Multiple-Choice Questions

GATE (2000–2009)

Year 2000

1. In a biaxial stress problem, the stresses in *x*- and *y*-directions are σ_x = 200 MPa, σ_Y = 100 MPa. The maximum principal stress in MPa is:
   (a) 50
   (b) 100
   (c) 150
   (d) 200

   Answer: (d)
   Reference: Chapter 4

2. A steel shaft 'A' of diameter *d* and *L* is subjected to a torque *T*. Another shaft '*B*' made of aluminium of the same diameter *d* and length 0.5*L* is also subjected to the same torque *T*. The shear modulus of steel is 2.5 times that of aluminium. The shear stress in the steel shaft is 100 MPa. The shear stress in the aluminium shaft, in MPa, is:
   (a) 40
   (b) 50
   (c) 100
   (d) 250

   Answer: (c)
   Explanation: From $\tau_{\max} = 16T/\pi d^3$ shows $\tau_{\max}$ depends only on torque and diameter. Since both shifts are of equal size and carry the same torque, shear stresses must be the same in both cases.

   Reference: Chapter 2

3. A simply supported beam carries a load *P* through a bracket as shown in Figure 1. The maximum bending moment in the beam is:

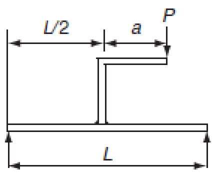


Figure 1 Question 3.

   (a) $\dfrac{PL}{4}$
   (b) $\dfrac{PL}{4} + \dfrac{Pa}{2}$
   (c) $\dfrac{PL}{2} + Pa$
   (d) $\dfrac{PL}{2} - Pa$

   Answer: (b)
   Explanation: We start drawing the free-body diagram of the beam as shown in Figure 2.

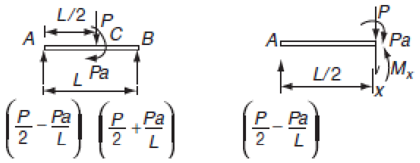


Figure 2 Explanation of Question 3.

Taking a section just right to the concentrated load P at C, we get:

$$\sum M_C = 0 \quad \Rightarrow \quad M_x = Pa + \frac{L}{2}\left(\frac{P}{2} - \frac{Pa}{L}\right) = \frac{PL}{4} + \frac{Pa}{2}$$

Reference: Chapter 5

4. The stress–strain behaviour of a material is shown in Figure 3.

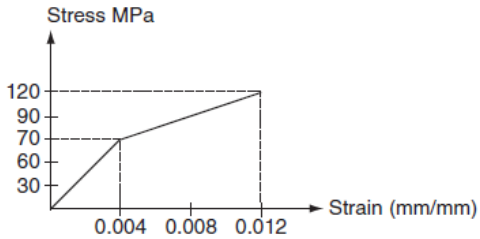


Figure 3 Question 4.

Its resilience and toughness, in Nm and m^3, respectively, are:
(a) 28×10^4, 76×10^4
(b) 28×10^4, 48×10^4
(c) 14×10^4, 90×10^4
(d) 76×10^4, 104×10^4

Answer: (c)

Explanation: The resilience is calculated as

$$\text{Resilience} = \frac{1}{2} \times 70(10)^6 \times 0.004\ \text{N m/m}^3 = 14 \times 10^4\ \text{N m/m}^3$$

The toughness is calculated as

$$\text{Toughness} = \frac{1}{2} \times 70(10^6) \times 0.004 + \frac{1}{2} \times (70 + 120) \times 0.008\ \text{Nm/m}^3 = 90 \times 10^4\ \text{Nm/m}^3$$

Reference: Chapter 10

Year 2001

5. The shape of the bending moment diagram for a uniform cantilever beam carrying a uniformly distributed load over its length is:
(a) a straight line.
(b) a hyperbola.
(c) an ellipse.
(d) a parabola.

Answer: (d)
Reference: Chapter 5

6. Bars AB and BC, each of negligible mass, support load *P* as shown in Figure 4. In this arrangement,

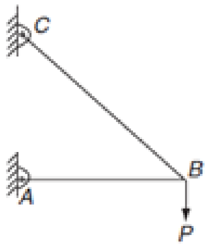


Figure 4 Question 6.

