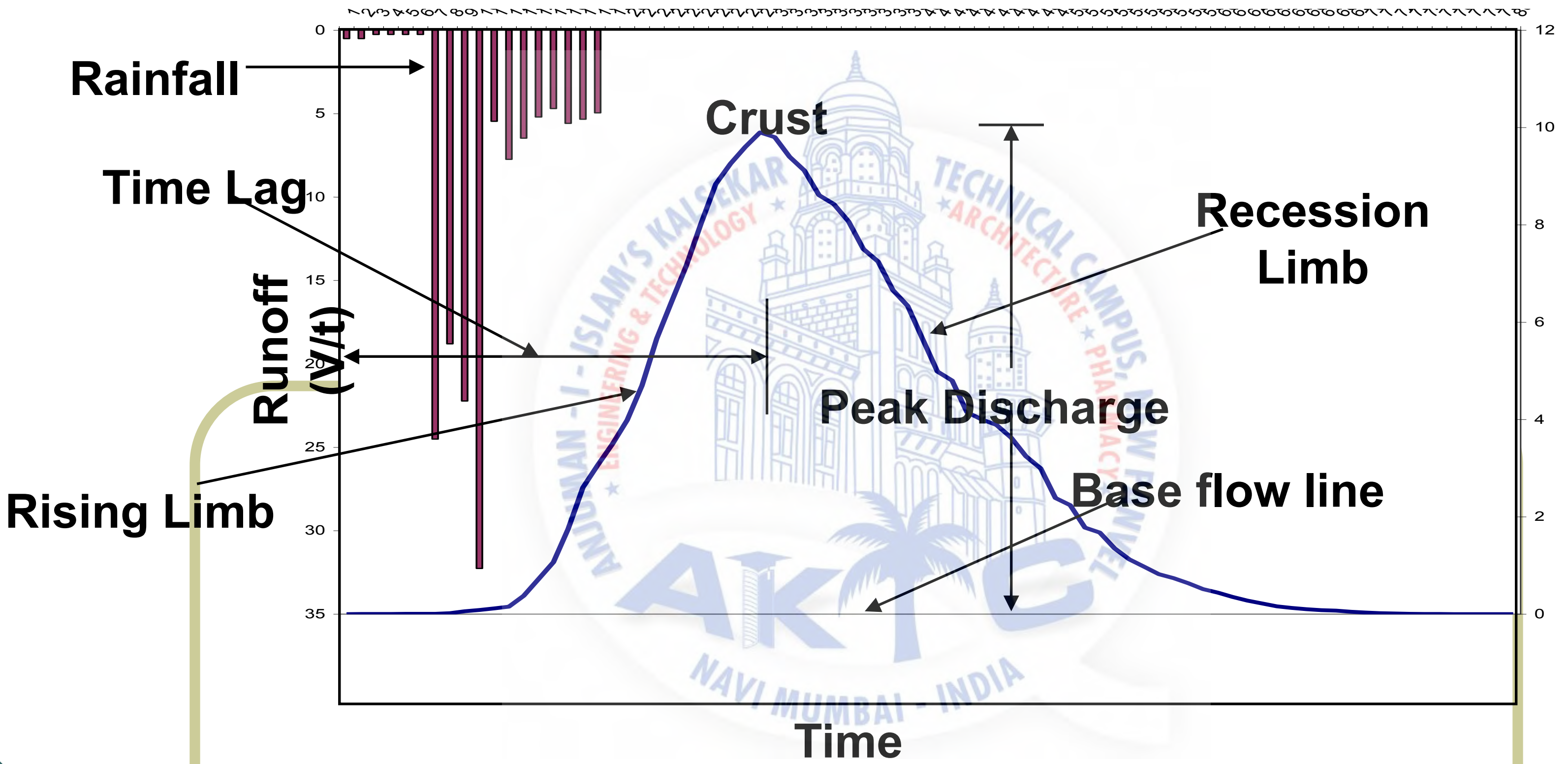
HYDROGRAPHS

B E CIVIL :

Subject; Irrigation Engineering and AHFC

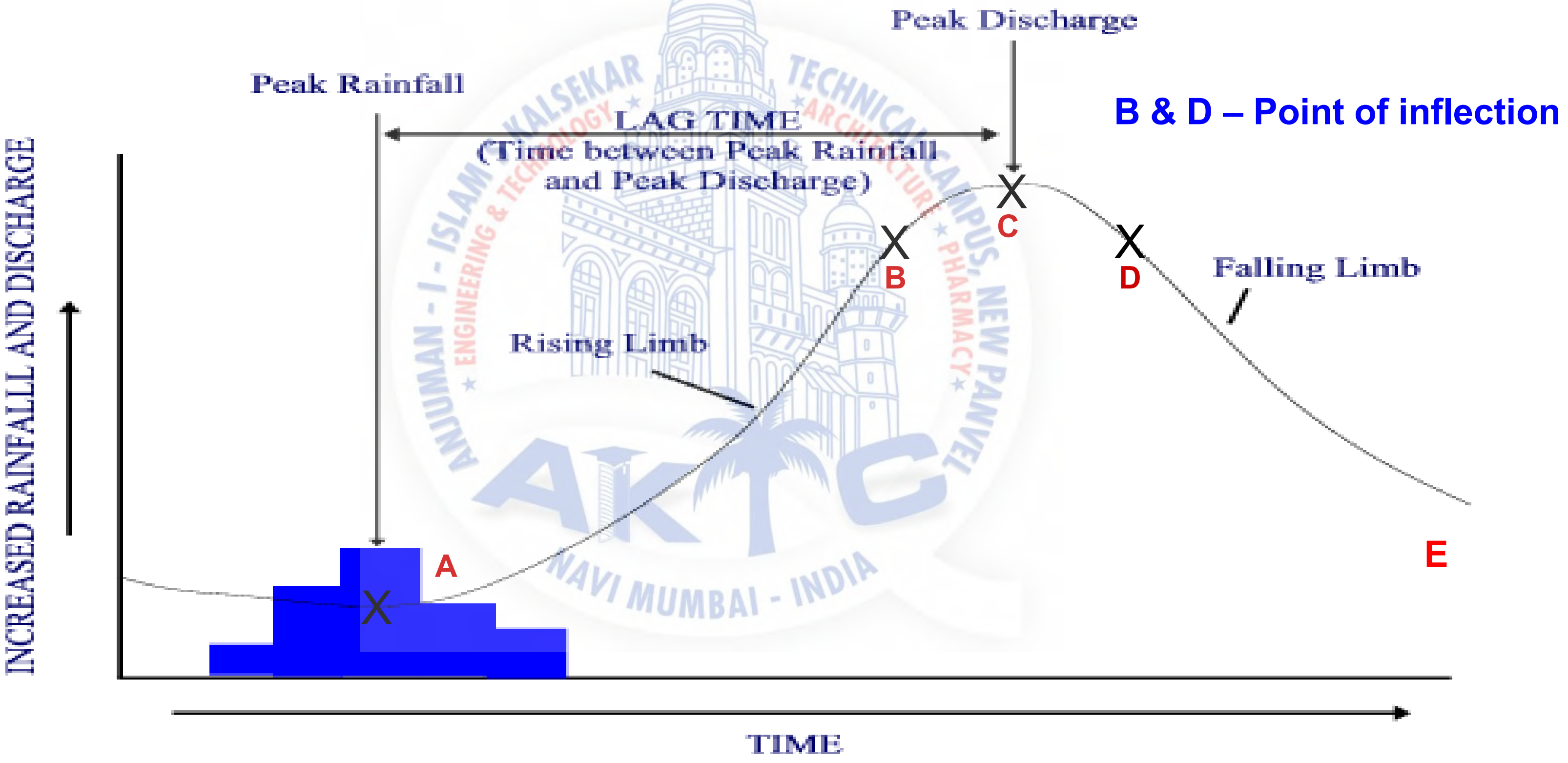
Hydrographs



Introduction

- Rainfall after initial losses and infiltration losses are met, reaches the stream as runoff.
- There is time lag between the occurrence of rainfall and the time, when it passes the gauging station.
- Hydrograph due to isolated storm is typically single peaked with skewed distribution of discharge.
- Commonly known as storm hydrographs, flood hydrographs or simply hydrograph.
- Hydrograph is the response of a given catchment to a rainfall input.
- It consists of all three phases of runoff viz.
    - Surface runoff
    - Interflow
    - Base flow

Elements of flood hydrograph



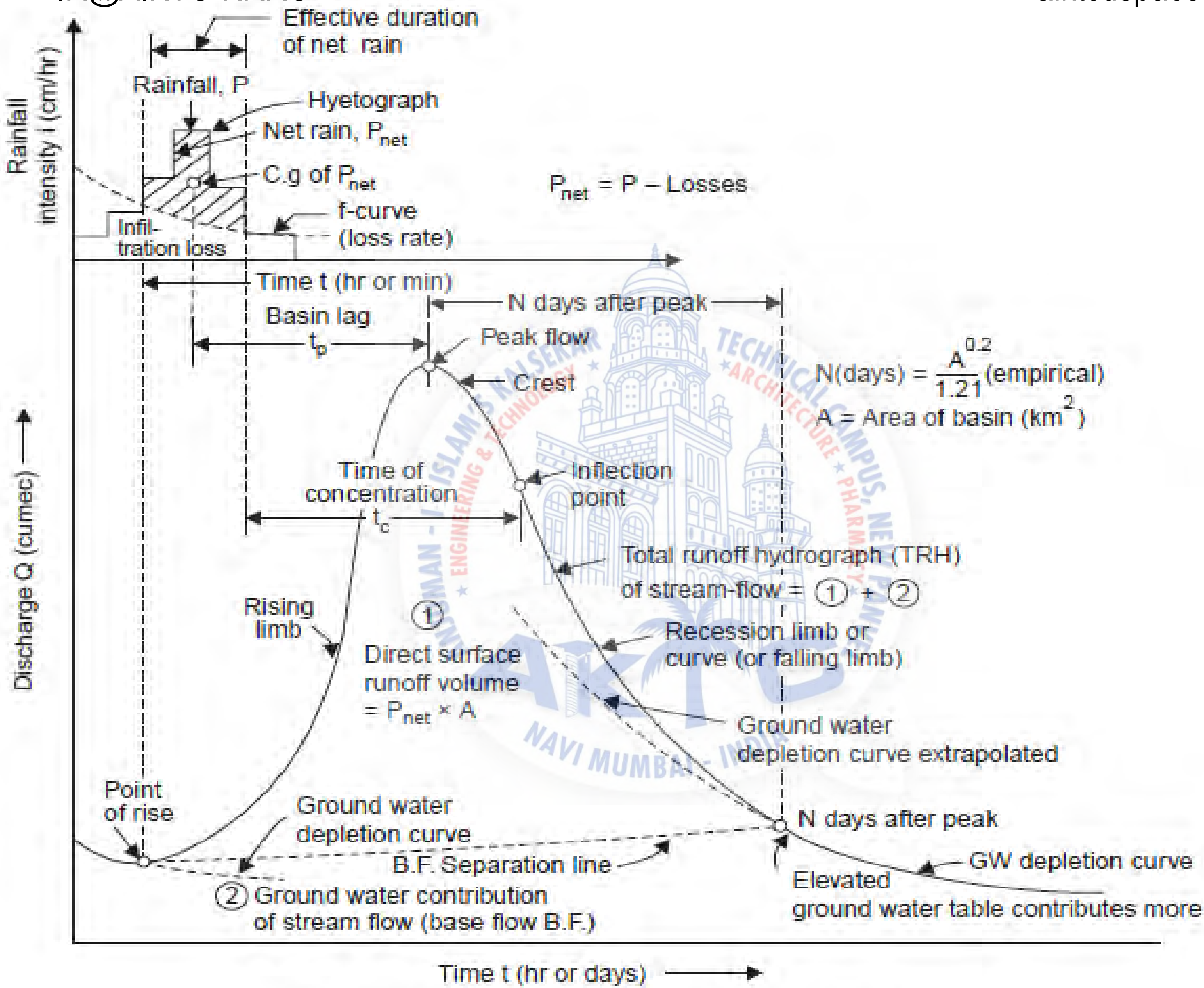


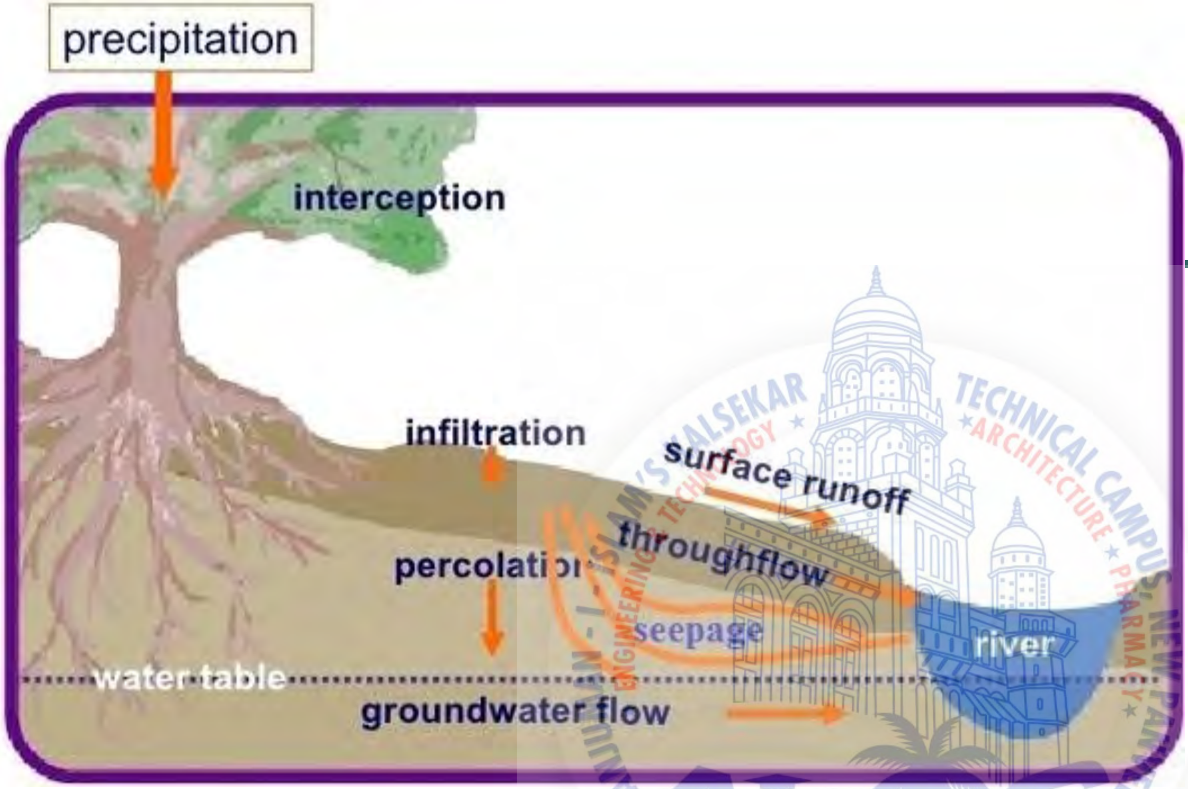
Fig. 5.1 Components of streamflow hydrograph

INFILTRATION

Water entering the soil at the ground surface is called infiltration. It replenishes the soil moisture deficiency and the excess moves downward by the force of gravity called deep seepage or percolation and builds up the ground water table.

The maximum rate at which the soil in any given condition is capable of absorbing water is called its infiltration capacity (*fp*).

Infiltration
(*f*) *often begins at a high rate (20 to 25 cm/hr) and decreases to a fairly steady state rate (fc) as* the rain continues, called the ultimate *fp (= 1.25 to 2.0 cm/hr) (F*


precipitation
interception
infiltration
surface runoff
throughflow
percolation
seepage
river
water table
groundwater flow

The infiltration rate (*f*) at any time *t is given by Horton's equation.*

$$f = f_c + (f_o - f_c)\, e^{-kt}$$

$$k = \frac{f_0 - f_c}{F_c}$$

where *f0 = initial rate of infiltration capacity*
fc = final constant rate of infiltration at saturation
k = a constant depending primarily upon soil and vegetation
e = base of the Napierian logarithm
Fc = shaded area in Fig. 3.6
t = time from beginning of the storm

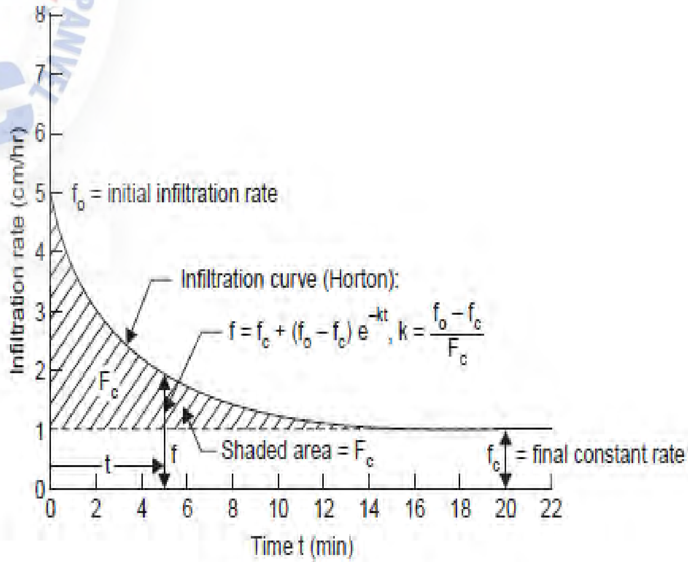


Fig. 3.6 Infiltration Curve (Horton)

INFILTRATION INDICES

Estimates of runoff volume from large areas are sometimes made by the use of infiltration indices, which assume a constant average infiltration rate during a storm, although in actual practice the infiltration will be varying with time.

This is also due to different states of wetness of the soil after the commencement of the rainfall.

