour.}$$

The slope of the cost curve $\frac{dC_i}{dP_{Gi}}$ is called the incremental fuel cost (IC) and is expressed in units of (Rs/MWh)



- A typical plot of incremental fuel cost versus power output is shown in the given figure
- If the cost curve is approximated as a quadratic can be expressed equation we have an linear relationship

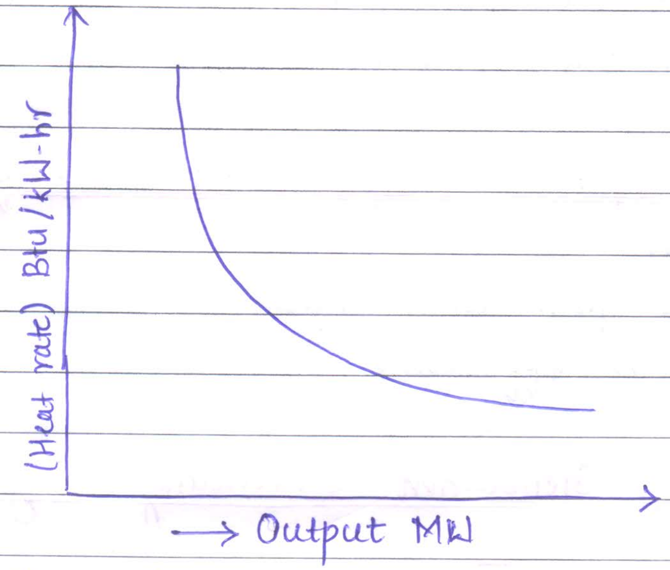
$$(IC)_i = a_i P_{Gi} + b_i$$

- For better accuracy incremental cost may be expressed by a number of short line segments which can be fitted by polynomial expression with a suitable degree to represent the incremental cost curve (IC)

$$P_{Gi} = \alpha_i + \beta_i (IC)_i + \gamma_i (IC)_i^2 \dots$$

Heat Rate curve

- The heat rate is the ratio of fuel input in Btu to energy output in KWh. It is the slope of the input-output curve at any point
- The reciprocal of heat rate is called fuel-efficiency.
- The heat rate curve is a plot of heat rate versus output in MW. A typical plot is shown in fig.



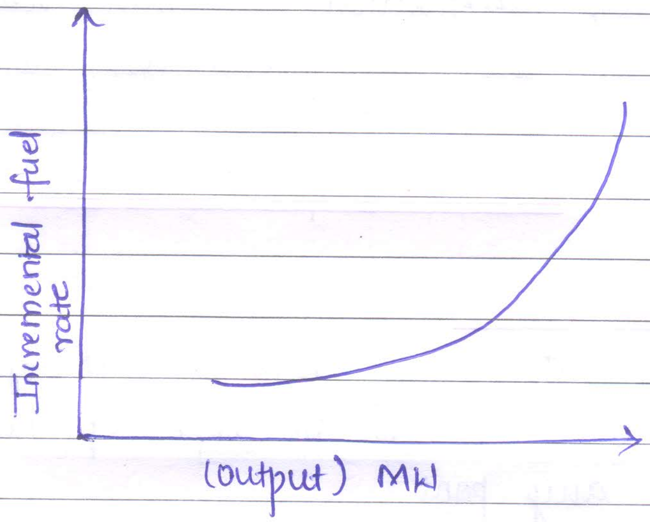
Heat Rate Curve.

Incremental Fuel Rate Curve.

- The incremental fuel rate is equal to a small change in input divided by the corresponding change in output

$$\text{Incremental fuel rate} = \frac{\Delta \text{Input}}{\Delta \text{Output}}$$

The unit is Btu/kWh. A plot of incremental fuel rate versus the ~~o/p~~ output is as shown in fig



Incremental fuel rate curve

~~Incremental Cost Curve~~

- ~~The incremental cost is the product of incremental fuel rate and fuel cost (Rs/Btu)~~

Cost of Operations depends upon the following aspects.

1) fuel Cost
2) Distance from load centre
3) Generation efficiency.

Economic Generation Scheduling Neglecting Losses and Generator Limits.

- Consider a system with (ng) number of generating plants supplying the total demand P_D
- If F_i is the cost of plant i in Rs/h, the mathematical formulation of economic scheduling can be stated as follows

$$\text{Minimize} \quad F_T = \sum_{i=1}^{ng} F_i \quad \text{—— (1)}$$

$$\text{such that} \quad \sum_{i=1}^{ng} P_{Gi} = P_D \quad \text{—— (2)}$$

where F_T = Total cost
P_{Gi} = Generation of plant i
P_D = Total demand

This is a constrained optimization problem, which can be solved by Lagrange's method.