(a) Bar AB is subjected to bending but bar BC is not.
(b) Bar AB is not subjected to bending but bar BC is.
(c) neither bar AB nor bar BC is subjected to bending
(d) both bars AB and BC are subjected to bending.

Answer: (d)
Reference: Chapter 5

7. Two helical tensile springs of the same material and also having identical mean coil diameter and weight, have wire diameters *d* and *d*/2.The ratio of their stiffness constants is:
(a) 16.0
(b) 4.0
(c) 64.0
(d) 128.0

Answer: (a)
Explanation: We know stiffness of spring, *k* is

$$k = \frac{Gd^4}{64nR^3}$$

Therefore, $k \propto d^4$
Hence,

$$\frac{k_1}{k_2} = \left(\frac{d_1}{d_2}\right)^4 = 2^4 = 16$$

Reference: Chapter 2

8. The maximum principal stress for the stress shown in Figure 5 is:

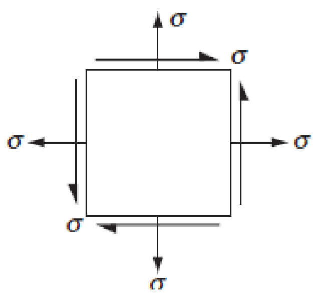


Figure 5 Question 8.

(a) σ
(b) 2σ
(c) 3σ
(d) 1.5σ

Answer: (b)
Explanation:

$$\sigma_{\max} = \frac{1}{2}\left(\sigma_{xx} + \sigma_{yy}\right) + \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sigma + \sqrt{0^2 + \sigma^2} = 2\sigma$$

Reference: Chapter 4

Year 2002

9. The total area under the stress–strain curve of a mild steel specimen tested up to failure under tension is a measure of:
(a) ductility.
(b) ultimate strength.
(c) stiffness.
(d) toughness.

Answer: (d)
Reference: Chapter 10

10. If the wire diameter of a closed-coil helical spring subjected to compressive load is increased from 1 cm to 2 cm, other parameters remaining the same, the deflection will decrease by a factor
(a) 16
(b) 8
(c) 4
(d) 2

Answer: (a)
Explanation: Refer the explanation of Question 7 above.
Reference: Chapter 2

11. The relationship between Young's modulus (*E*), bulk modulus (*K*) and Poisson's ratio (ν) is given by:
(a) $E = 3K(1-2\nu)$
(b) $K = 3E(1-2\nu)$
(c) $E = 3K(1-\nu)$
(d) $K = 3E(1-\nu)$

Answer: (a)
Explanation:
We know that volumetric strain,

$$\epsilon_{\text{V}} = \frac{3\sigma_{\text{h}}}{E}(1-2\nu)$$

where σ_{h} is the hydrostatic stress or mean stress. Hence,

$$K = \frac{\sigma_{\text{h}}}{\epsilon_{\text{V}}} = \frac{E}{3(1-2\nu)} \Rightarrow E = 3K(1-2\nu)$$

Reference: Chapters 1 and 9

Year 2003

12. The second moment of a circular area about the diameter is given by (*D* is the diameter)

(a) $\dfrac{\pi D^4}{4}$

(b) $\dfrac{\pi D^4}{16}$

(c) $\dfrac{\pi D^4}{32}$

(d) $\dfrac{\pi D^4}{64}$

Answer: (d)
Reference: Chapter 6

13. A concentrated load of *P* acts on a simply supported beam of span *L* at a distance *L*/3 from the left support. The bending moment at the point of application of the load is given by:

(a) $\dfrac{PL}{3}$

(b) $\dfrac{2PL}{3}$

(c) $\dfrac{PL}{9}$

(d) $\dfrac{2PL}{9}$

Answer: (d)
Explanation: Refer to Figure 6. Considering free-body diagram of AB [Figure 6(b)], we get

$$\sum M_{\mathrm{B}} = 0 \Rightarrow M_x = \frac{2PL}{9}$$

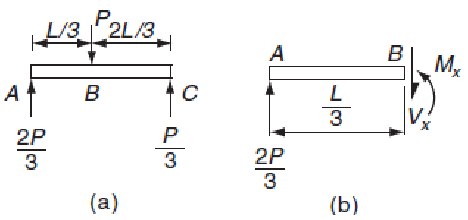


Figure 6 Explanation for Question 13.