(i) φ-index (ii) W-index

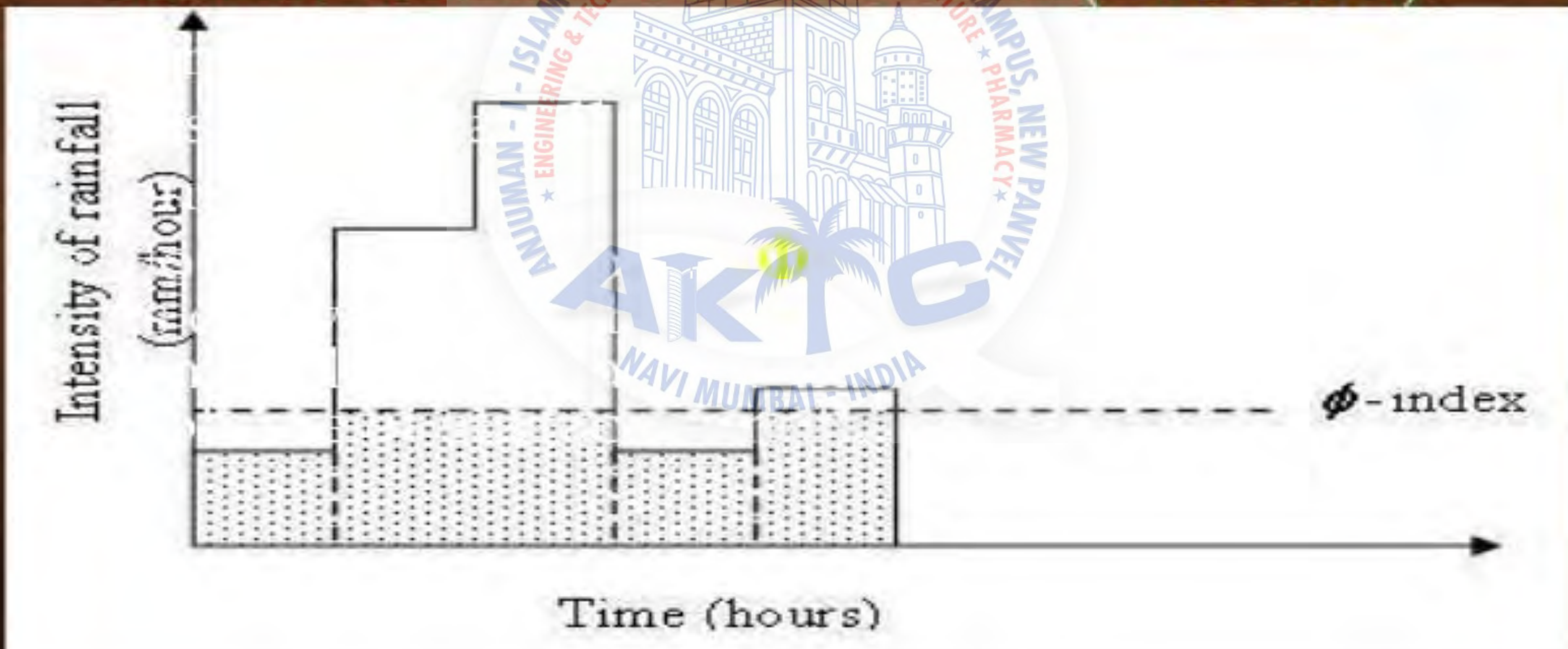
INFILTRATION INDICES

- For consistency in hydrological calculations, a constant value of infiltration rate for the entire storm duration is adopted. The average infiltration rate is called the **INFILTRATION INDEX**.
- The two commonly used infiltration indices are the following:
  - *φ – index*
  - *W – index*

There are extremely used for the analysis of major floods when the soil is wet and the infiltration rate becomes constant.

Φ – INDEX

- This is defined as the rate of infiltration above which rainfall volume = runoff volume(saturation).



- *Φ – INDEX for a catchment, during a storm depends on*

- Soil type
- vegetation cover
- Initial moisture condition

- *Application – Estimation of flood magnitudes due to critical storms.*

W – INDEX

- This is the average infiltration rate during the time when the rainfall intensity > infiltration rate.

$$W\text{-index} = (P - R - I_a)/t_f = (F/t_f)$$

where P = Total storm precipitation (cm)

R = Total surface runoff (cm)

I_a = Depression and interception losses (cm)

t_f = Time period of runoff (in hours)

- The w- index is more accurate than Φ – index because it excludes the *Depression* & *interception*.

INTERCEPTION : it is a part of water caught by the vegetation and subsequently evaporated as

a) Surface flow
b) Stem flow
c) Evapotranspiration

For a given storm, the interception loss is estimated as

$$I_i = S_i + K_i E_t$$

Where

- I_i = Interception loss in mm.
- S_i = Interception storage varies from 0.25 to 1.25 mm depending on the nature of vegetation
- K_i =Ratio of vegetal surface area to its projected area.
- E_t = Evaporation rate in mm/h during the precipitation.
- t = Duration of rainfall in hours.

- W-index is the refined version of Φ – INDEX.
- Initial losses I_a are separated from total abstractions.
- W-index = Φ–index – I_a
- The accurate estimation of W-index is rather difficult to obtain hence Φ – index is most commonly used.
- Since retention rate is very low both index W & Φ are almost same.

φ-*index*—*The φ-index is defined as that rate of rainfall above which the rainfall* volume equals the runoff volume. The φ-index is relatively simple and all losses due to infiltration, interception and depression storage (*i.e., storage in pits and ponds) are accounted for;*

hence,

$$\phi\text{-index} = \frac{\text{basin recharge}}{\text{duration of rainfall}}$$

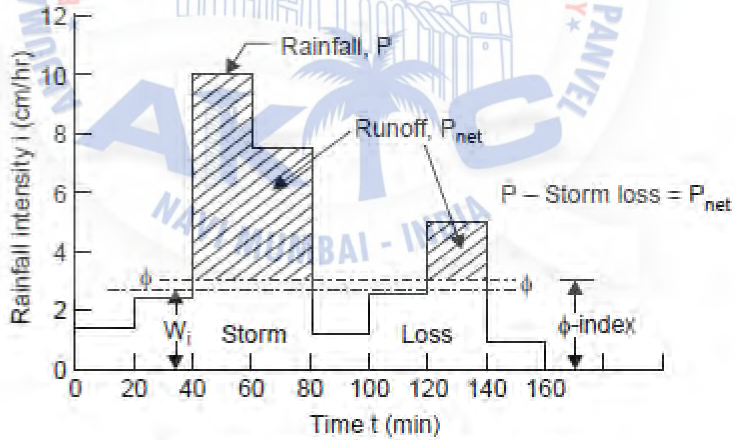


Fig. 3.12 Infiltration loss by φ-index

(*ii*) *W-index*—The *W*-index is the average infiltration rate during the time rainfall intensity exceeds the infiltration capacity rate, *i.e.*,

$$W\text{-index} = \frac{F_p}{t_R} = \frac{P - Q - S}{t_R} \qquad \text{...(3.15)}$$

where P = total rainfall

Q = surface runoff

S = effective surface retention

t_R = duration of storm during which $i > f_p$

F_p = total infiltration

The *W*-index attempts to allow for depression storage, short rainless periods during a storm and eliminates all rain periods during which $i < f_p$. Thus, the *W*-index is essentially equal to the ϕ-index minus the average rate of retention by interception and depression storage, *i.e.*, $W < \phi$.

Example 3.6 ***The rates of rainfall for the sucessive 30 min period of a 3-hour storm are:***

1.6, 3.6, 5.0, 2.8, 2.2, 1.0 cm/hr. The corresponding surface runoff is estimated to be 3.6 cm. Establish the φ-index. Also determine the W-index.

Solution Construct the hyetograph as shown in Fig. 3.13 (*a*)

$$\Sigma(i - \phi)t = P_{net}, \quad \text{and thus it follows}$$

$$[(3.6 - \phi) + (5.0 - \phi) + (2.8 - \phi) + (2.2 - \phi)]\,\frac{30}{60} = 3.6$$

$$\therefore \qquad \phi = \mathbf{1.6\ cm/hr}$$

$$P = (1.6 + 3.6 + 5.0 + 2.8 + 2.2 + 1.0)\,\frac{30}{60} = \mathbf{8.1\ cm}$$

$$W\text{-}index = \frac{P - Q}{t_R} = \frac{8.1 - 3.6}{3} = \mathbf{1.5\ cm/hr}$$

Suppose the same 3-hour storm had a different pattern as shown in Fig. 3.13 (*b*) producing the same total rainfall of 8.1 cm. To obtain the same runoff of 3.6 cm (shaded area), the φ-index can be worked out as 1.82 cm/hr. Hence, it may be seen that a single determination of φ-index is of limited value and many such determinations have to be made and averaged, before the index is used. The determination of φ-index for a catchment is a trial and error procedure.

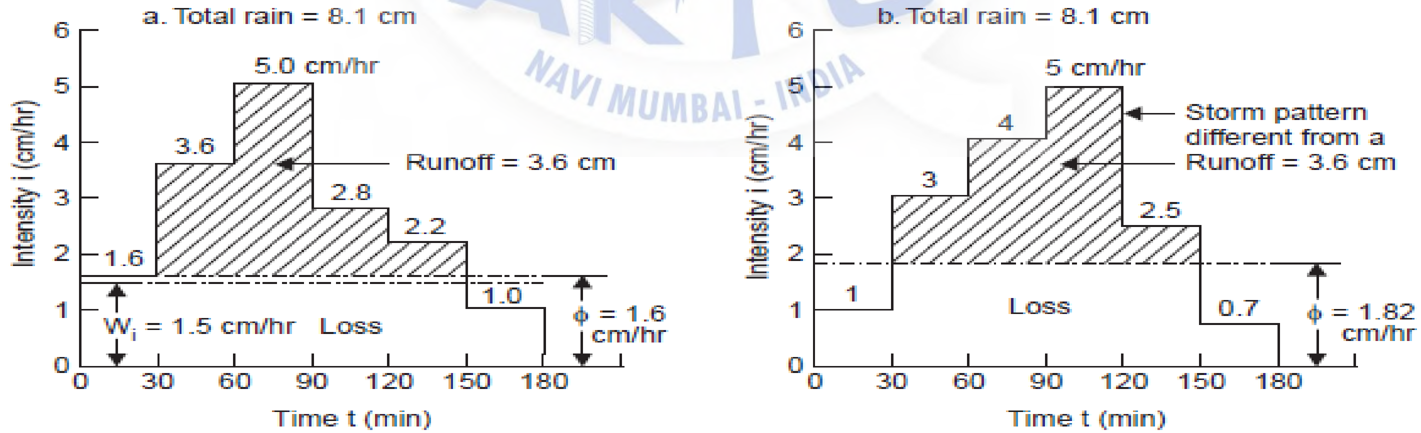
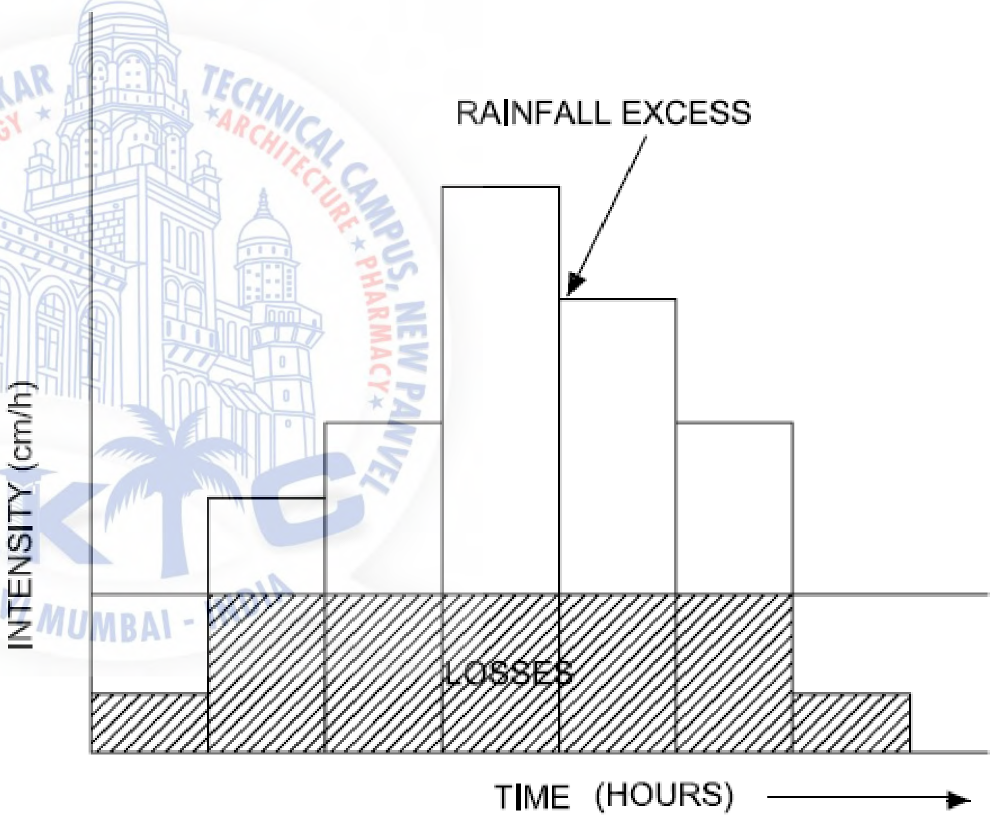


Fig. 3.13 φ-Index computations

Effective rainfall or Rainfall Excess

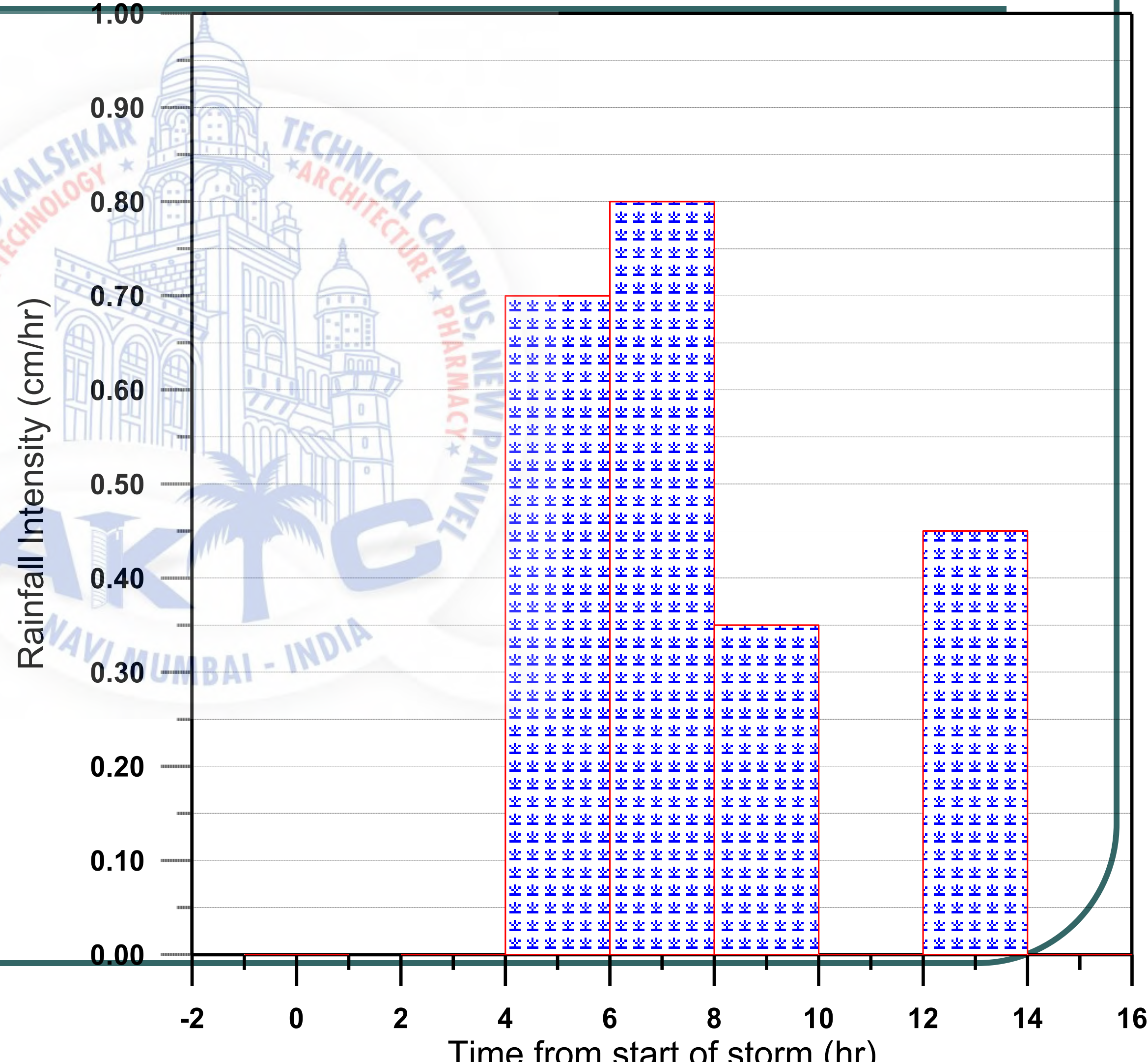
- Hyetograph (effective runoff hydrograph) showing the direct runoff (initial losses and infiltration losses are separated).
  - Called as *effective rainfall hyetograph* or *hyetograph of rainfall excess*
- It is the rainfall which contributed for direct runoff



1 A storm over a catchment of area 5 km^2 had a 14 hrs duration. The mass curve of rainfall is given. If ϕ index is 0.4 cm/h, determine the effective rainfall hyetograph and the volume of direct runoff from the catchment.

Time from start of storm (hr)	Accumulated rainfall (cm)	depth of rainfall in cm	ϕ index	Effective rainfall (cm)	Rainfall intensity cm/hr
0	0	0	-	-	-
2	0.6	0.6	0.8	0	0
4	2.8	2.2	0.8	1.4	0.7
6	5.2	2.4	0.8	1.6	0.8
8	6.7	1.5	0.8	0.7	0.35
10	7.5	0.8	0.8	0	0
12	9.2	1.7	0.8	0.9	0.45
14	9.6	0.4	-	0	0

- Total effective rainfall= area of ER hyetograph
  - =(0.7+0.8+0.35+0.45)*2
  - =4.6 cm
- Volume of direct runoff = rainfall excess X area of the basin
  - $=0.046*5*(1000)^2$
  - $=23000\ m^3$



W_{index} and ϕ_{index} are the two indices which are commonly used.

W_{index} is the average infiltration rate or the infiltration capacity averaged over the whole storm period, and is given by

$$W_{index} = \frac{F}{t_r} = \frac{P-Q}{t_r} \qquad \ldots(7.101)$$

where F = Total infiltration including initial basin recharge, called **potential infiltration**

P = Total precipitation

Q = Total runoff

t_r = Duration of rainfall in hr.

ϕ_{index} is defined as the average rate of loss such that the volume of rainfall in excess of that rate will be equal to the volume of direct runoff.

It can be defined, the other way round, as the rate of rainfall above which the rainfall volume equals the runoff volume.

ϕ_{index} can be represented graphically as shown in Fig. 7.86.