Constrained Parameter Optimization

Consider the following optimization problem:

$$\text{Minimize} \quad f(x_1, x_2, \cdots x_n) \quad \text{—— (3)}$$

$$\text{such that} \quad \left. \begin{array}{l} g_j(x_1, x_2, \cdots x_n) = 0 \\ j = 1 \cdots k \end{array} \right\} \text{—— (4)}$$

- Here $x_1, x_2, \cdots x_n$ are the variables and g_j is the equality constraint and f is the cost function.
- In the Lagrange multiplier method, the cost function is augmented by a k-vector λ of undetermined quantities
- The augmented, unconstrained cost function is given by

$$L = f(x_1, x_2, \cdots x_n) + \sum_{j=1}^{k} g_j \lambda_j \quad \text{—— (5)}$$

The necessary condition for local minima of L are the following

$$\frac{\partial L}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{j=1}^{k} \lambda_j \frac{\partial g_i}{\partial x_i} = 0 \; ; \quad i = 1 \cdots n \quad \text{—— (6)}$$

$$\frac{\partial L}{\partial \lambda_j} = g_j = 0 \quad \text{—— (7).}$$

From the constrained parameter optimization the solution for eqn (1) subject to constraint eqn (2) can be easily applied.

Minimize $F_T = \sum_{i=1}^{ng} F_i$

such that $P_D - \sum_{i=1}^{ng} P_{Gi} = 0.$

The augmented cost function is given by

$$L = F_T + \lambda \left(P_D - \sum_{i=1}^{ng} P_{Gi} \right) \quad \text{—— (8)}$$

The minimum is obtained when

$$\frac{\partial L}{\partial P_{Gi}} = 0 \quad \text{and} \quad \frac{\partial L}{\partial \lambda} = 0$$

From eq^n (8) $$\frac{\partial L}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0 \quad \text{—— (9)}$$

$$\frac{\partial L}{\partial \lambda} = P_D - \sum_{i=1}^{ng} P_{Gi} = 0 \quad \text{—— (10)}$$

The cost of plant F_i depends only on its own output P_{Gi}, we have

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} \quad \text{—— (11)}$$

Using eq^n (11) relation, Then the eq^n (9) can be written as follows

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda_i \qquad i = 1 \ldots ng. \quad \text{—— (12)}$$

Since we know that incremental cost function

6 is given as

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad \text{—— (13)}$$

$\therefore \quad b_i + 2c_i P_{Gi} = \lambda_i \qquad i = 1 \cdots ng.$ —(14)

The above equation is called as the co-ordination equation

- For economic generation scheduling to meet a particular load demand when transmission losses are neglected and generation limits are not imposed all plants must operate at equal incremental production cost, subject to the constraint that the total generation be equal to demand.

From eqn (14) we get

$$P_{Gi} = \frac{\lambda - b_i}{2c_i} \quad \text{—(15)}$$

We know that when the system is without losses then

$$\sum_{i=1}^{ng} P_{Gi} = P_D \quad \text{—(16)}$$

Substituting the value of P_{Gi} in eqn (16)

$$\sum_{i=1}^{ng} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of λ can be obtained as follows.

$$\boxed{\lambda = \frac{P_D + \sum_{i=1}^{ng} \frac{b_i}{2c_i}}{\sum_{i=1}^{ng} \frac{1}{2c_i}}}$$

It can be seen that λ is dependent on the demand and coefficient of the cost function.

Economic Dispatch inculding Transmission losses

- The transmission losses are a significant part of generation and have to be considered in the generation schedule for economic operation

The mathematical formulation can be stated as

$$\text{Minimize} \quad F_T = \sum_{i=1}^{ng} F_i \quad -(1)$$

Such that

$$\sum_{i=1}^{ng} P_{Gi} = P_D + P_L \quad -(2)$$

ng = number of generating plants.

where, P_L = Total loss, P_{Gi} = Generation of i^{th} plant

P_D = sum of load demand at all buses

The Lagrange function can be written as

$$\mathcal{L} = F_T - \lambda \left[\sum_{i=1}^{ng} P_{Gi} - P_D - P_L \right] = 0 \quad -(3)$$

The minimum or optimum solution can be obtained when

$$\frac{\partial \mathcal{L}}{\partial P_{Gi}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda \left[1 - \frac{\partial P_L}{\partial P_{Gi}} \right] = 0 \quad -(4)$$

$$i = 1, \ldots ng.$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i=1}^{ng} P_{Gi} - P_D - P_L = 0 \quad -(5)$$

Since

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} \quad -(6)$$

Equation (4) can be written as

$$\frac{dF_i}{dP_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda \quad -(7).$$

$$\boxed{\lambda = \frac{dF_i}{dP_{Gi}} \left[\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} \right]} \quad \text{— (8).}$$

The term $\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}$ is called penalty factor of plant i, L_i

The coordination equation including losses are given by

$$\boxed{\lambda = \frac{dF_i}{dP_{Gi}} L_i \quad i = 1, \ldots ng.}$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same when losses are considered.

