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SCHOOL OF ENGINEERING & TECHNOLOGY
SCHOOL OF PHARMACY
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Laplase Transform

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AIKTC SCHOOL OF ENGINEERING

Use of Laplace Transforms

- Find solution to differential equation using algebra
- Relationship to Fourier Transform allows easy way to characterize systems
- Useful with multiple processes in systems Analysis

What we study

- Obtain Laplace transform of f(t) using standerd results &properties
- Obtain Inverse Laplace transforms of f(s) using i) properties ii) convolution Theorem iii) partial fractions
- To solve Differential equations using Laplace Transforms

We Define Laplace Transform as

- ‘t’ is real and ‘s’ is complex.

$$L\{F(t)\} = \int_0^{\infty} e^{-st} F(t)\,dt$$

- The answer if this integral exists is Laplace transform and is denoted by f(s)
- F(t) is inverse Laplace transform of f(s).

Some Standard formulae

- These results can be obtained by direct integration

L { 1} = $\int_0^\infty e^{-st}$ 1 dt = 1/s

L { e^{at}} = $\int_0^\infty e^{-st}$ e^{at} dt = 1/s-a

L { sin at} = $\int_0^\infty e^{-st}$ sin at dt = $\frac{a}{s^2+a^2}$

L { cos at} = $\int_0^\infty e^{-st}$ cos at dt = $\frac{s}{s^2+a^2}$

L { t^n} = $\int_0^\infty e^{-st}$ t^n dt = $\frac{\Gamma n+1}{s^{n+1}}$

Properties of Laplace Transforms

- Linearity property : L[A f(t) + B g(t)] = A f(s)+B g(s)
- Multiplication by 't' property :L[t F(t)] = - d/ds f(s)
- Division by 't' property: L[F(t)/t] =
- $$\int_{s}^{\infty} f(s)ds$$

Properties of Laplace Transforms

- Shifting Property : L[e^at F(t)] = f(s-a)
- Change of scale property: L[F(at)] = 1/a f(s/a)
- L[d/dt (F(t))] = s f(s) – F(0)
- L[∫F(t) dt] from 0 to t = f(s)/s

Some Examples

- How to apply properties to obtain L.T.

$L[e^{at} \cos 3t] = \frac{s-a}{(s-a)^2+3^2}$

L[d/dt sin t] = s { 1/(s^2 +1)} – sin0

$L\,[\int_0^t e^{3t} \, cost \, dt\,] = \frac{1}{s}\left(\frac{(s-3)}{(s-3)^2+1}\right)$

$L[\,t \cos 2t] = -\frac{d}{ds}\left\{\frac{s}{s^2+4}\right\} = \frac{s^2-4}{(s^2+4)^2}$

$L[\frac{sint}{t}] = \int_s^{\infty} \frac{1}{s^2+1} ds = \frac{\pi}{2} - \tan^{-1} s$

Inverse Laplace Transform

- Using convolution Theorem-

$$L^{-1}[f(s)*g(s)] = \int_0^t F(u)G(t-u)du$$

$$L^{-1}\left[\frac{1}{s^2+1}*\frac{s}{s^2+4}\right] = \int_0^t \cos 2u \; \sin(t-u)du$$

Inverse Laplace Transforms

- Using Partial fractions
- Put the function into partial fractions and use properties to write inverse Laplace transforms.
- To solve a differential equation take Laplace transform of both sides and the equation becomes a linear equation. Find y(s) and take inverse Laplace transform to get y(t).

THANK YOU