Reference: Chapter 5

14. Two identical circular rods of same diameter and same length are subjected to same magnitude of axial tensile force. One of the rods is made of mild steel having modulus of elasticity 206 GPa. The other rod is made out of cast iron having the modulus of elasticity of 100 GPa. Assume both the material to be homogeneous and isotropic and the axial force causes the same amount of uniform stress in both the rods. The stresses developed are within the proportional limit of the respective materials.
Which of the following observations is correct?
(a) Both rods elongate by the same amount.
(b) Mild steel rod elongates more than the cast iron.
(c) Cast iron rod elongates more than the mild steel rod.
(d) Stresses are equal and strains are also equal in both the rods.

Answer: (c)
Explanation:

$$\delta_{\mathrm{St}} = \frac{P \cdot L}{AE_{\mathrm{St}}},\ \delta_{\mathrm{CI}} = \frac{PL}{AE_{\mathrm{CI}}} \quad (\text{as } E_{\mathrm{St}} > E_{\mathrm{CI}},\ \text{so } \delta_{\mathrm{CI}} > \delta_{\mathrm{St}})$$

Reference: Chapter 1

15. The beams, one having square cross-section and another circular cross-section, are subjected to the same amount of bending moment. If the cross-sectional area as well as the material of both the beams are the same then

(a) maximum bending stress developed in both the beams is the same.
(b) the circular beam experiences more bending stress than the square one.
(c) the square beam experiences more bending stress than the circular one.
(d) as the material is same both beams will experience same deformation.

Answer: (b)
Explanation: Here for square beam:

$$\sigma_{\max} = \frac{6M}{a^3} = \frac{6M}{Aa^2}$$

for circular beam:

$$\sigma'_{\max} = \frac{32M}{\pi d^3} = \frac{8M}{Ad^2}$$

But,

$$\frac{\pi d^2}{4} = a^2 \Rightarrow \frac{d^2}{a^2} = \frac{4}{\pi}$$

Therefore,

$$\frac{\sigma_{\max}}{\sigma'_{\max}} = \frac{6M}{Aa^2} \cdot \frac{Ad^2}{8M} = \frac{3}{4} \cdot \frac{d^2}{a^2} = \frac{3}{\pi} \quad \left(\text{as} \frac{d^2}{a^2} = \frac{4}{\pi}\right)$$

and so, $\sigma_{\max} < \sigma'_{\max}$.

Reference: Chapter 6

16. Maximum shear stress developed on the surface of a solid circular shaft under pure torsion is 240 MPa. If the shaft diameter is doubled, then the maximum shear stress developed corresponding to the same torque will be

(a) 120 MPa
(b) 60 MPa
(c) 30 MPa
(d) 15 MPa

Answer: (c)
Explanation:

$$\tau_{\max} = \frac{16T}{\pi d^3} \Rightarrow \tau_{\max} \propto \frac{1}{d^3}$$

Therefore,

$$\frac{\tau'_{\max}}{\tau_{\max}} = \frac{d^3}{d'^3} = \frac{d^3}{(2d)^3} = \frac{1}{8} \Rightarrow \tau'_{\max} = \frac{\tau_{\max}}{8}$$

or $\tau'_{\max} = 30 \text{ MPa}$.

Reference: Chapter 2

17. A simply supported laterally loaded beam was found to deflect more than a specified value. Which of the following measures will reduce deflection?

(a) Increase the area moment of inertia
(b) Increase the spam of the beam
(c) Select a different material having lesser modulus of elasticity
(d) Increase the magnitude of the load.