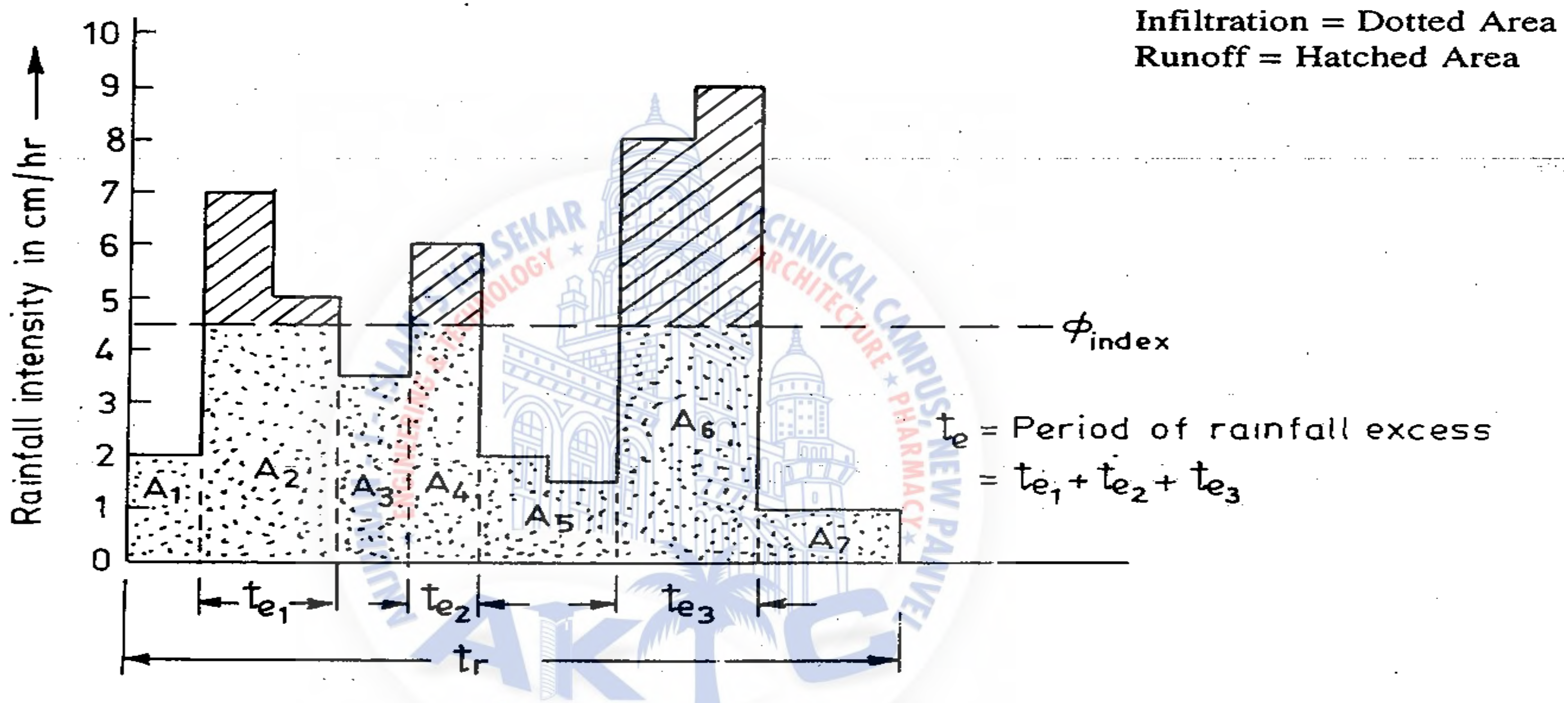


Fig. 7.86

It can be seen from this figure, that ϕ_{index} would be equal to :

$$\phi_{index} = \frac{\text{Total infiltration during period of rainfall excess}}{\text{Period of rainfall excess } (t_e)} \quad \ldots(7.102)$$

$$= \frac{A_2 + A_4 + A_6}{t_e} \quad \ldots[7.102\ (a)]$$

whereas, $$W_{index} = \frac{A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7}{t_r} \quad \ldots[7.101\ (a)]$$

Example 7.24. *The following are the rates of rainfall for successive 20 minutes period of a 140 minutes storm: 2.5, 2.5, 10.0, 7.5, 1.25, 1.25, 5.0 cm/hr. Taking the value of ϕ_{index} as 3.2 cm/hr, find out the net runoff in cm, the total rainfall and the value of W_{index}.*

Solution. The rain intensity pattern (*rainfall hyetograh*) can be plotted from the given rainfall rates, as shown in Fig. 7.88.

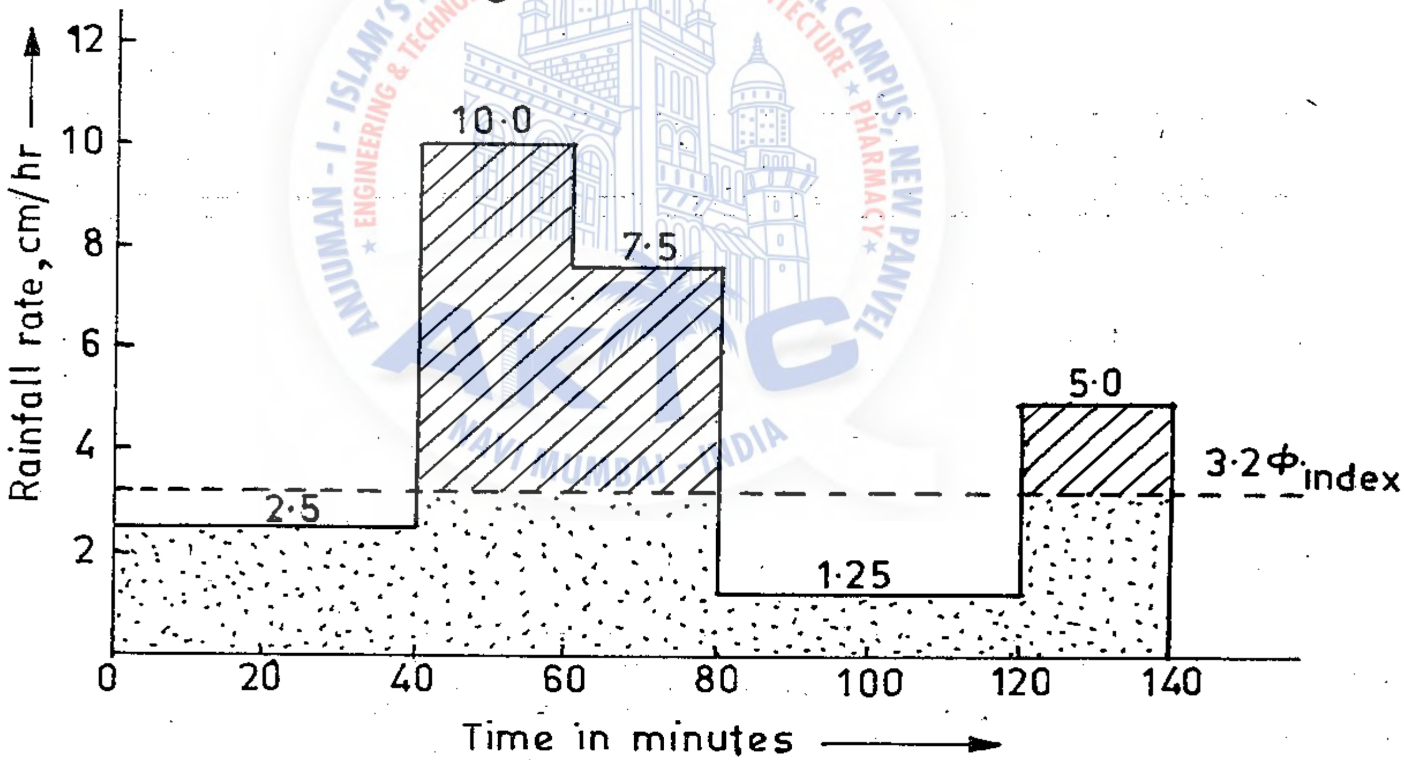


Fig. 7.88

ϕ_{index} line at a height of 3.2 cm/hr is superimposed. The hatched area is calculated, so as to obtain the value of runoff :

$$\text{Total Runoff } (Q) = (10-3.2)\frac{20}{60} + (7.5-3.2)\frac{20}{60} \times (5-3.2)\frac{20}{60}$$

$$= 6.8 \times \frac{20}{60} + 4.3 \times \frac{20}{60} + 1.8 \times \frac{20}{60}$$

$$= \frac{20}{60}(6.8+4.3+1.8) = \frac{20}{60} \times 12.9$$

$$= 4.3 \text{ cm.} \quad \textbf{Ans.}$$

Total precipitation (P)

$$= 2.5 \times \frac{40}{60} + 10 \times \frac{20}{60} + 7.5 \times \frac{20}{60} + 1.25 \times \frac{40}{60} + 5 \times \frac{20}{60}$$

$$= \frac{20}{60}(5.0+10+7.5+2.5+5) = \frac{20}{60} \times 30$$

$$= 10 \text{ cm} \quad \textbf{Ans.}$$

$$W_{index} = \frac{P-Q}{t_r \text{ in hr}} = \frac{10-4.3}{\left(\frac{140}{60}\right)} = \frac{5.7 \times 60}{140} = 2.44 \text{ cm/hr.} \quad \textbf{Ans.}$$

Example 7.26. *A storm with a 15.0 cm precipitation produced a direct runoff of 8.7 cm. The time distribution of the storm is as follows :*

Time from start in hr	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Incremental rainfall in each hr in cm	*0.6*	*1.35*	*2.25*	*3.45*	*2.7*	*2.4*	*1.5*	*0.75*

Estimate the ϕ_{index} *of the storm.* *(Civil Services, 1987)*

Solution. The hyetograph of rainfall is drawn in Fig. 7.89, by using the given values of rainfall of each hr, as they represent the rainfall intensity in cm/hr.

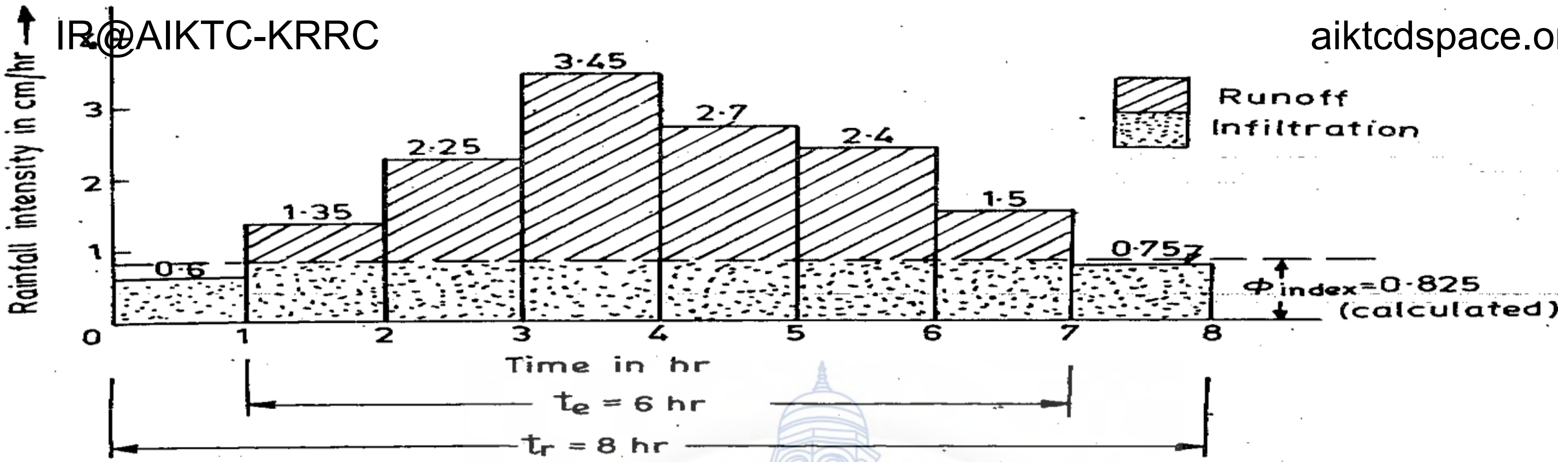


Fig. 7.89

Now, total precipitation $= P = 15.0$ cm

Total runoff $= Q = 8.7$ cm

$$W_{index} = \frac{P - Q}{t_r} = \frac{15 - 8.7}{8} \quad [\because \quad t_r = \text{Rainfall duration} = 8 \text{ hr}]$$

$$= 0.7875 \text{ cm/hr}$$

Since ϕ_{index} has to be somewhat more than W_{index}, we can conclude that ϕ_{index} would be a little more than 0.7875 cm/hr. When this is so, evidently, the first hour rainfall and 8th hour rainfall would become ineffective in producing excess rain, because the rain intensity in those two hours would be less than ϕ_{index}.

In other words, the period (t_e) during which excess rain occurs, would be only $8 - 2 = 6$ hr. In that case, using equation (7.102), we have

$$\phi_{index} = \frac{\text{Total infiltration during period of excess rainfall}}{\text{Period of rainfall excess}}$$

$$= \frac{[\text{Total infiltration} - \text{Infiltration during the period when no excess rain occurs}]}{t_e}$$

$$= \frac{6.3 - 0.6 - 0.75}{6} \text{ cm/hr}$$

$$= \frac{6.30 - 1.35}{6} \text{ cm/hr}$$

$$= \frac{4.95}{6} = 0.825 \text{ cm/hr.}$$

This constant value of ϕ_{index} is marked as a straight line in Fig. 7.89, which confirms that there is no other period, except 1st and 8th hr, during which excess rain does not occur. Hence, the above calculated value of ϕ_{index} must be correct.

Hence, $\phi_{index} = 0.825$ cm/hr. **Ans.**

Time Characteristics of a hydrograph

1. Time to peak
   - From beginning of rising limb to peak discharge
     - Function of basin characteristics
       - Drainage density, slope channel size, roughness and soil infiltration characteristics
2. Time of concentration
   - Time required for the farthermost rain to reach the outlet
3. Lag time or Basin lag time
   - Between centre of mass of rainfall and runoff hydrograph mass
   - Since it is very difficult to find the center then it is the time between centre of mass of effective rainfall to peak discharge
4. Duration of rainfall
5. Base time of hydrographs

Other time periods (where attention is needed while deriving the flood hydrograph)

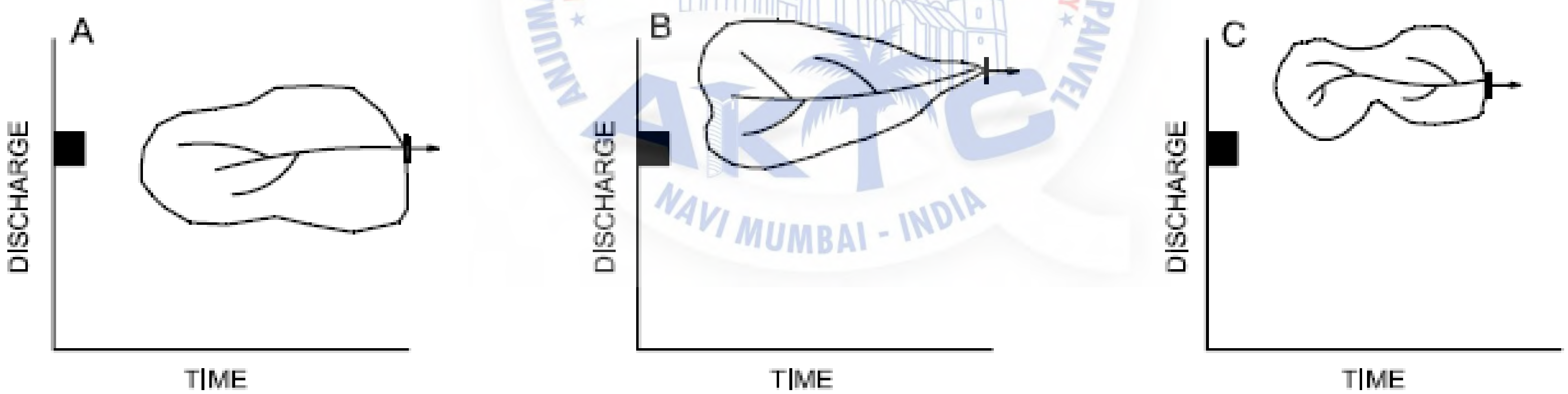
1. Time of measurement of discharge
2. Rainfall intensity
3. Rainfall duration
4. Discharge rate

Factors affecting flood hydrographs

S. No	Physiographic Factors	S.No	Climatic factors
1.	**Basin characteristics**	1.	**Storm characteristics**: precipitation,
	(a) Shape		Intensity, duration, magnitude and
	(b) Size		movement of storm.
	(c) slope		
	(d) Nature of the valley		
	(e) Elevation		
	(f) Drainage density		
2.	**Infiltration characteristic**	2.	**Initial loss**
	(a) Land use and cover		
	(b) Soil type and geological condition		
	(c) Lakes, swamps and other storage		
3.	**Channel characteristics**: cross section, roughness and storage capacity.	3.	**Evapotranspiration**

Factors affecting flood hydrographs

- Shape of the basin
  - It influences the time taken for water from the remote parts of the catchment to arrive at the outlet.
- Size
  - Small basins and large basins behave differently due to relative importance of overland flow and channel flow.



- Slope

  It controls the velocity of the flow in the channel and affects the steepness of recession limb.

- Drainage density

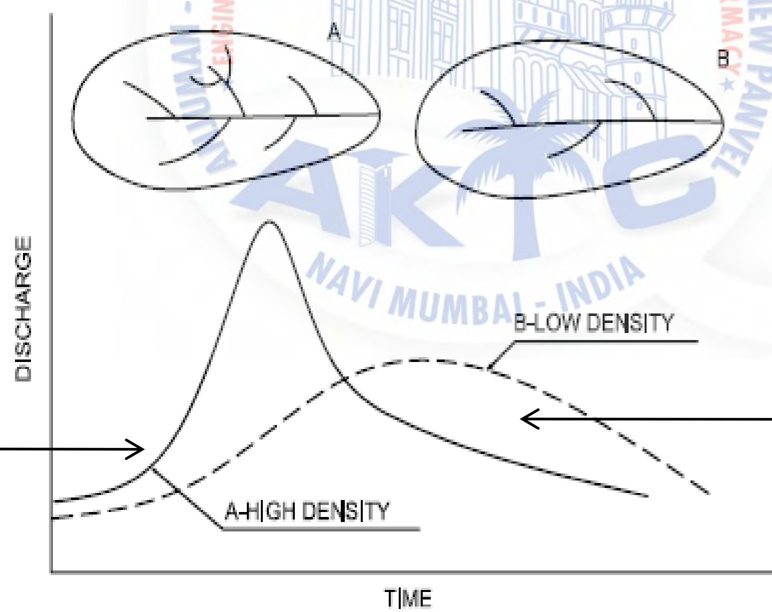
  It is the ratio of the total channel length to total channel area. High drainage density reflected in a pronounced peak.

- Land use

  Vegetation and forests increase the infiltration and storage capacity of soils and retards the overland flow.