Equation (8) can also be written as

$$\boxed{(IC_i) = \lambda [1 - (ITL_i)]} \quad i = 1, 2, \ldots ng. \quad \text{— (9)}$$

The above equation is referred as the exact coordination equation.

Derivation of Transmission Loss Formula (B-coefficient).

- A precise method of obtaining a general formula for transmission loss has been given by Kron which is however quite complicated
- The aim of deriving the transmission loss formula is to make it simpler by considering certain assumptions
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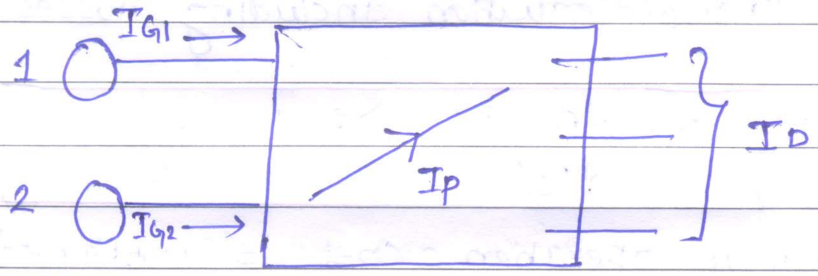
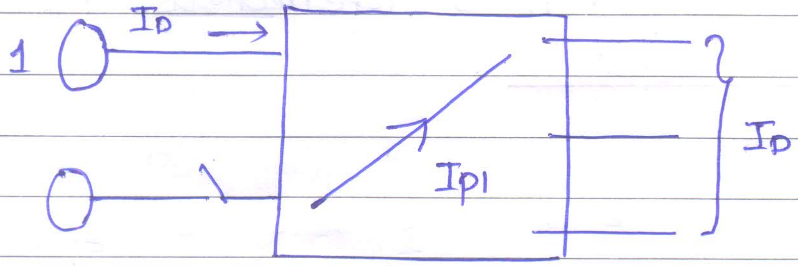


fig (a)

- Two generating plants connected to an arbitary load through a transmission network.
- One line is within the network is designated as branch p.

- Let us assume that the load current I_D is supplied to plant 1 as shown in fig (b)
- let the current in line p be I_{p1}



Then the Current distribution factor

$$M_{p1} = \frac{I_{p1}}{I_D} \quad \text{— (1)}$$

- Similarly for plant 2 is alone supplying the total load current we may define the current distribution factor for plant 2 as follows.

$$M_{p2} = \frac{I_{p2}}{I_d} \quad -(2)$$

- The values of current distribution factor depends upon the impedances of the lines, their interconnection and they are independent of the current I_D

- When both generators 1 and 2 are supplying current into the network as shown in fig (a)
Then by applying the principle of superposition the current in the line p can be expressed as

$$\boxed{I_p = M_{p1} I_{G1} + M_{p2} I_{G2}} \quad -(3)$$

where I_{G1} and I_{G2} are the currents supplied by the plant 1 and 2 respectively.

- For simplication we obtain certain assumptions which are outlined below.

(1) All load currents have some phase angle with respect to common refrence

$$|I_{Di}| \angle(\delta_i - \phi_i) = |I_{Di}| \angle \theta_i$$

where δ_i - phase angle of bus voltage
ϕ_i - lagging phase angle of the load

Since δ_i & ϕ_i vary with a narrow range at a various buses
where as θ_i remains same for all load currents

(2) Ratio of X/R is same for all network branches

These two assumptions lead us to conclusion that

- I_{p1} and I_0 from fig (b) and I_{p2} and I_0 from fig (c) have the ~~ph~~ same phase angle

- Such that the current distribution factors M_{p1} and M_{p2} are real rather than complex

let $I_{G1} = |I_{G1}| \angle \sigma_1$

$I_{G2} = |I_{G2}| \angle \sigma_2$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} respectively with respect to the common refrence.