Answer: (a)
Reference: Chapter 7

18. A shaft subjected to torsion experiences a pure shear stress on the surface. The maximum principal stress on the surface which is at 45° to the axis will have a value

(a) $\tau \cos 45^\circ$

(b) $2\tau \cos 45^\circ$

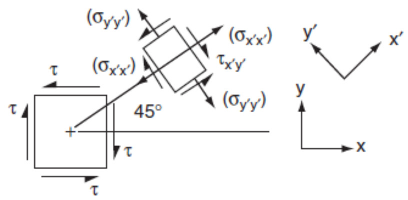
(c) $2\tau \sin 45^\circ$

(d) $2\tau \sin 45^\circ \cos 45^\circ$

Answer: (d)
Explanation:

$$\sigma_{\max} = \frac{1}{2}(\sigma_{x'x'} + \sigma_{y'y'}) + \sqrt{\left(\frac{\sigma_{x'x'} + \sigma_{y'y'}}{2}\right)^2 + \tau_{x'y'}^2}$$

Now, $\sigma_{x'x'}$ is the stress in x'–x' direction [Figure 7(a)] and $\sigma_{y'y'}$ is the stress in y'–y' direction.



(a)

Clearly,

$$\sigma_{x'x'} = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) + \frac{1}{2}(\sigma_{xx} - \sigma_{yy})\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$= \frac{1}{2}(0) + 0 - \tau \sin 90° = -\tau \quad (\text{as } \sigma_{xx} = \sigma_{yy} = 0;\ \tau_{xy} = \tau)$$

Similarly,

$$\sigma_{y'y'} = \tau \quad (\text{as } \sigma_{x'x'} + \sigma_{y'y'} = \sigma_{xx} + \sigma_{yy} = 0, \text{ etc.})$$

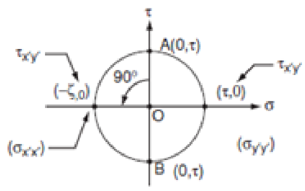
Finally, shear stress in x'–y' axes, $\tau_{x'y'}$ is given by

$$\tau_{x'y'} = \frac{1}{2}(\sigma_{xx} - \sigma_{yy})\sin 2\theta + \tau_{xy}\cos 2\theta$$
$$= 0 \quad (\text{for } \theta = 45°)$$

Therefore,

$$\sigma_{\max} = 0 + \sqrt{\tau^2 + 0^2} = \tau$$
$$= \tau \sin 90° = 2\tau \sin 45° \cos 45°$$

Alternative explanation: You can arrive at the above results also very quickly by drawing the Mohr's circle of stresses as shown in Figure 7(a).



(b)

Figure 7 (a) Explanation, (b) alternative explanation for Question 18.

Thus, from the above figure,

$$\sigma_{x'x'} = -\tau; \quad \sigma_{y'y'} = \tau; \quad \tau_{x'y'} = 0$$

Putting these in the equation of $\sigma_{\max}$, we get the same result as above.

Reference: Chapter 4

19. The state of stress at a point P in a two-dimensional loading is such that the Mohr's circle is a point located at 175 MPa on the positive normal stress axis.

(i) Determine the maximum and minimum principal stresses with respect from the Mohr's circle.

(a) (+175, –175) MPa
(b) (+175, +175) MPa
(c) (0, –175) MPa
(d) (0,0)

(ii) Determine the directions of maximum and minimum principal stresses at the point P from the Mohr's circle:

(a) 0°, 90°
(b) 90°, 0°
(c) 45°, 135°
(d) All directions

Answer: (i) (b); (ii): (d)
Explanation:
(i) For stress in one direction and zero stress in its orthogonal direction (i.e., uniaxial stress consideration) in biaxial stress case, Mohr's circle will degenerate to a point.
Reference: Chapter 4

Year 2004

20. In terms of Poisson's ratio (v), the ratio of Young's modulus (E) to shear modulus (G) of elastic material is

(a) $2(1+\nu)$
(b) $2(1+\nu)$
(c) $(1+\nu)/2$
(d) $(1-\nu)/2$

Answer: (a)
Reference: Chapters 1 and 9

21. The following figure (Figure 8) shows the state of stress at a point in a stressed body. The magnitudes of normal stresses in the x and y direction are 100 MPa and 20 MPa, respectively. The radius of Mohr's stress circle representing this state of stress is:

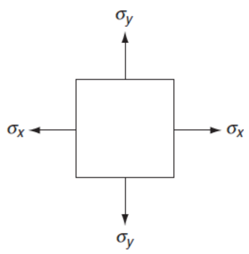


Figure 8 Question 21.