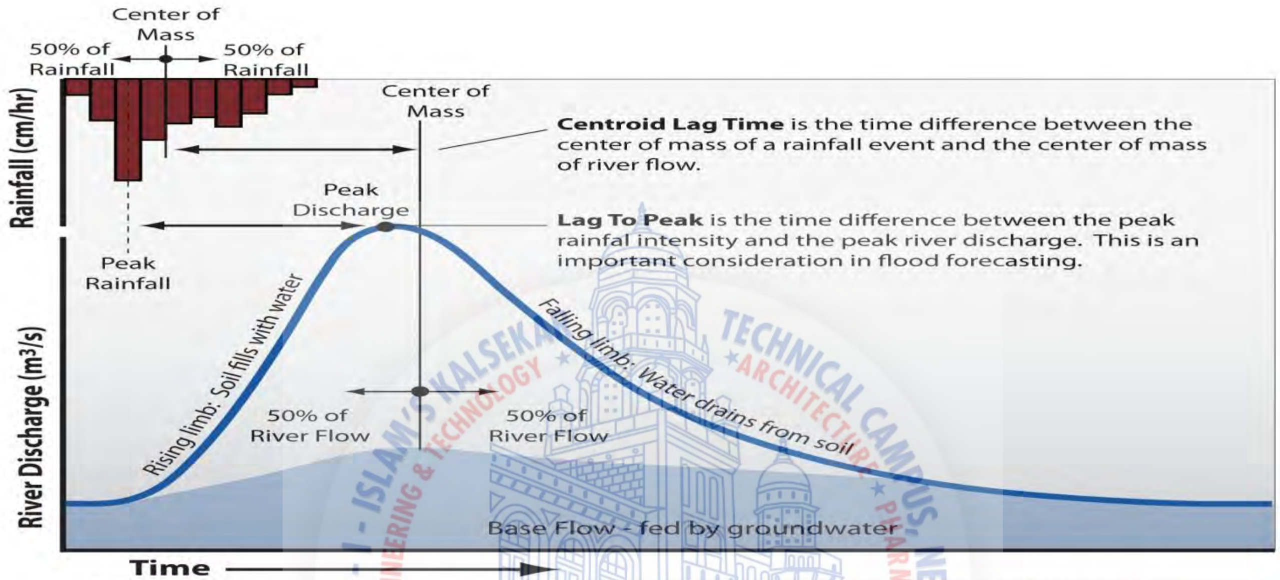
- Climatic factors
  - Intensity, duration and direction of storm movement are three important climatic factors, which affects the shape flood hydrographs.

Controlled mostly by climatic factors



Controlled mostly by basin factors

Figure 4.9 - Hydrograph Definitions
(chapter 4 - Hydrology)



Rainfall

During a **rainfall event**, the intensity at which precipitation falls on a landscape is often variable. The **peak rainfall**, a measure of the greatest intensity of rainfall, does not necessarily occur in the middle of a rainfall event. Therefore, the peak and the **center of mass** of rainfall (the point at which half of the total rainfall has fallen) often do not correspond.

Discharge

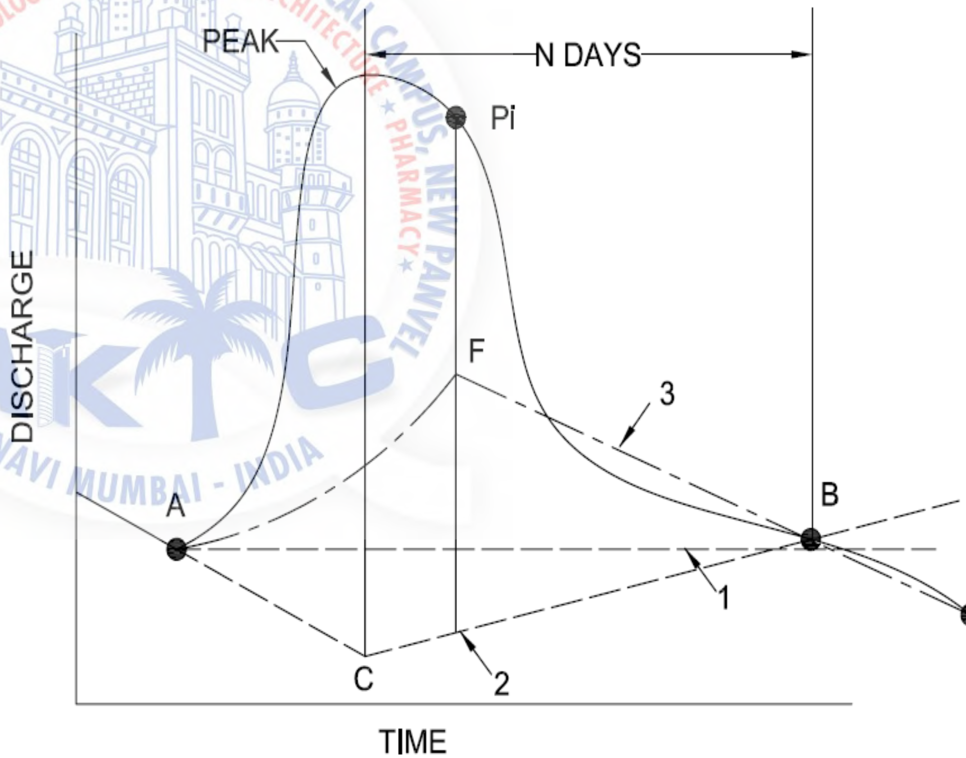
As precipitation falls across a landscape, some **infiltrates** the soil and some **runs off** and enters river channels causing discharge and river stage to rise. As the soil fills with water, more precipitation enters river channels as runoff, groundwater flow and subsurface stormflow. Stage rises until reaching the **peak**, or maximum discharge resulting from a rainfall event. As water drains from soils and from the landscape, the river stage begins to fall, eventually returning to **base flow**, which reflects normal groundwater discharge to rivers in humid regions.

Notes:
centroid lag (lc)...center of mass to center of mass.
Dunne and Leopold, p. 326
lag to peak, Dunne and Leopold, pg. 258

SES.06
SES.14

Base flow separation

- To draw surface runoff hydrograph, it is required to separate base flow
- Methods
  - Method I – Straight line method
    $N=0.83A^{0.2}$
    - Line AB
  - Method II
    - Line ACB
  - Method III
    - Line AFE



6.4 Base Flow Separation

- The surface hydrograph is obtained from the total storm hydrograph by separating the quick-response flow from the slow response runoff.

- The base flow is to be deducted from the total storm hydrograph to obtain the surface flow hydrograph in three methods

Method I: Straight line method

- Draw a horizontal line from start of runoff to intersection with recession limb (Point A).
- Extend from time of peak to intersect with recession limb using a lag time, N.

$$N = 0.83\ A^{0.2}$$

Where: A = the drainage area in Km^2 and N = days where Point B can be located and determine the end of the direct runoff .

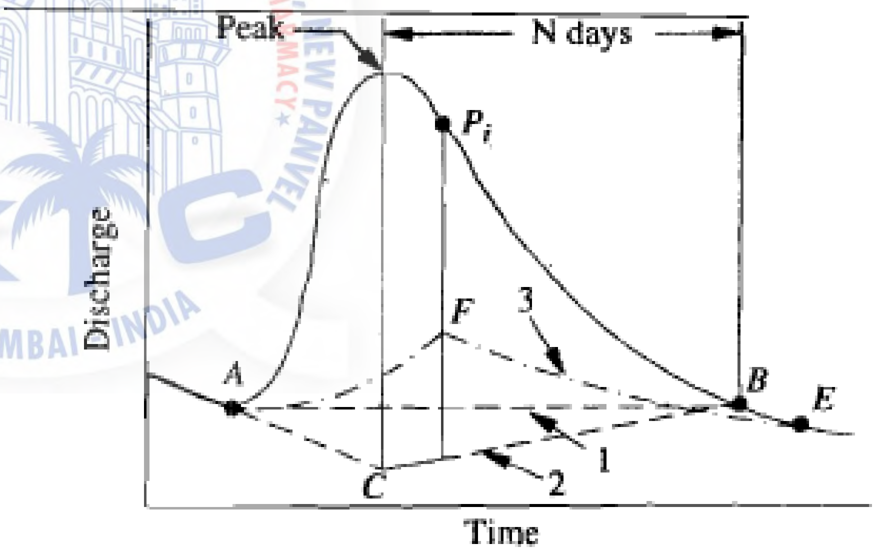


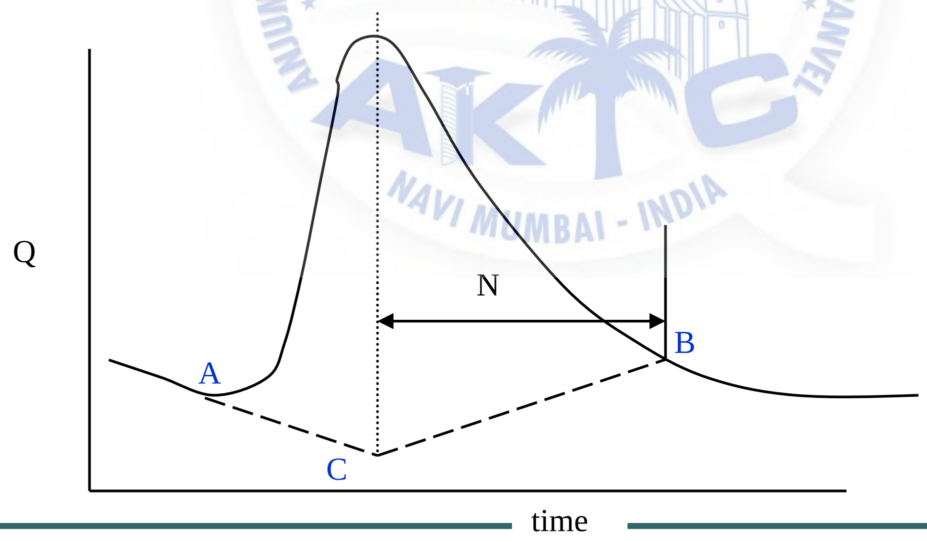
Fig. 6.5 Base flow seperation methods

Method II:

- In this method the base flow curve existing prior to the beginning of the surface runoff is extended till it intersects the ordinate drawn at the peak (point C in Fig, 6.5). This point is joined to point *B* by a straight line.

- Segment *AC* and *CB* separate the base flow and surface runoff.

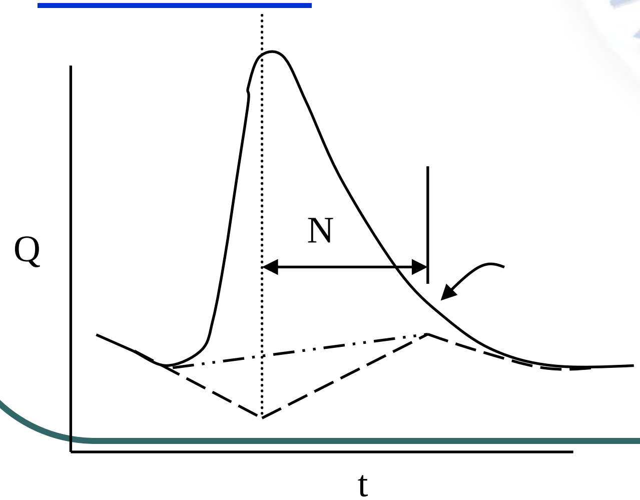
- This is probably the most widely used base-flow separation procedure.



Method III

- In this method the base flow recession curve after the depletion of the flood water is extended backwards till it intersects the ordinate at the point of inflection (line *EF* in Fig. 6.5), Points *A* and *F* are joined by an arbitrary smooth curve.
- This method of base-flow separation is realistic in situations where the groundwater contributions are significant and reach the stream quickly.
- The selection of anyone of the three methods depends upon the local practice and successful predictions achieved in the past.
- The surface runoff hydrograph obtained after the base-flow separation is also known as *direct runoff hydrograph* (DRH).

Method V



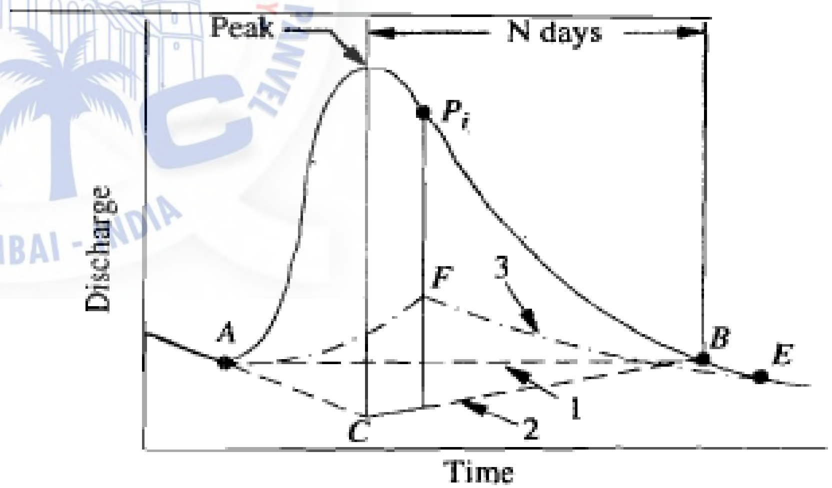


Fig. 6.5 Base flow seperation methods

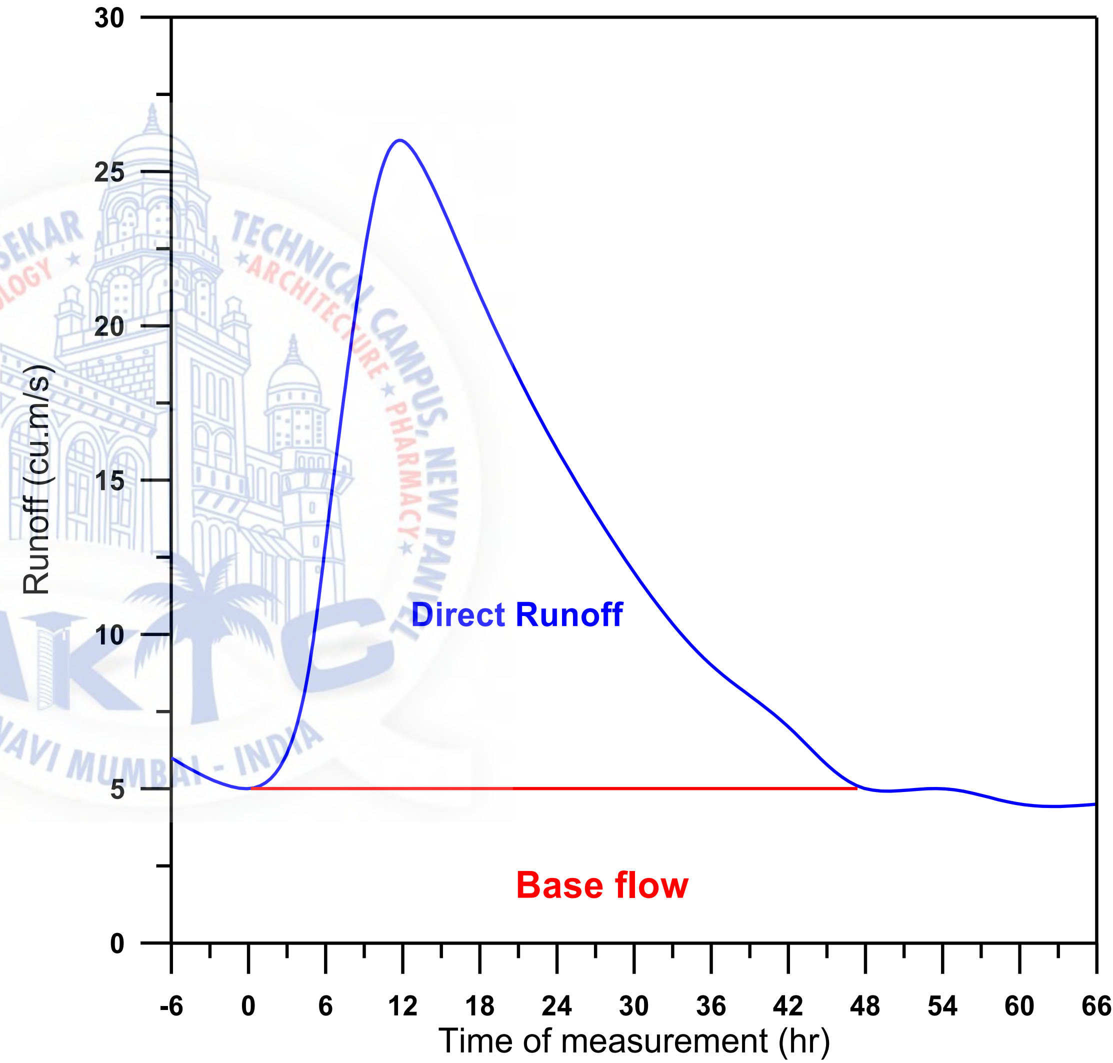
? A rainfall of 3.8 cm and 2.8 cm occurred on two consecutive 4-hr duration over an area of 27 km^2. Estimate the rainfall excess and ϕ-index from the measured runoff.

Time from start of rainfal l (h)	Observed runoff (m^3/s)	Base flow (m^3/s)	Direct runoff (m^3/s)
-6	6	5	-
0	5	5	0
6	13	5	8
12	26	5	21
18	21	5	16
24	16	5	11
30	12	5	7
36	9	5	4
42	7	5	2
48	5	5	0
54	5	5	0
60	4.5	5	-
66	4.5	5	-

STEPS

1. Draw the runoff hydrograph
2. Separate the base flow (any one technique) $N=0.83A^{0.2}$
3. Or assume a base flow of 5 m^3/s
4. Calculate the DRH
5. Estimate the runoff depth = runoff volume/area
5. Use ϕ index formula to determine the loss
6. Then estimate the rainfall excess

- Direct runoff volume
  - = ΣDRH ordinates x time
  - =(8+21+16+11+7+4+2)*6*3600
  - $=1.49 \times 10^6$ m^3
  - (remember Trapezoidal formula)
- Runoff depth
  - = Volume of runoff /area
  - $= (1.49 \times 10^6)/(27 \times 10^6)$
  - 5.52 cm
- Total rainfall = 3.8+2.8=6.6 cm
- ϕ-index = $(P-R)/t_e$
  - (6.6-5.52)/8 = 0.135 cm/h
- Hence rainfall excess
  - 3.8-(0.135x4)=3.26 cm for first 4 hr
  - 2.8-(0.135x4)=2.26 cm for second 4 hr



Unit hydrograph

- The measurement of runoff works out to be a costly and more time consuming process
- Hence various methods has been used like RR models, empirical, rational, soft computing etc…
- UNIT HYDROGRAPH METHOD – Introduced by Sherman (1932)
- The unit hydrograph represents the **lumped response** of the catchment to a **unit rainfall excess** of **D-h duration** to produce a direct-runoff hydrograph.
- It relates only the direct runoff to the rainfall excess.
- Hence the volume of water contained in the unit hydrograph must be equal to the volume of rainfall excess.
- As 1 cm depth of rainfall excess is considered, the area of the unit hydrograph is equal to a volume given by 1 cm over the catchment for that duration.
- The rainfall is considered to have an average intensity of excess rainfall (ER) of 1 cm/h for the D hr duration of the storm.
- The distribution of the storm is considered to be uniform all over the catchment.

