



ANJUMAN-I-ISLAM'S KALSEKAR TECHNICAL CAMPUS NEW PANVEL

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Laplase Transform Rhuta Mahajan, Asst. Professor

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Use of Laplace Transforms

- Find solution to differential equation using algebra
- Relationship to Fourier Transform allows easy way to characterize systems
- Useful with multiple processes in systems Analysis

What we study

- Obtain Laplace transform of f(t) using standerd results & properties
- Obtain Inverse Laplace transforms of f(s) using i) properties ii) convolution Theorem iii) partial fractions
- To solve Differential equations using Laplace Transforms

We Define Laplace Transform as

- 't' is real and 's' is complex. $L\{F(t)\} = \int_0^\infty e^{-st} F(t) dt$
- The answer if this integral exists is Laplace transform and is denoted by f(s)
- F(t) is inverse Laplace transform of f(s).

Some Standard formulae

These results can be obtained by direct integration

L { 1} =
$$\int_0^\infty e^{-st} 1 \, dt = 1/s$$

L { e^{at} } = $\int_0^\infty e^{-st} e^{at} \, dt = 1/s$ -a

L { sin at} = $\int_0^\infty e^{-st}$ sin at dt = $\frac{a}{s^2 + a^2}$

L { cos at} =
$$\int_0^\infty e^{-st} \cos at dt = \frac{s}{s^2 + a^2}$$

$$L \{ t^n \} = \int_0^\infty e^{-st} t^n dt = \frac{\Gamma n + 1}{s^{n+1}}$$

Properties of Laplace Transforms

- Linearity property : L[A f(t) + B g(t)] = A f(s)+B g(s)
- Multiplication by 't' property :L[t F(t)] = - d/ds f(s)

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f(s)ds

• Division by 't' property: L[F(t)/t] =

Properties of Laplace Transforms

- Shifting Property : L[e^at F(t)] = f(s-a)
- Change of scale property: L[F(at)] = 1/a f(s/a)
- L[d/dt(F(t))] = sf(s) F(o)
- L[$\int F(t) dt$] from 0 to t = f(s)/s

Some Examples

 How to apply properties to obtain L.T. $L[e^{at} \cos 3t] = \frac{s-a}{(s-a)^2+3^2}$ $L[d/dt sin t] = s \{ 1/(s^2 + 1) \} - sin 0$ $L\left[\int_{0}^{t} e^{3t} \cos t dt\right] = \frac{1}{s} \left(\frac{(s-3)}{(s-3)^{2}+1}\right)$ L[t cos 2t] = $-\frac{d}{ds} \{\frac{s}{s^2+4}\} = \frac{s^2-4}{(s^2+4)^2}$ $L[\frac{sint}{t}] = \int_{s}^{\infty} \frac{1}{s^{2}+1} ds = \frac{\pi}{2} - \tan^{-1}s$

Inverse Laplace Transform

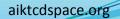
Using convolution Theorem-

 $L^{-1} [f(s)^*g(s)] = \int_0^t F(u)G(t-u)du$ $L^{-1} [\frac{1}{s^2+1}^* \frac{s}{s^2+4}] = \int_0^t \cos 2u \, \sin(t-u)du$

Inverse Laplace Transforms

- Using Partial fractions
- Put the function into partial fractions and use properties to write inverse Laplace transforms.
- To solve a differential equation take Laplace transform of both sides and the equation becomes a linear equation. Find y(s) and take inverse Laplace transform to get y(t).







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