From Eqn (3) we can write.

$$|I_p|^2 = (M_{p1}|I_{G1}|\cos\sigma_1 + M_{p2}|I_{G2}|\cos\sigma_2)^2 + (M_{p1}|I_{G1}|\sin\sigma_1 + M_{p2}|I_{G2}|\sin\sigma_2)^2 \quad \text{—(4)}$$

Expanding and simplifying the above Eqn.

$$|I_p|^2 = M_{p1}^2|I_{G1}|^2 + M_{p2}^2|I_{G2}|^2 + 2M_{p1}M_{p2}|I_{G1}||I_{G2}|\cos(\sigma_1-\sigma_2) \quad \text{—(5)}.$$

$$|I_{G1}| = \frac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1} \quad \text{—(6)} \qquad |I_{G2}| = \frac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2} \quad \text{—(7)}$$

- where P_{G1} and P_{G2} are three phase real power outputs of plant 1 and 2 at power factors of $\cos\phi_1$ and $\cos\phi_2$

- V_1 and V_2 are the bus voltages at the both plants.

If R_p is the resistance of branch p, the total transmission loss is given by

$$P_L = \cancel{\sum_p} \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2} \sum_p M_{p1}^2 R_p + \cdots$$

$+$

$$\boxed{\lambda = \frac{dF_i}{dP_{Gi}} \left[\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}} \right]} \quad \text{--- (8).}$$

The term $\frac{1}{1 - \frac{\partial P_L}{\partial P_{Gi}}}$ is called penalty factor of plant i, L_i

The coordination equation including losses are given by

$$\boxed{\lambda = \frac{dF_i}{dP_{Gi}} L_i \quad i = 1, \ldots ng.}$$

The minimum operation cost is obtained when the product of the incremental fuel cost and the penalty factor of all units is the same when losses are considered.

Equation (8) can also be written as

$$\boxed{(IC_i) = \lambda \left[1 - (ITL_i) \right]} \quad i = 1, 2, \ldots ng. \quad \text{--- (9)}$$

The above equation is referred as the exact coordination equation.

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_p M_{p1}^2 R_p + \cdots + \frac{P_{Gng}^2}{|V_k|^2(\cos\phi_k)^2}\sum_p M_{png}^2 R_p + \cdots$$

$$+2\sum_{m,n=1}^{ng}\left\{\frac{P_{Gm}P_{Gn}\cos(\sigma_m-\sigma_n)}{|V_m||V_n|\cos\phi_m\cos\phi_n}\sum_p M_{pm}M_{pn}R_p\right\} \quad \text{— (8)}$$

$$P_L = \sum_p 3|I_p|^2 R_p \quad \text{— (9)}$$

Substituting for $|I_p|^2$ from eqn (5) and $|I_{G1}|$ and $|I_{G2}|$ from eq (6) & (7) we obtain

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_p M_{p1}^2 R_p + \frac{2P_{G1}P_{G2}\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_p M_{p1}M_{p2}R_p + \frac{P_{G2}^2}{|V_2|^2(\cos\phi_2)^2}\sum_p M_{p2}^2 R_p \quad \text{— (10)}$$

From eq (10) it can be seen that

$$\boxed{P_L = P_{G1}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}^2 B_{22}}$$

where

$$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_p M_{p1}^2 R_p$$

$$B_{12} = \frac{\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_p M_{p1}M_{p2}R_p$$

$$B_{22} = \frac{1}{|V_2|^2(\cos\phi_2)^2}\sum_p M_{p2}^2 R_p$$

- The terms B_{11}, B_{12}, and B_{22} are called loss coefficients or B-coefficients
- If voltages are line to line kV with resistance in ohms the unit of B-coefficient are in MW^{-1}
  P_{G1} and P_{G2} will be expressed in MW
  P_L will be expressed in MW

The above results can be expressed for the general case considering ng = number of generating plants with transmission loss can be expressed as

$$\boxed{P_L = \sum_{m=1}^{ng} \sum_{n=1}^{ng} P_{Gm} B_{mn} P_{Gn}}$$

where $B_{mn} = \dfrac{\cos(\sigma_m - \sigma_n)}{|V_m||V_n| \cos\phi_m \cos\phi_n} \sum_{p} M_{pm} M_{pn} R_p$

$$P_L = P_{G1}^2 B_{11} + \cdots + P_{GK}^2 B_{KK} + 2 \sum_{\substack{m,n=1 \\ m \neq n}}^{ng} P_{Gm} B_{mn} P_{Gn}$$

If B-coefficients are to be treated as constants as total load and load sharing between plants vary.

These assumptions are as follows:

1. All load currents maintain a constant ratio to the total current.
2. Voltage magnitude at all plants remain constant
3. Ratio of reactive and real power i.e. the power factor at each plant remains constant
4. Voltage phase angles at plant buses remains fixed.