Basic assumptions of unit hydrograph

- **Time invariance**

  This first basic assumption is that the direct-runoff response to a given effective rainfall in a catchment is time-invariant.

  This implies that the DRH for a given ER in a catchment is always the same irrespective of when it occurs.

- **Linear Response**

  The direct-runoff response to the rainfall excess is assumed to be linear.

  This is the most important assumption of the unit-hydrograph theory.

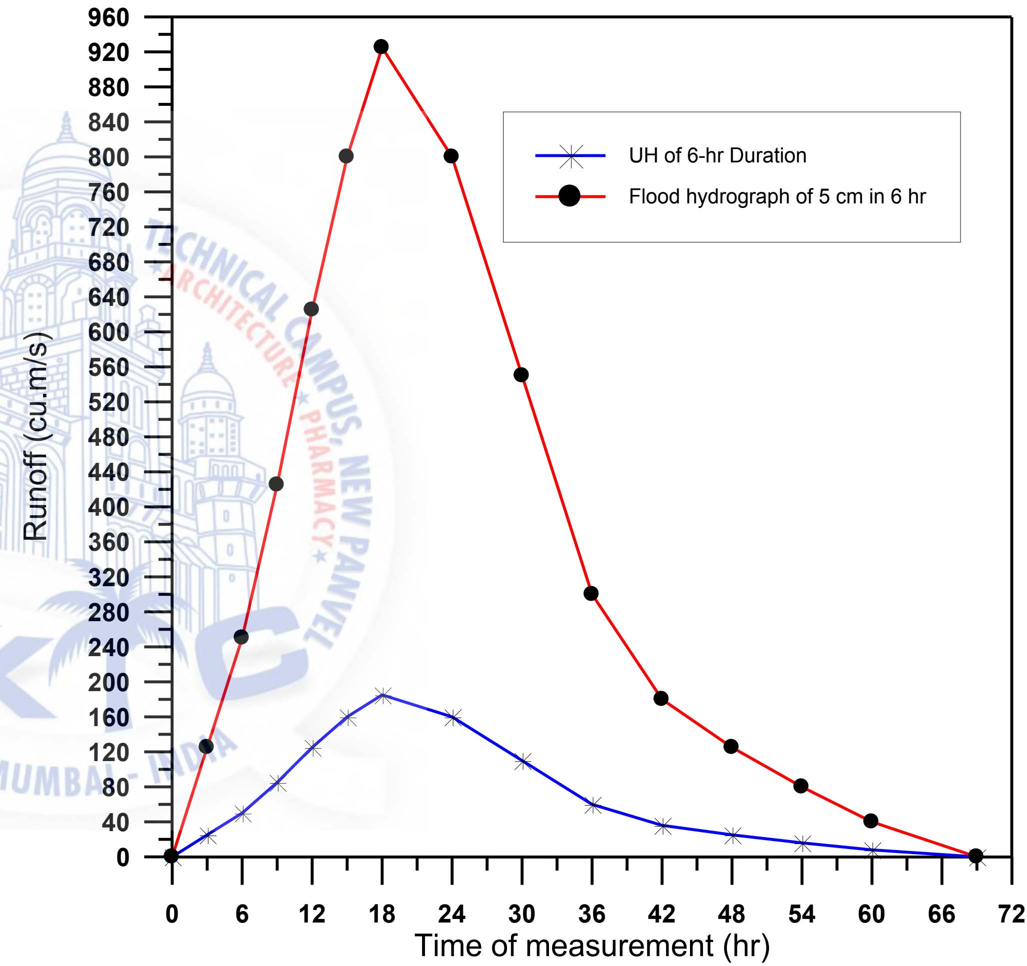
  The Rainfall –runoff obeys the principles of superposition

Application of unit hydrograph

- ✓ It is useful in calculating DRH (flood as well as storm hydrograph) of a given storm occurred for D hr.
  - ✓ If a D hr duration UH is available, it is possible to derive multiples of D hr duration unit hydrograph

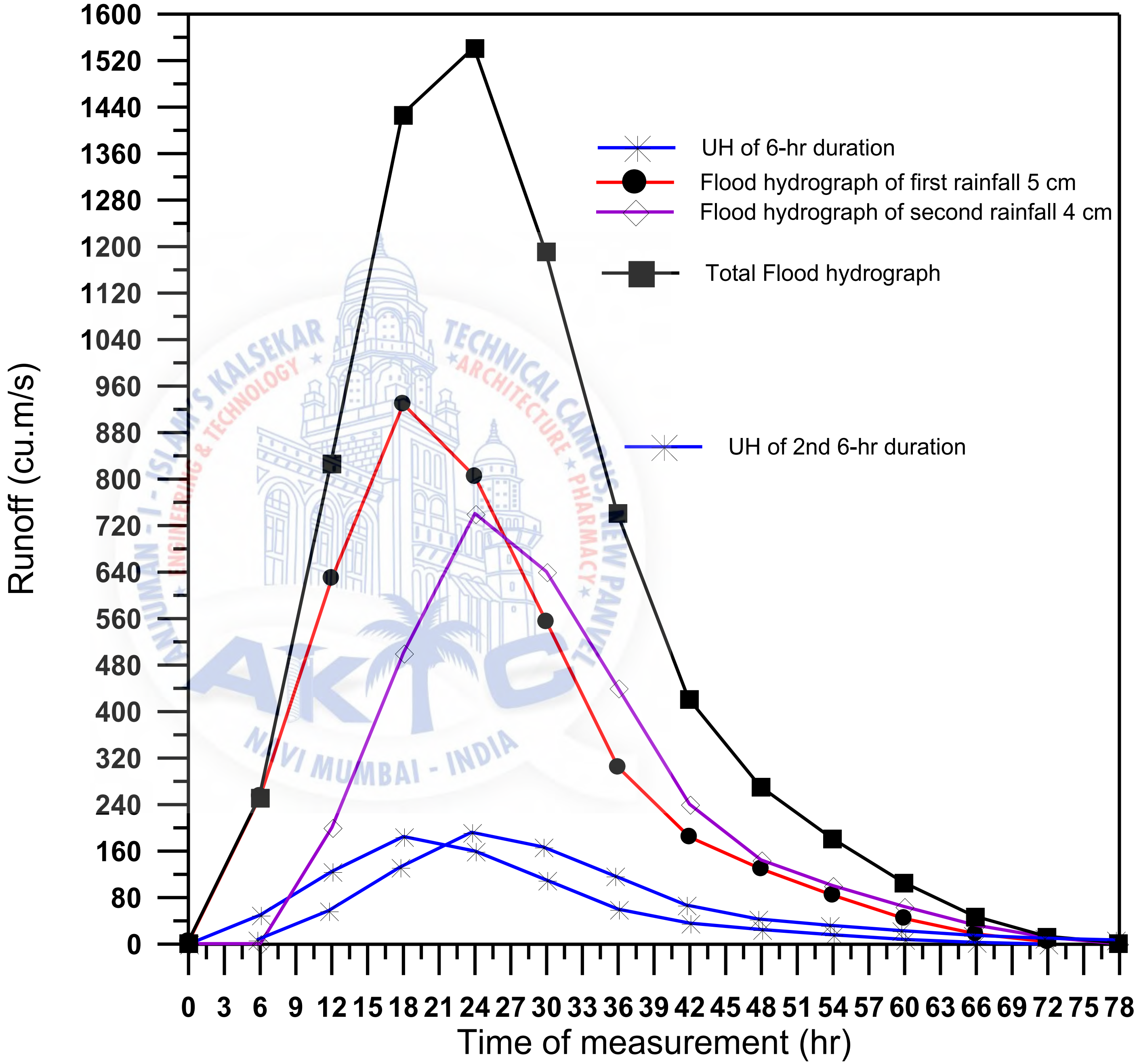
? Given below are the ordinates of 6-hr unit hydrograph for a catchment. Calculate and draw the DRH to a rainfall excess of 5 cm occurred in 6-hr

Time (h)	UH ordinates (m^3/s)	Ordinates of 5cm DRH (m^3/s)
0	0	0
3	25	125
6	50	250
9	85	425
12	125	625
15	160	800
18	185	925
24	160	800
30	110	550
36	60	300
42	36	180
48	25	125
54	16	80
60	8	40
69	0	0



Observe the time of measurement of runoff

? Two storms each of 6 hr duration having rainfall excess of 5 cm and 4 cm respectively has occurred concurrently in the same basin. Calculate the resulting DRH?



Time (h)	UH ordinates (m^3/s)	Ordinates of 5 cm DRH (m^3/s)	Ordinates of 4 cm DRH (m^3/s)	Total DRH (m^3/s)
0	0	0	0	0
6	50	250	0	250
12	125	625	200	825
18	185	925	500	1425
24	160	800	740	1540
30	110	550	640	1190
36	60	300	440	740
42	36	180	240	420
48	25	125	144	269
54	16	80	100	180
60	8	40	64	104
66	2.7	13.5	32	45.5
72	0	0	10.8	10.8
78			0	0

Derivation of unit hydrographs

Flood hydrographs used in the analysis should be selected to meet the following desirable features with respect to the storms responsible for them:

- The storms should be an isolated storm.
- The rainfall should be fairly uniform during the duration and should cover the entire catchment area.
- The duration of the rainfall should be 1/5 to 1/3 of the basin lag. (how will you estimate this?????)
- The rainfall excess of the selected storm should be high. A range ER values of 1.0 to 4.0 cm is sometimes preferred.

Steps on derivation of unit hydrographs

1. Collect the rainfall intensity, duration, and runoff values over a period of time.

2. Plot the storm hydrograph.

3. Separate the base flow if any (use any one of the four methods) and find the direct runoff

4. Find the volume of direct runoff from the DRH

5. Divide the volume of Direct runoff by area of the catchment to find the rainfall excess.

6. Divide each and every DRH ordinates using the rainfall excess

7. The resulting ordinates are the ordinates of the UNIT HYDROGRAPH

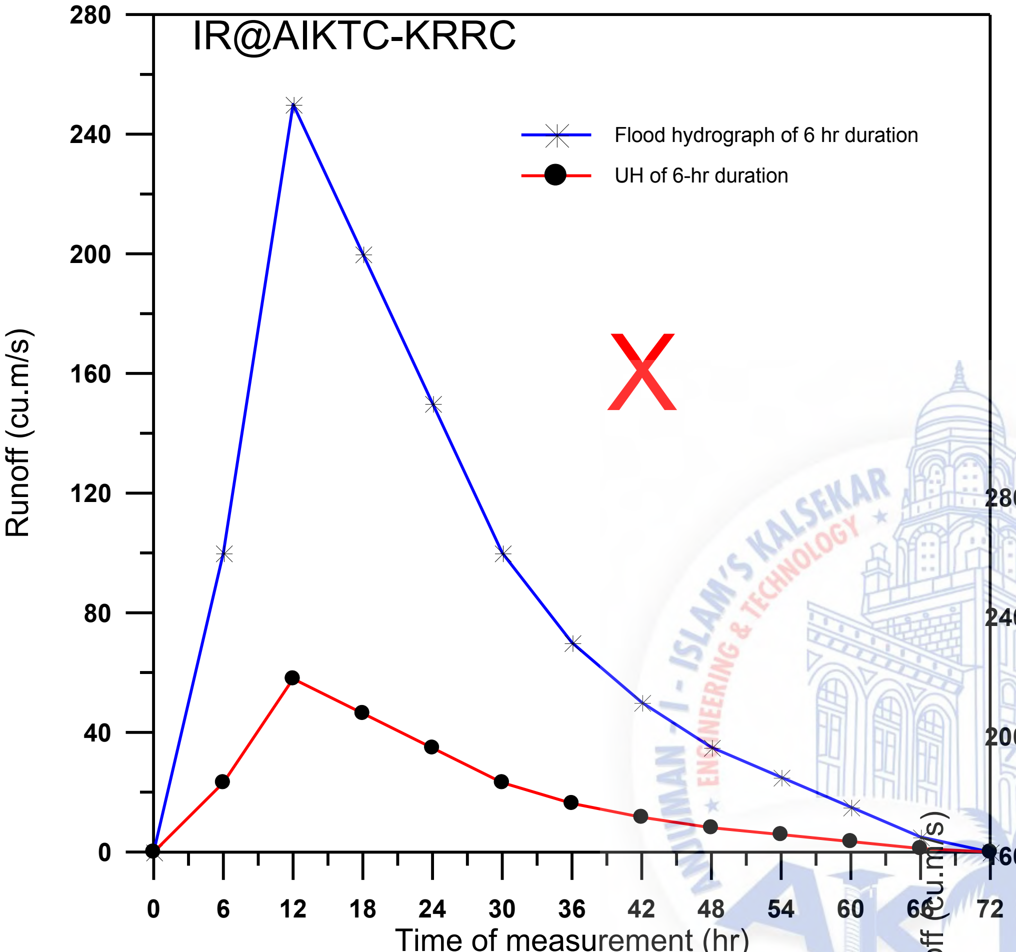
Possible rainfall events to derive Unit Hydrographs

1. Single storm of D hr duration
2. Complex storm or multiple storm
   a. Multiple storm of each D hr duration (only magnitude varies)
   b. Multiple storm of varying magnitude and varying duration

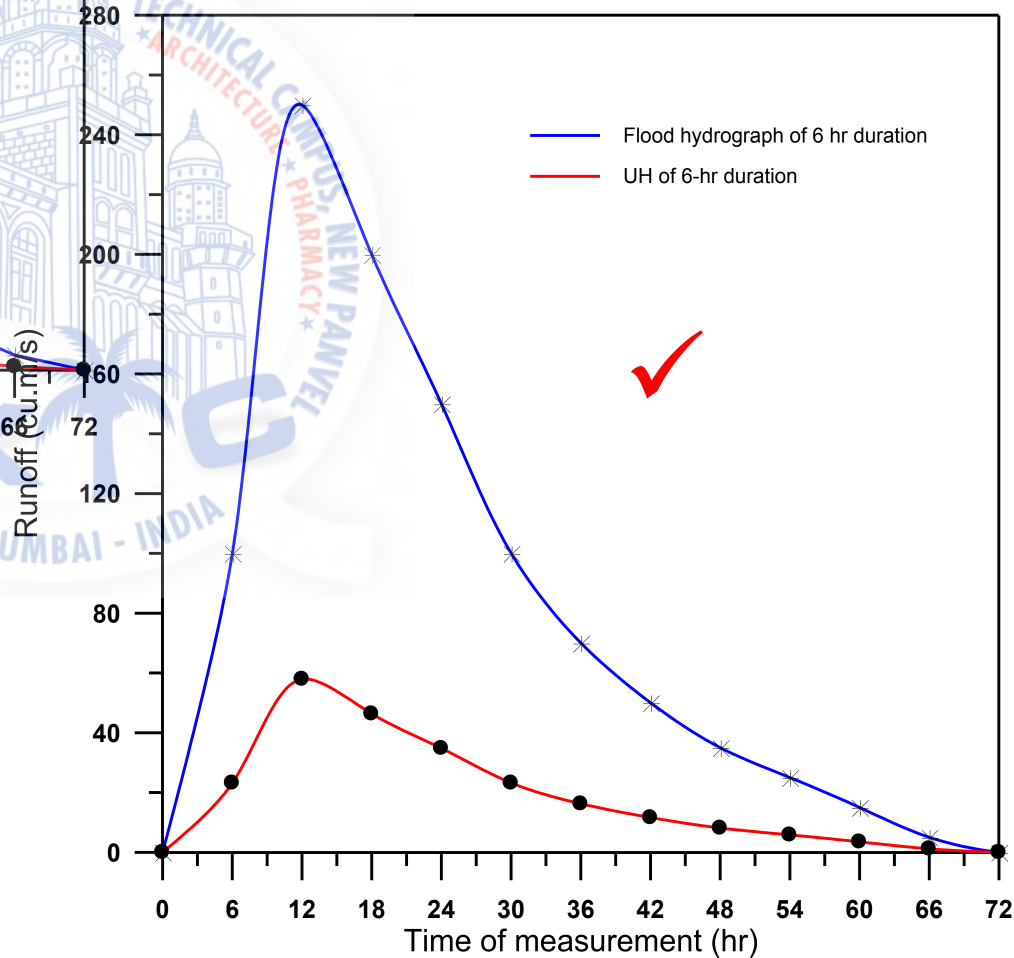
Given below are observed flows from a storm of 6hr duration on a stream with a catchment area of 500 km². Derive the ordinates of 6-hr unit hydrograph

Time (h)	Observed flow (m^3/s)	DRH = Observed-base flow (m^3/s)	UH = DRH/4.32 (m^3/s)
0	0	0	0.0
6	100	100	23.1
12	250	250	57.9
18	200	200	46.3
24	150	150	34.7
30	100	100	23.1
36	70	70	16.2
42	50	50	11.6
48	35	35	8.1
54	25	25	5.8
60	15	15	3.5
66	5	5	1.2
72	0	0	0.0

- Draw the flood hydrograph
- Estimate the base flow (in this case it is zero)
- Calculate the DRH
  - = FH-base flow
- Estimate the volume of runoff = DRH ordinates*time interval of measurement
  - $=1000*6*3600 = 21.6x10^6\ m^3$
- Estimate rainfall excess = volume of runoff /area of catchment
  - $= 21.6x10^6 / 500x10^6$
  - = 0.0432 m
  - = 4.32 cm
- Ordinates of UH
  - = DRH / 4.32

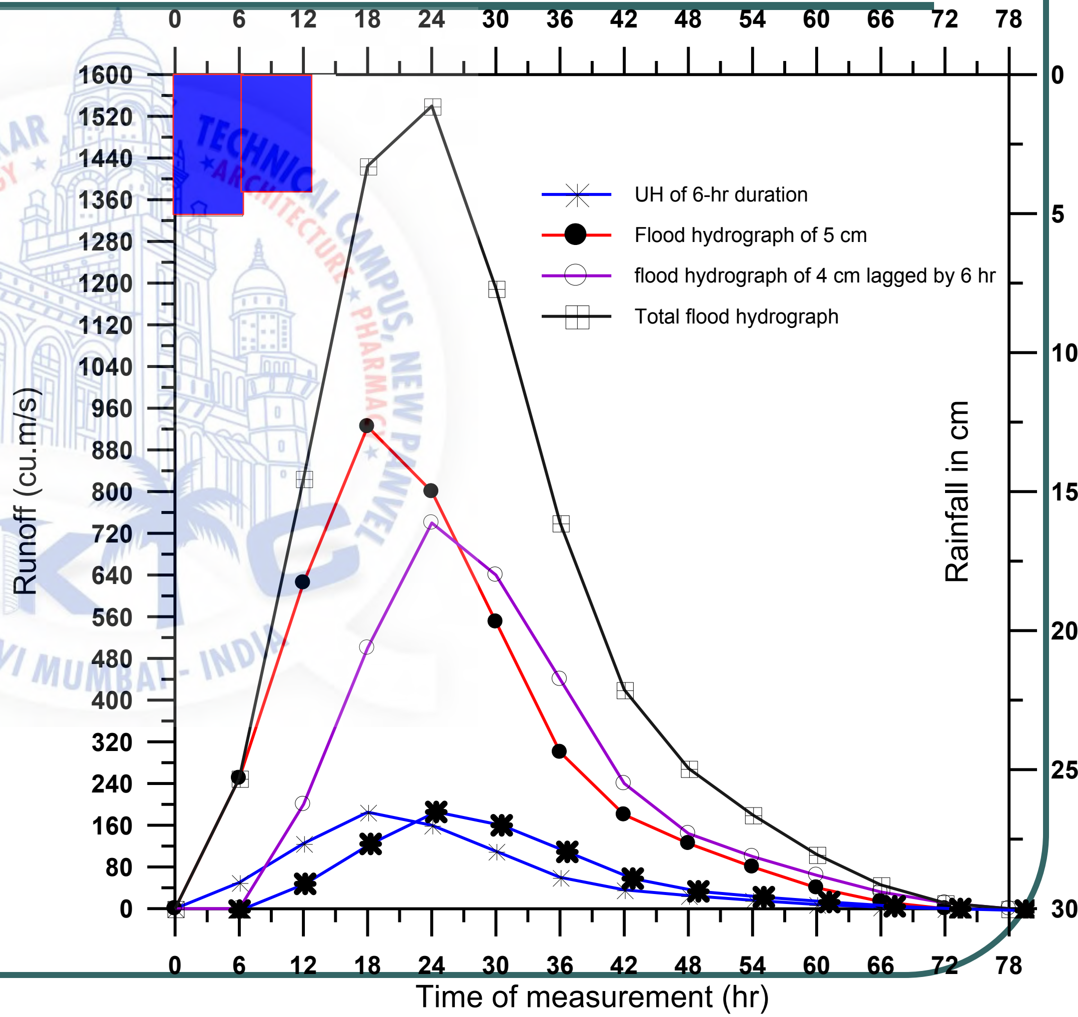


? Redraw hydrographs of the previous problems



Derivation of Unit hydrograph from a complex storm of constant duration

- This is inverse of the problem already solved.
- In this case rainfall of varying magnitude has occurred for constant duration of D hr
- Assume that a Unit Hydrograph for that D hr duration is available, let the ordinates be u_1, u_2, u_3, u_4, u_5, ….



Derivation of Unit hydrograph from a complex storm of constant duration (method of least squares)

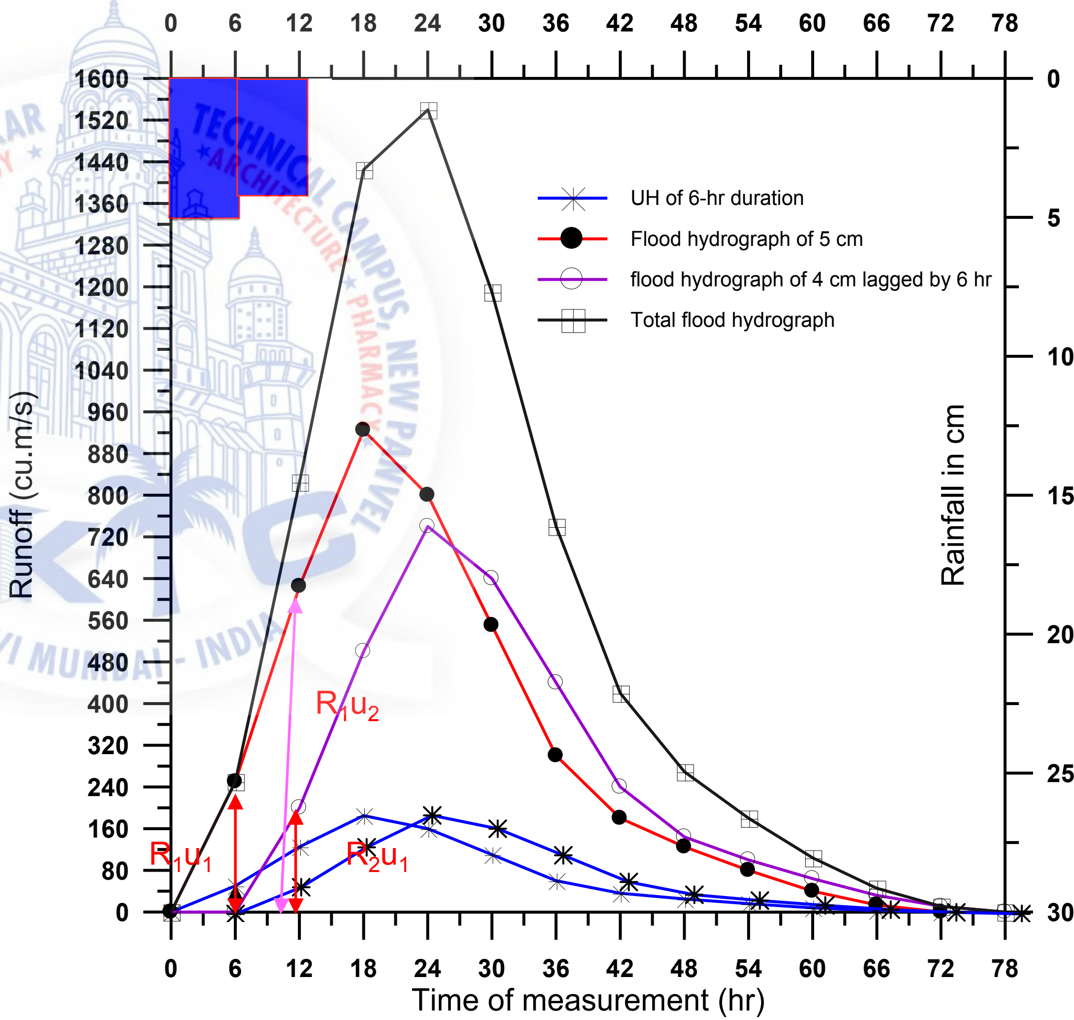
$Q_1 = R_1 u_1$

$Q_2 = R_1 u_2 + R_2 u_1$

$Q_3 = R_1 u_3 + R_2 u_2 + R_3 u_1$

$Q_4 = R_1 u_4 + R_2 u_3 + R_3 u_2 + R_4 u_1$

$$Q_t = \sum_{i=1}^{M} R_i - u\left[t - (i-1)\right]$$



t- at any measured time.
Where M = rainfall events
Sum only positive u values
This equation holds good if discharge measured time interval is equal to unit hydrograph time duration.
This can be converted into a matrix form and then solve, to arrive the UH ordinates

Derivation of Unit hydrograph from a complex storm of constant duration

Time (h)	Total DRH (m^3/s)	R_1 = 5 cm R_2 = 4 cm		UN ordinates (m^3/s)
0	0	0		0
6	Q_1 250	$Q_1 = R_1 u_1 = 250$	$u_1 = 50$	50
12	Q_2 825	$Q_2 = R_1 u_2 + R_2 u_1 = 825$	$u_2 = 125$	125
18	Q_3 1425	$Q_3 = R_1 u_3 + R_2 u_2 = 1425$	$u_3 = 185$	185
24	Q_4 1540	$Q_4 = R_1 u_4 + R_2 u_3 = 1540$	$u_4 = 160$	160
30	Q_5 1190	$Q_5 = R_1 u_5 + R_2 u_4 = 1190$	$u_5 = 110$	110
36	Q_6 740	$Q_6 = R_1 u_6 + R_2 u_5 = 740$	$u_6 = 60$	60
42	Q_7 420	$Q_7 = R_1 u_7 + R_2 u_6 = 420$	$u_7 = 36$	36
48	Q_8 269	$Q_8 = R_1 u_8 + R_2 u_7 = 269$	$u_8 = 25$	25
54	Q_9 180	$Q_9 = R_1 u_9 + R_2 u_8 = 180$	$u_9 = 16$	16
60	Q_{10} 104	$Q_{10} = R_1 u_{10} + R_2 u_9 = 104$	$u_{10} = 8$	8
66	Q_{11} 45.5	$Q_{11} = R_1 u_{11} + R_2 u_{10} = 45.5$	$u_{11} = 2.7$	2.7
72	Q_{12} 10.8	$Q_{12} = R_1 u_{12} + R_2 u_{11} = 10.8$	$u_{12} = 0$	0
78	0	0		

? The following table gives the ordinates of a direct runoff hydrograph resulting from two successive 3-hr duration of rainfall excess values of 2 and 4 cm respectively. Derive 3-hr unit hydrograph for the catchment

Time Hr	0	3	6	9	12	15	18	21	24	27	30
DRH m^3/s	0	120	480	660	460	260	160	100	50	20	0

Time Hr	0	3	6	9	12	15	18	21	24	27	30
DRH m^3/s	0	120	480	660	460	260	160	100	50	20	0
UN 3hr	0	60	120	90	50	30	20	10	5	0	

Derivation of Unit hydrograph from a complex storm of variable duration (trial and error procedure)

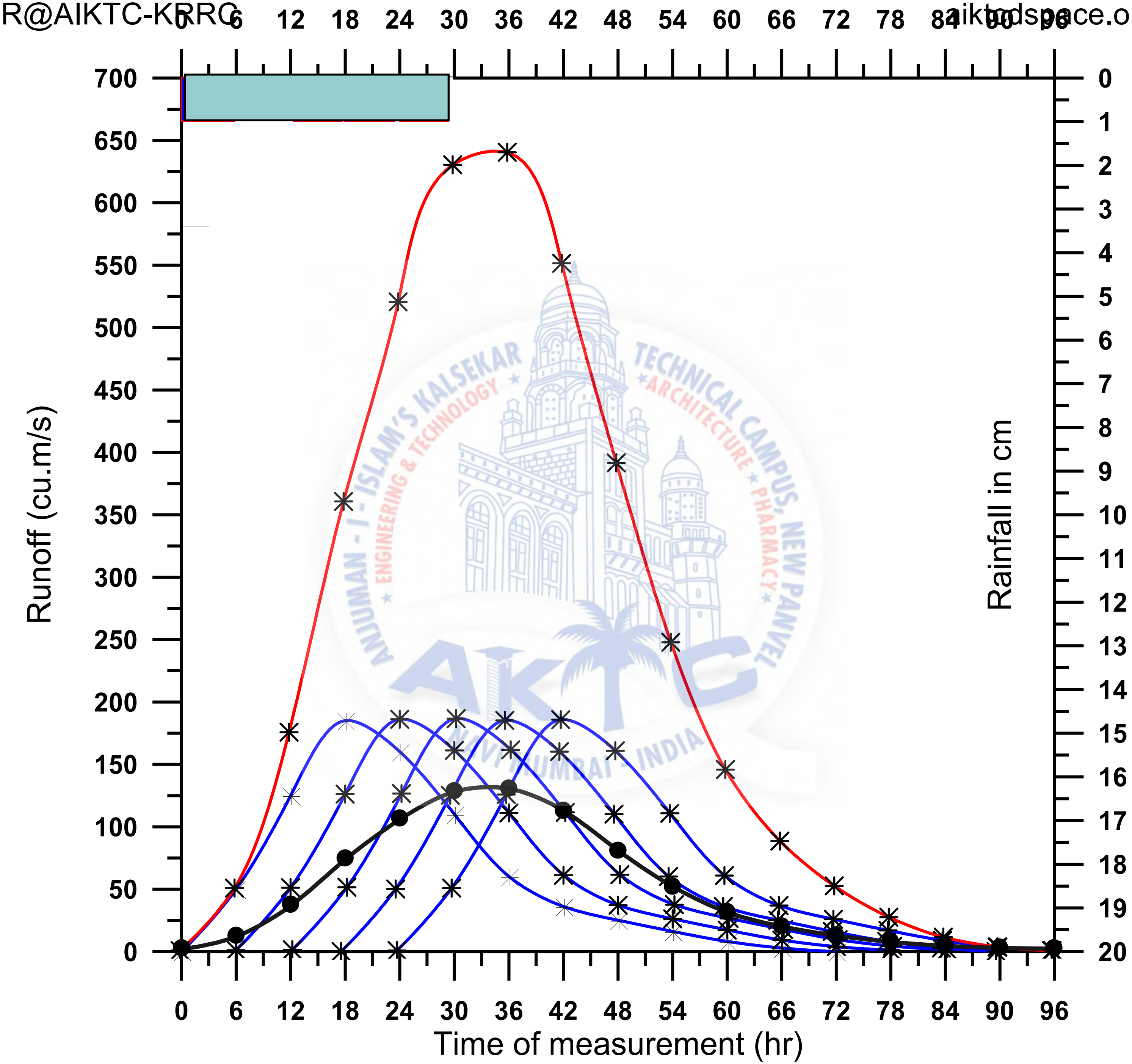
- In real life if rainfall of constant duration is not available
  - Try to adjust the rainfall within a period of ±10%D
  - Else derive various *mD* hr duration Unit hydrograph and then try to super impose
    - Thus to solve this problem we need to convert the available D hr unit hydrograph into *mD* hr unit hydrograph.
      - *m* may be an integer or a fraction

Derivation of Unit hydrograph of different durations from available D hr unit hydrograph.

- Method of superposition
  - Is suitable when *m* is an integer
- Method of S- curve
  - Is suitable when *m* is a fraction

Derivation of Unit hydrograph of different durations from available D hr unit hydrograph.

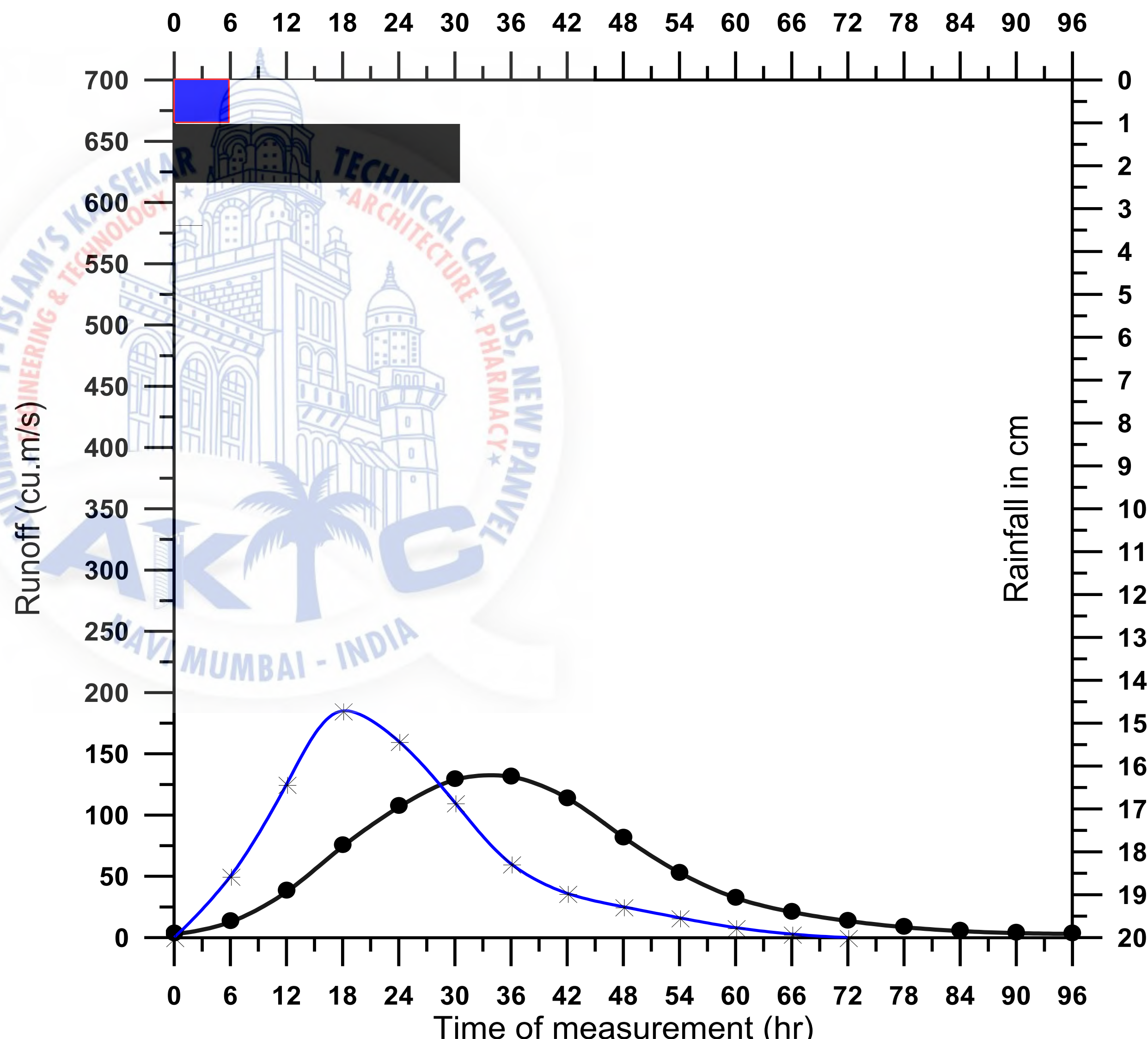
1. Method of superposition
   - Is suitable when *m* is an integer
   - To solve this method a D hr Unit hydrograph is available
   - Super impose m unit hydrographs with each hydrograph lagged by D hr from the previous unit hydrograph
   - The sum of ordinates of lagged hydrograph gives rise to DRH of *m* cm in *mD* hr duration
   - Thus to derive the Unit hydrograph of *mD* hr divide the DRH ordinates by *m* times. The resulting UH is for *mD* hr UH
   - Example: derive 30hr UH from 6 hr unit hydrograph
     - Plot the 6 hr UH
     - Plot 4 more such UH each one lagged by 6 hrs
     - Add the ordinates- the resulting hydrograph is DRH for 5 cm in 30 hrs
     - Divide the ordinates of DRH by 5 to arrive the UN of 30 hr duration.


0 6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
Runoff (cu.m/s)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Rainfall in cm
0 6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96
Time of measurement (hr)

Time (h)	UH ordinates (m^3/s)	Lag 1 by 6 hr	Lag 2 by 12 hr	Lag 3 by 18 hr	Lag 4 by 24 hr	DRH of 5 cm rainfall in 30 hr	UH of 30 hr duration
0	0					0	0
6	50	0				50	10
12	125	50	0			175	35
18	185	125	50	0		360	72
24	160	185	125	50	0	520	104
30	110	160	185	125	50	630	126
36	60	110	160	185	125	640	128
42	36	60	110	160	185	551	110.2
48	25	36	60	110	160	391	78.2
54	16	25	36	60	110	247	49.4
60	8	16	25	36	60	145	29
66	2.7	8	16	25	36	87.7	17.54
72	0	2.7	8	16	25	51.7	10.34
78		0	2.7	8	16	26.7	5.34
84			0	2.7	8	10.7	2.14
90				0	2.7	2.7	0.54
96					0	0	0

? What happens when mD hr unit hydrograph is derived from D hr unit hydrograph in case of m is an integer

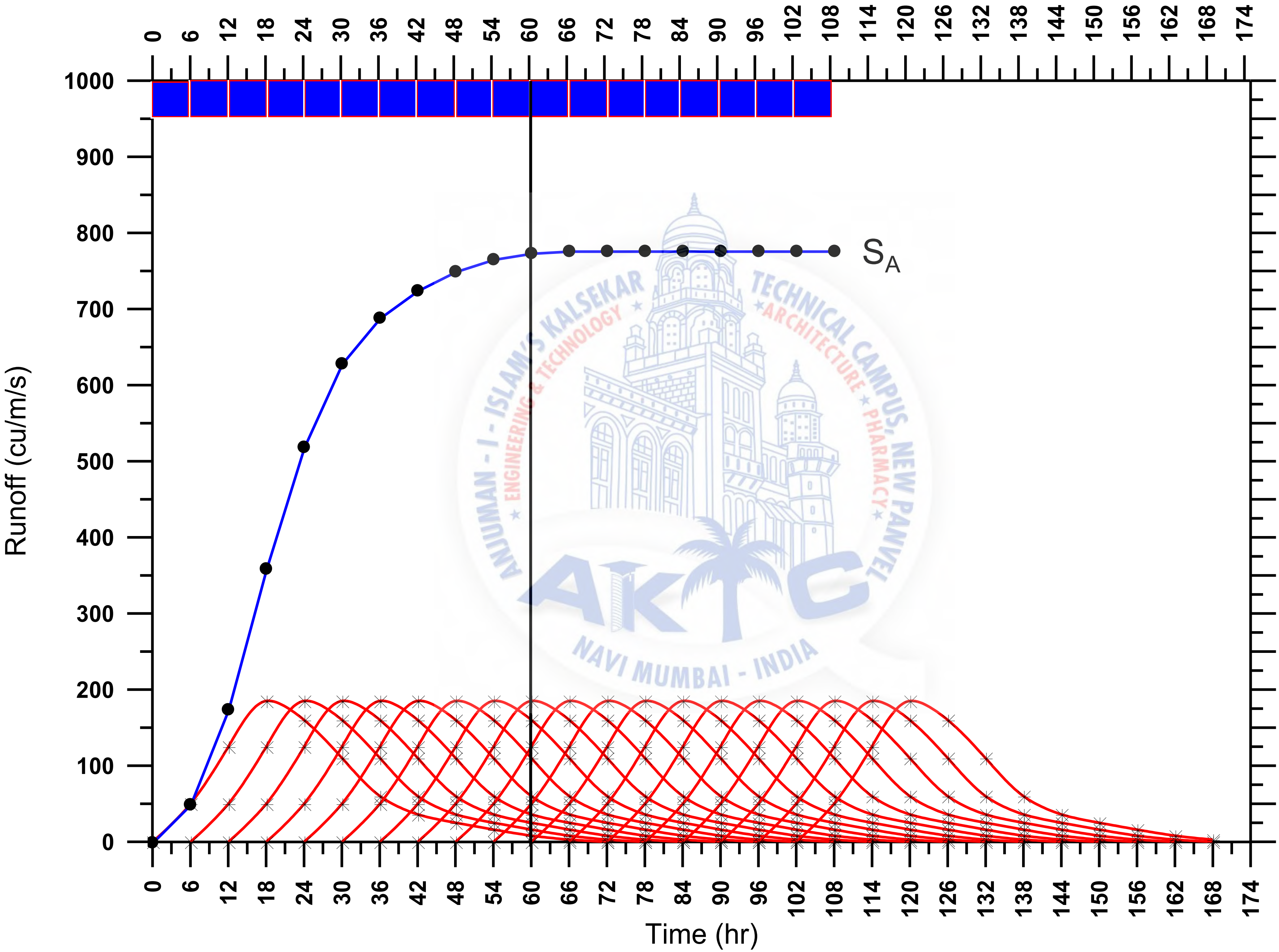
- 1 cm in D hr duration
- 1 cm in mD hr duration
- The magnitude of the peak discharge gets reduced
- The base time increases
- Time to peak increases
- Over all the hydrograph gets flattened
- However the volume of runoff remains the same.



Derivation of Unit hydrograph of different durations from available D hr unit hydrograph (contd…)

2. S-Curve technique

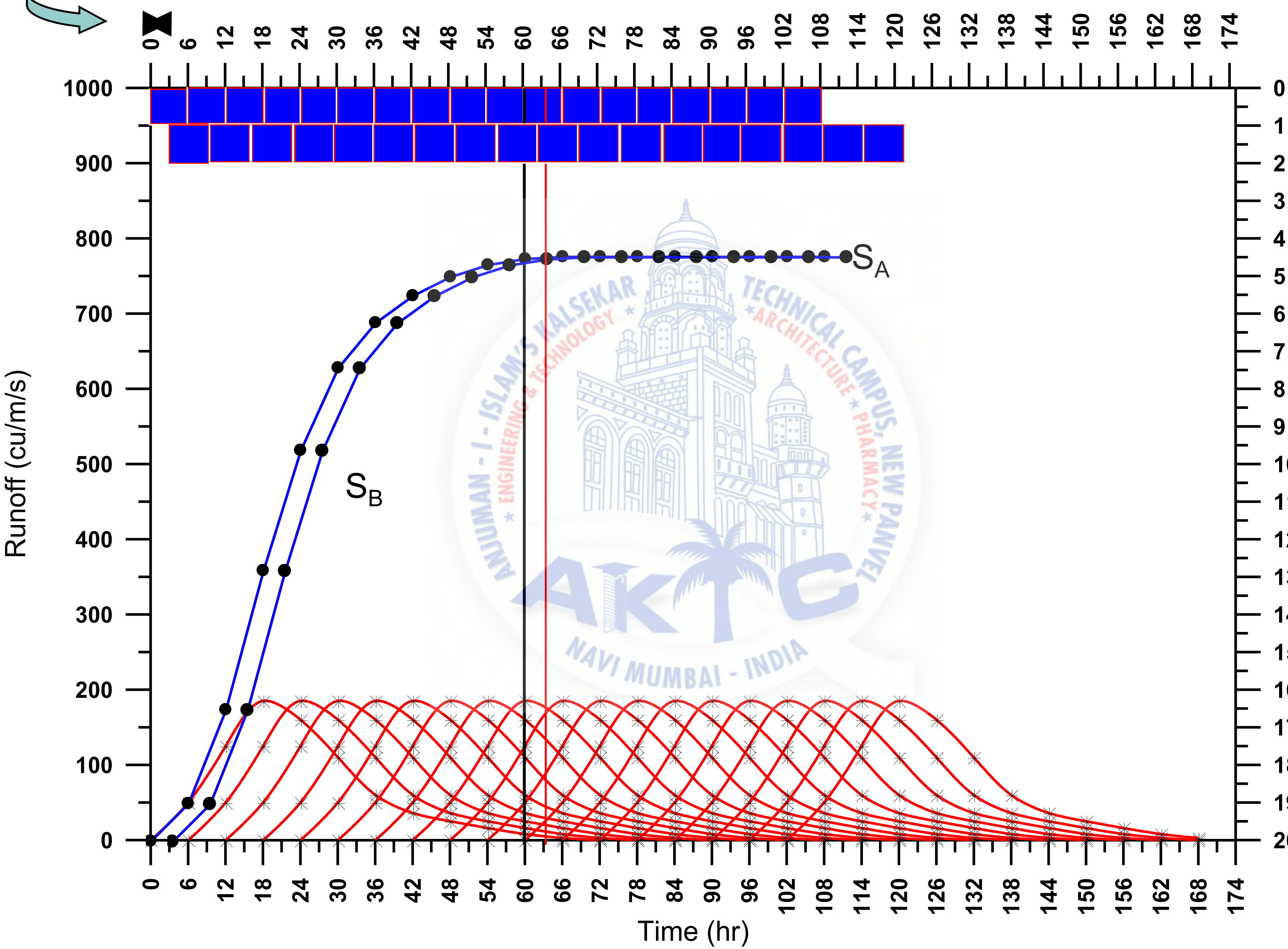
- Is suitable when *m* is a fraction (it works for integer also)
- To solve this method a D hr Unit hydrograph is available
- Develop a hydrograph produced by a continuous effective rainfall of D hr for a infinite period.
- Then get the summation hydrograph (S curve ordinates) obtained by summation of the infinite series of D hr UH spaced D hr apart.
  - The maximum summation value is
    - = (Area X 1 cm) / D hr


S_A
Runoff (cu/m/s)
Time (hr)
0
100
200
300
400
500
600
700
800
900
1000
0 6 12 18 24 30 36 42 48 54 60 66 72 78 84 90 96 102 108 114 120 126 132 138 144 150 156 162 168 174
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Derivation of Unit hydrograph of different durations from available D hr unit hydrograph (contd…)

- This S curve represents the maximum rate at which an Effective rainfall intensity of 1 cm/hr in D hr duration can drain out from a catchment of area A km^2. (S_A)
- Suppose we lag this S curve by mD hrs (S_B)
- And take the difference between the two S curves lagged at mD hr intervals, (S_A-S_B)

mD hr duration for a rainfall of *m* cm/hr

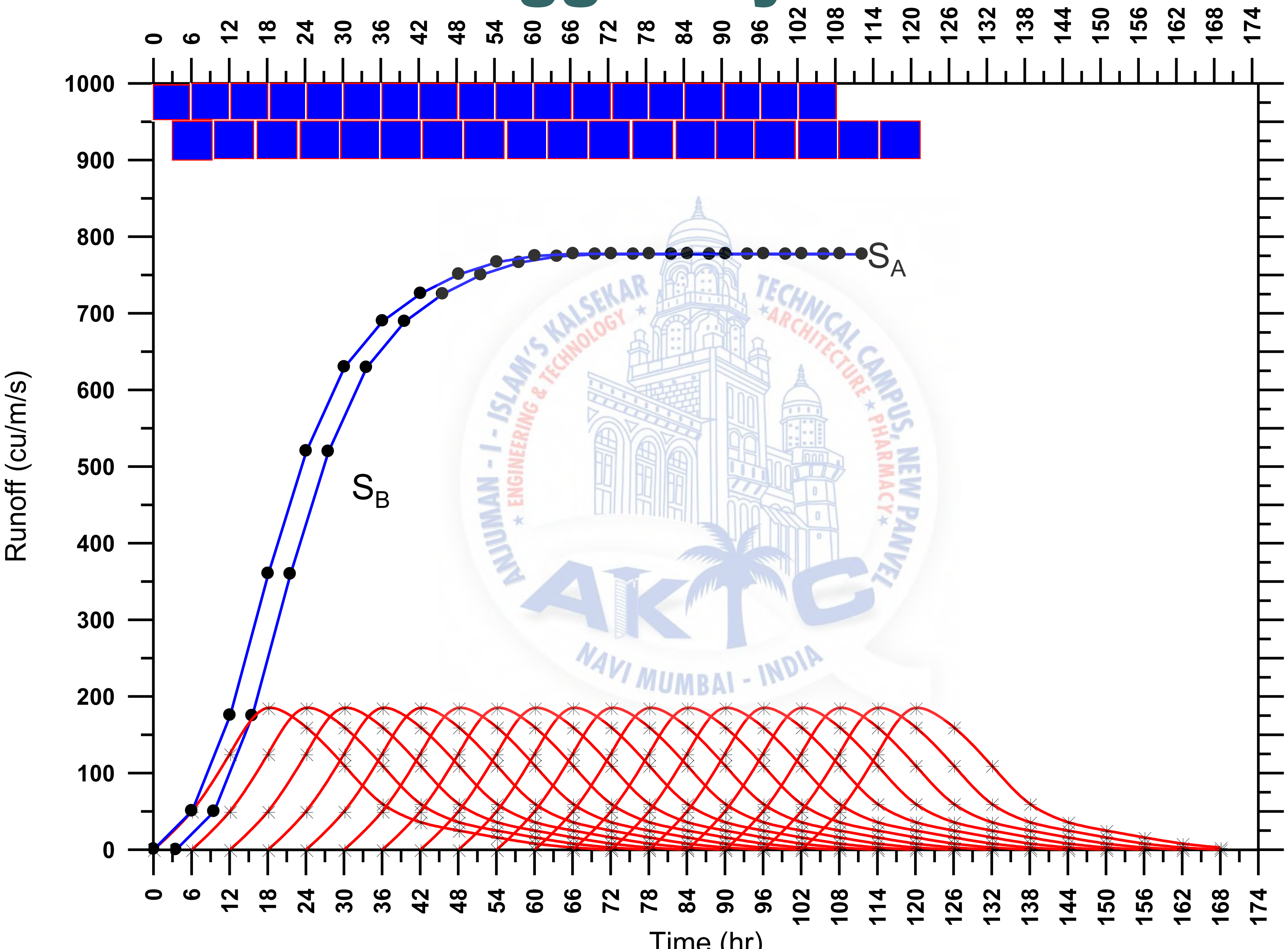


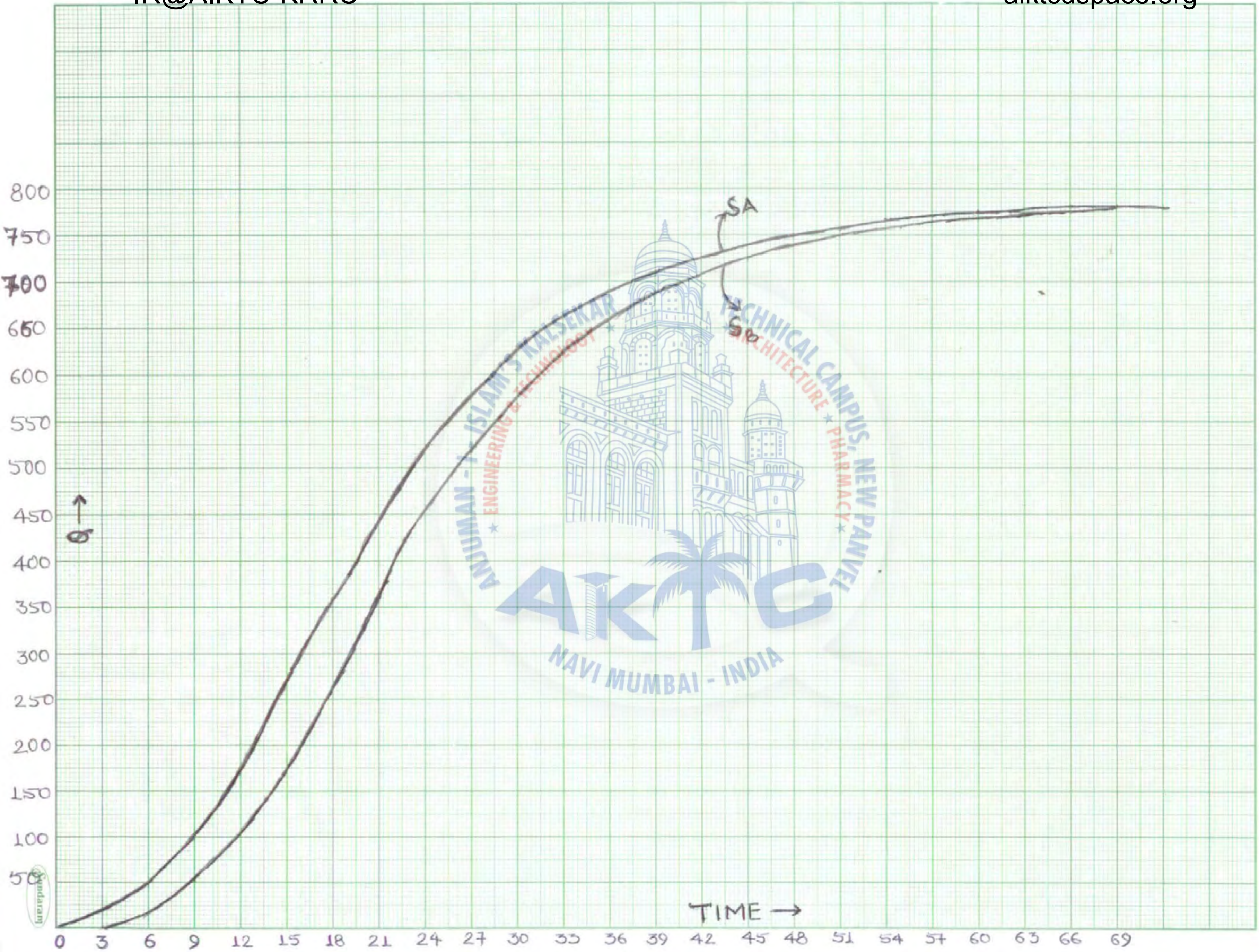
Derivation of Unit hydrograph of different durations from available D hr unit hydrograph (contd…)

- It will be the DRH produced by a rainfall excess of duration mD hrs and magnitude equal to *m* cm/hr (or also equal to $\frac{mD}{D}$)
- Thus the ordinate difference (S_A-S_B) divided by $\frac{mD}{D}$ (or *m*) results in a UH of mD hr duration.

? From the 6 hr UH derive 3 hr and 12 hr unit hydrographs

S curves lagged by 3 hr duration





SA
Sb
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
Ø
TIME →
0
3
6
9
12
15
18
21
24
27
30
33
36
39
42
45
48
51
54
57
60
63
66
69

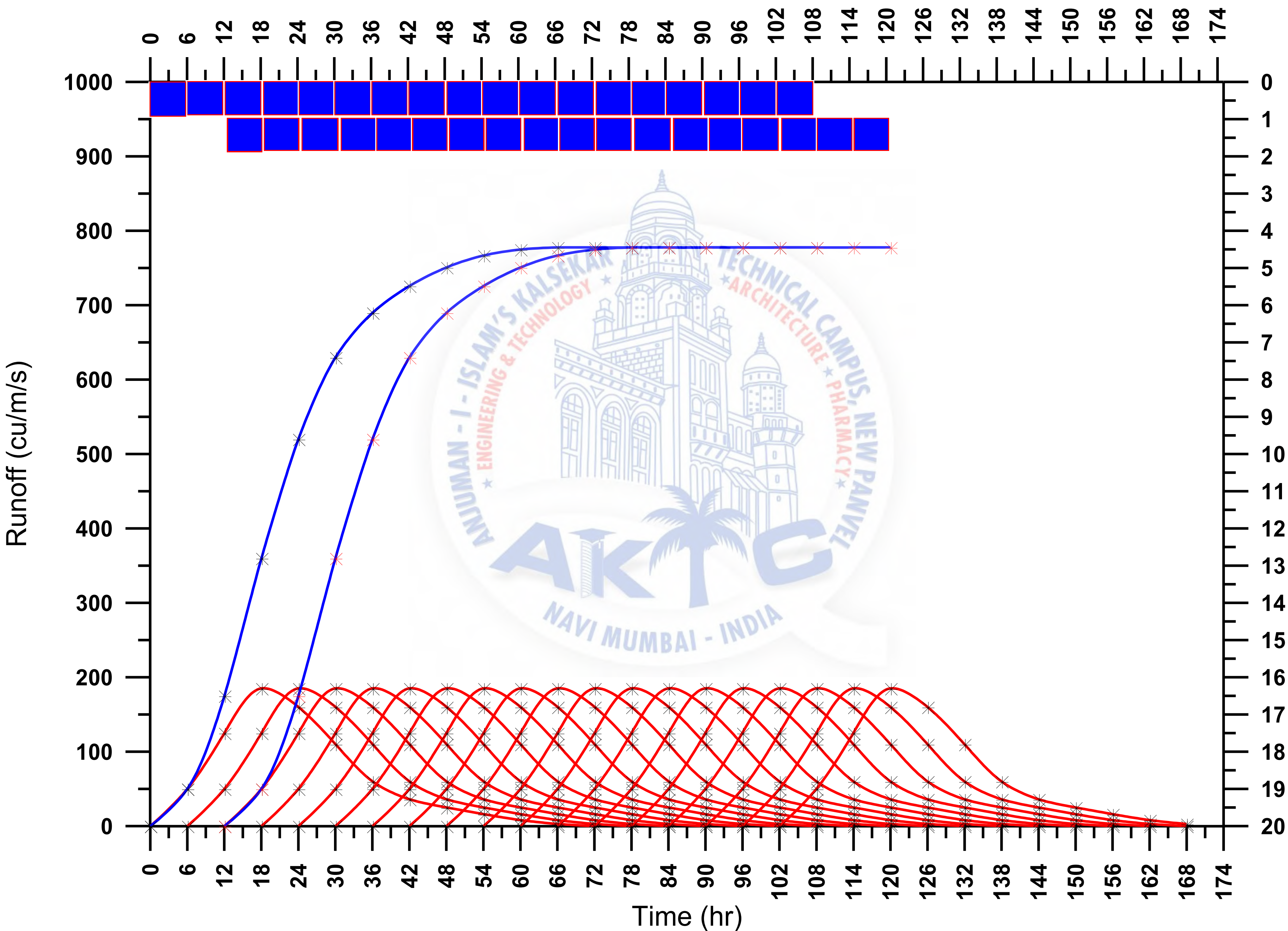
UH of 3 hr duration

Time hr	SA-SB	3 hr UH
0	0.0	0.0
3	20.0	40.0
6	32.5	65.0
9	52.5	105.0
12	67.5	135.0
15	100	200
18	95.0	190.0
21	77.5	155.0
24	65.0	130.0
27	55.0	110.0
30	50.0	100.0
33	32.5	65.0
36	30.0	60.0
39	20.0	40.0
42	16.3	32.5
45	15.0	30.0
48	12.5	25.0
51	10.0	20.0
54		15.0
57		10.0
60		7.0
63		4.0
66		2.0
69	0.0	0.0

After getting the difference of S_A-S_B
The ordinates of 3 hr UH = $(S_A-S_B)/(mD/D)$

In this case it is $(S_A-S_B)/0.5$

S curves lagged by 12 hr duration



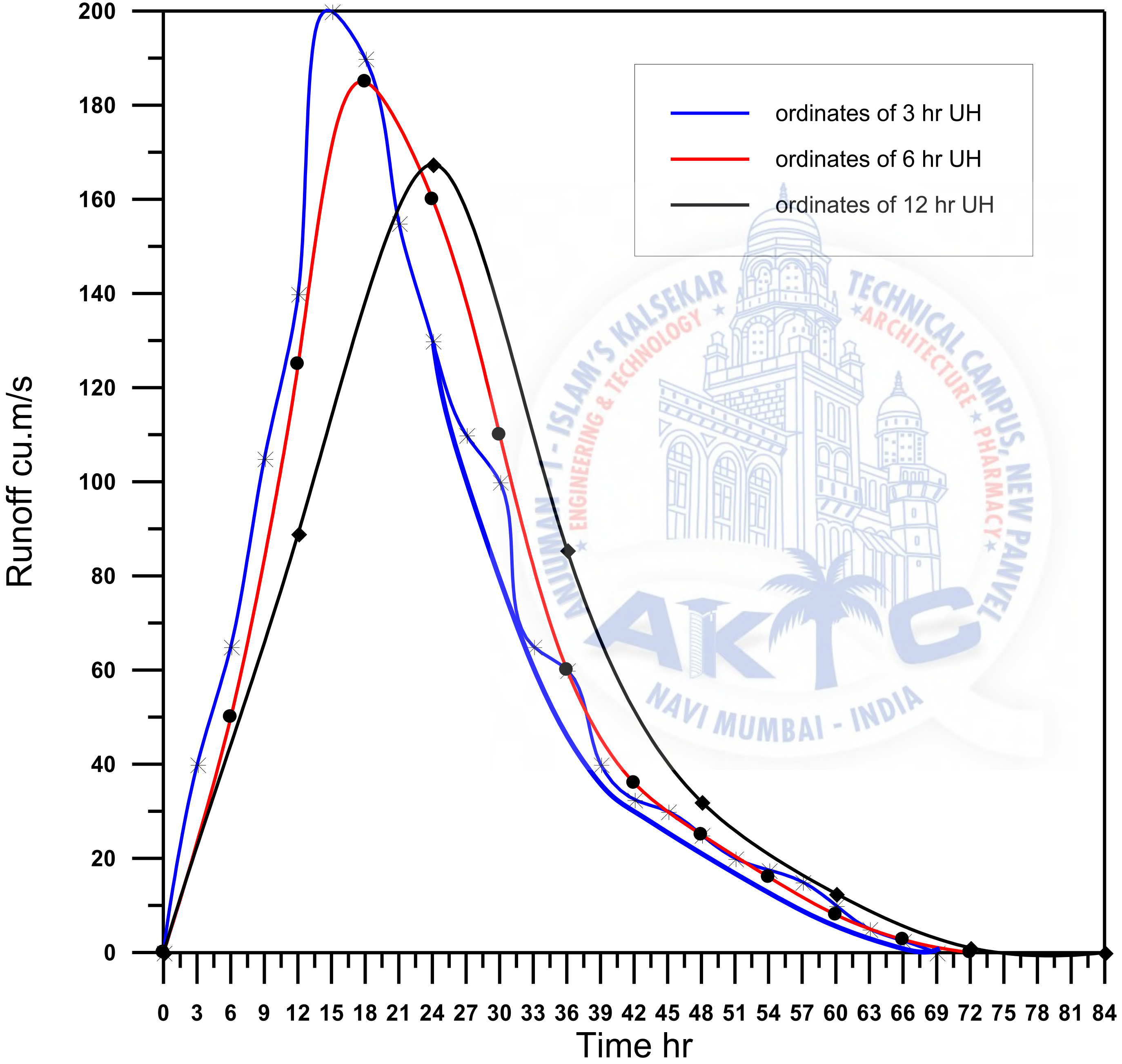
UH of 12 hr duration

Time hr	S_A	S_B	S_A-S_B	12 hr UH	S_A-S_B (from S curve graph)	12 hr UH ordinates
0	0		0	0	0	0
6	50		50	25		
12	175	0	175	87.5	178	89
18	360	50	310	155		
24	520	175	345	172.5	335	167.5
30	630	360	270	135		
36	690	520	170	85	171	85.5
42	726	630	96	48		
48	751	690	61	30.5	64	32
54	767	726	41	20.5		
60	775	751	24	12	25	12.5
66	777.7	767	10.7	5.35		
72	777.7	775	2.7	1.35	2	1
78	777.7	777.7	0	0		
84	777.7	777.7	0	0	0	0

After getting the difference of S_A-S_B The ordinates of 3 hr UH = (S_A-S_B)/(mD/D)

In this case it is (S_A-S_B)/2

? What happens when mD hr unit hydrograph is derived from D hr unit hydrograph in case of m is an integer and or m is a fraction



? Check the volume of runoff from each UH??? Are they equal???

Use of hydrographs

- The development of flood hydrographs for extreme rainfall magnitudes for use in the design of hydraulic structures
- Extension of flood-flow records based on rainfall records and
- Development of flood forecasting and warning systems based on rainfall.

Limitations of hydrographs

- It assumes uniform distribution of rainfall over the catchment.
- Intensity of rainfall is assumed constant for the duration of the rainfall excess.
- Precipitation must be from rainfall only. Snow-melt runoff cannot be satisfactorily represented by unit hydrograph.
- The catchment should not have unusually large storages in terms of tanks, ponds, large flood-bank storages, etc. which affect the line relationship between storage and discharge.
- For derivation of a Unit hydrograph the upper limit of basin area is 5000 km^2 and lower limit is 200 ha.

Rainfall duration of a Unit Hydrograph

A rough guide for the choice of duration D is that it should not exceed the least of

(i) Time to peak or time of rising limb

(ii) basin lag – best time is ¼ of basin lag

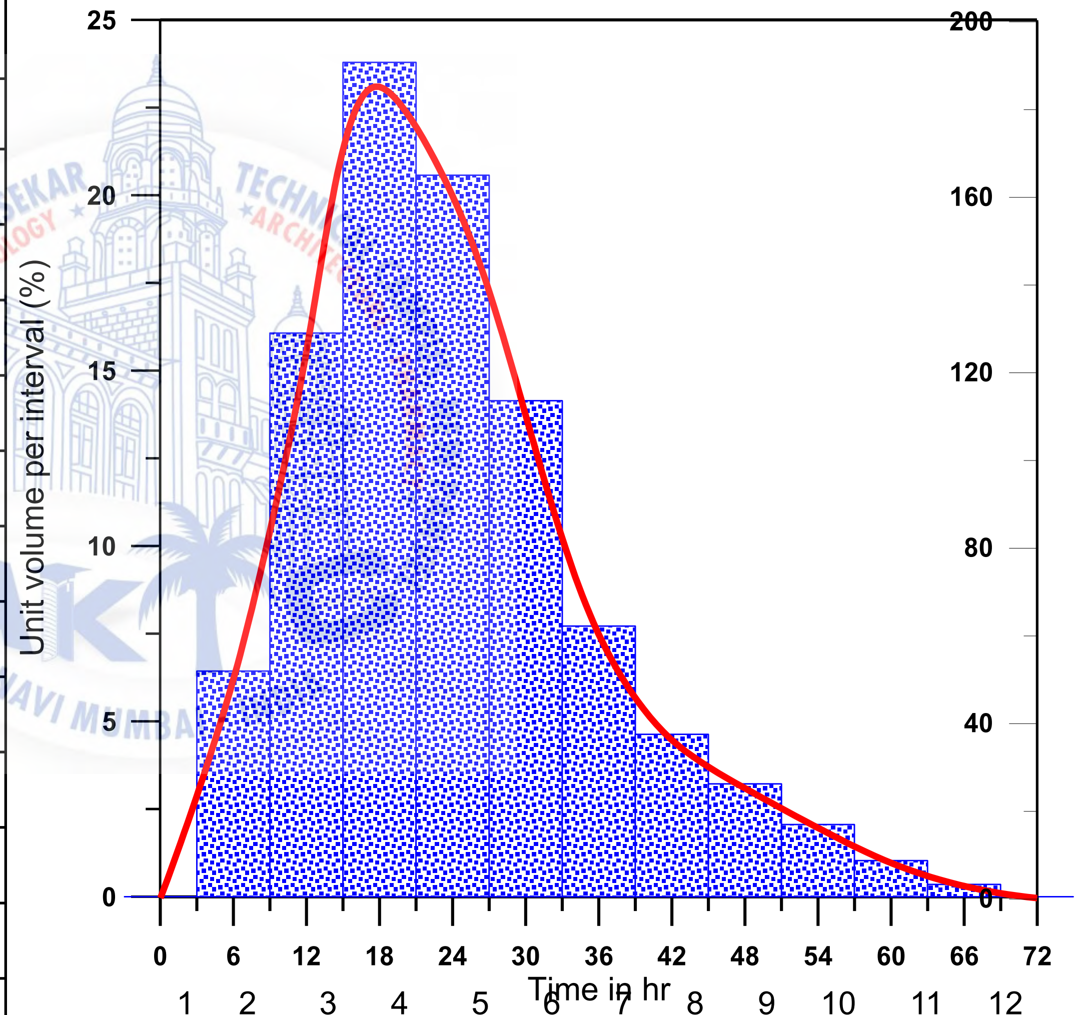
(iii) time of concentration

Distribution Graph

- Introduced by Bernad (1935)
  - Is the variation of the unit hydrograph ordinates with respect to the total runoff
- D hr UH with ordinates showing the percentage of the surface runoff.
- In this graph the base time interval must be equal to the duration of rainfall.
- The total are of the distribution graph works out to be 100%

Derive a distribution Graph from the 6-hr UH ordinates in a basin

Time (h)	UH ordinates (m^3/s)	Distribution Ratio %
0	0	0.00
6	50	6.43
12	125	16.07
18	185	23.79
24	160	20.57
30	110	14.14
36	60	7.72
42	36	4.63
48	25	3.21
54	16	2.06
60	8	1.03
66	2.7	0.35
72	0	0.00
Total	777.7	100.00



What is the use of these distribution graph

- These are more useful in comparing the runoff characteristics of different catchments, which are hydrologically similar.
- Also useful in establishing the UH for the un-gauged basins

A 4 hr distribution graph of a basin is available. Another similar catchment with an area of 50 km² has rainfalls of 3.5, 2.2 and 1.8 cm in consecutive 4 hr periods. Assuming an average ϕ-index of 0.25 cm/hr, determine the resulting direct runoff hydrograph

Unit periods (4 hr)	1	2	3	4	5	6
Distribution %	5	20	40	20	10	5

Time interval hr	Time of measurement hr	Rainfall cm	Infiltration loss cm	Effective rainfall	Distribution %	Runoff			Total runoff in cm	Runoff in m^3/s
						2.5 cm	1.2 cm	0.8 cm		
0-4	2	3.5	1	2.5	5	0.125			0.125	4.34
4-8	6	2.2	1	1.2	20	0.5	0.06		0.56	19.44
8-12	10	1.8	1	0.8	40	1	0.24	0.04	1.28	44.44
12-16	14				20	0.5	0.48	0.16	1.14	39.58
16-20	18				10	0.25	0.24	0.32	0.81	28.13
20-24	22				5	0.125	0.12	0.16	0.405	14.06
24-28	26					0	0.06	0.08	0.14	4.86
28-32	30						0	0.04	0.04	1.39
32-36	34							0	0	0.00

Synthetic Unit Hydrograph

- Is a technique to derive UH for a poorly gauged or un gauged basin from a known (gauged) hydrologicaly similar basins
- Synder's (1938) method is first method and widely used method
- How ever to derive this SUH for the un-gauged basin atleast the basin characteristics are available.
- Synder first studied the Appalachian Highlands in USA and developed the SUH
- The same was modified to suit other basin in other countries and was named as Synder's Unit Hydrograph
- In this method, estimate the parameters of the UH from the gauged Basin and apply that parameters to the un-gauged basin

Important parameters and constants in SUH

1. Time to peak
2. Standard duration of the rainfall
3. Peak discharge estimation
4. Base time of the unit hydrograph
5. Width of UH at 50% and 75% of the peak discharge

Important parameters and constants in SUH

1. Lag time
   - Between centre of mass of rainfall and runoff hydrograph mass
   - Since it is very difficult to find the center then it is the time between centre of mass of effective rainfall to peak discharge
   - Physically it is the average time of travel of water from all parts of the basin to the outlet.
   - It is function of basin characteristics such as basin size, length, stream density and vegetation.

$$t_p = C_t (LL_{ca})^{0.3}$$

$$t_p = C_t \left(\frac{LL_{ca}}{\sqrt{S}}\right)^n$$

t_p=lag time

L=basin length measured along the water course from the basin divide to the outlet point

L_{ca} = distance along the main water course from gauging station to a point of the watershed centroid in km

Ct = a regional constant representing watershed slope and storage

C_t-varies between 0.3 to 6

n is 0.38, C_t=1.715 (mountains), 1.03 (foot hills) and 0.5 (valley)

Important parameters and constants in SUH (contd...)

2. Standard duration of effective rainfall

$$t_r = \frac{t_p}{5.5}$$

If this works out to be non standard duration then rework out the basin lag time

$$t_p' = t_p + \frac{t_R - t_r}{4}$$

$$t_p' = \frac{21}{22} t_p + \frac{t_R}{4}$$

Where t_R=is the standard rainfall duration

Important parameters and constants in SUH (contd...)

3. Peak discharge estimation

$$Q_p = \frac{2.78 * C_p * A}{t_p^{'}}$$

Q_p-peak discharge in m^3/s

A – area of the basin in km^2

C_p is the regional constant (varies from 0.31 to 0.91)

Important parameters and constants in SUH (contd...)

4. Base time of the unit hydrograph

$$t_b = 72 + t_p^{'} \quad or$$

$$t_b = 5\left(t_p^{'} + \frac{t_R}{2} \right)$$

One has to select the suitable base time from this estimates

Generally second one is selected, this shows that base time is five times the time to peak.

Important parameters and constants in SUH (contd…)

5. Width of UH at 50% and 75% of Q_p

$$W_{50} = \frac{5.87}{q^{1.08}}$$

$$W_{75} = \frac{W_{50}}{1.75}$$

W_{50} is the width (time period at 50% of the peak discharge)

W_{75} is the width (time period at 75% of the peak discharge)

$q = Q_p/A$ =discharge per unit area $m^3/s/m^2$

? Two catchments A and B are considered meteorologically similar.

Item	Catchment A	Catchment B
A	2718 km^2	1400 km^2
L	148 km	106 km
L_{ca}	76 km	52 km

For a 6-hr UH in catchment A, the peak discharge is at 37 h from start of the rainfall excess and its value is 200 m^3/s . Determine the elements of 6-hr SUH for catchment B

Solve this in class room