(a) 120
(b) 80
(c) 40
(d) 60

Answer: (d)
Explanation:

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau^2_{xy}} = \sqrt{\left(\frac{100+20}{2}\right)^2 + 0} = 60$$

Reference: Chapter 4

22. A torque of 10 N m is transmitted through a stepped shaft as shown in Figure 9. The torsional stiffness of individual sections of lengths MN, NO and OP are 20 N m/rad, 30 N m/rad and 60 N m/rad, respectively. The angular deflection between the ends M and P of the shaft is:

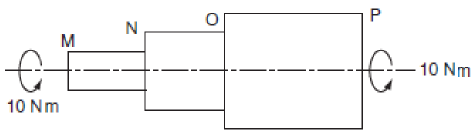


Figure 9 Question 22.

(a) 0.5 rad
(b) 1.0 rad
(c) 5.0 rad
(d) 10.0 rad

Answer: (b)
Explanation: Torsional springs are in series. Therefore, equivalent torsional stiffness, $(k_t)_e$ is given by

$$\frac{1}{(k_t)_e} = \frac{1}{(k_t)_{MN}} + \frac{1}{(k_t)_{NO}} + \frac{1}{(k_t)_{OP}}$$

Thus,

$$\frac{1}{(k_t)_e} = \frac{1}{20} + \frac{1}{30} + \frac{1}{60} = \frac{3+2+1}{60} = \frac{1}{10}$$

Hence,

$$(k_t)_e = 10 \text{N m/rad}$$

$$\text{or} \quad \theta_{M/P} = \frac{T}{(k_t)_e} = \frac{10}{10} = 1 \text{rad}$$

Reference: Chapter 2

23. The figure below (Figure 10) shows a steel rod of 25 mm^2 cross-sectional area. It is loaded at four points, K, L, M and N. Assume E = 200 GPa. The total change in length of the rod due to loading is:

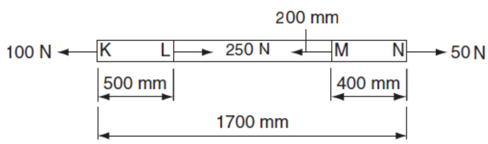


Figure 10 Question 23.

(a) 1 μm
(b) –10 μm
(c) 16 μm
(d) –20 μm

Answer: (b)
Explanation: Refer to free-body diagram of different segments:

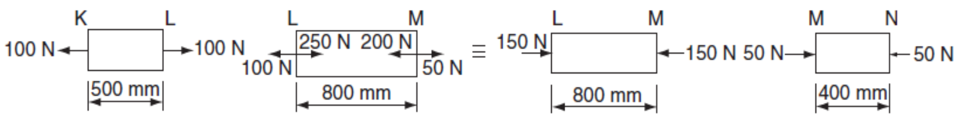


Figure 11 Explanation for Question 23.

Therefore,

$$\delta_{L/K} = \frac{(100)(500)}{(25)(200)(10^3)} \text{mm} = +10 \mu\text{m}$$

$$\delta_{M/L} = \frac{(150)(800)}{(25)(200)(10^3)} \text{mm} = -24 \mu\text{m}$$

and $$\delta_{N/M} = +\frac{(50)(400)}{(25)(200)(10^3)}\,\text{mm} = +4\,\mu\text{m}$$

Therefore,

$$\begin{aligned}\delta_{N/K} &= \delta_{L/K} + \delta_{M/L} + \delta_{N/M} \\ &= (10 - 24 + 4)\,\mu\text{m} = -10\,\mu\text{m}\end{aligned}$$

Reference: Chapter 1

24. A solid circular shaft of 60 mm diameter transmits a torque of 1600 N m. The value of maximum shear stress developed is:

(a) 37.72 MPa
(b) 47.72 MPa
(c) 57.72 MPa
(d) 67.72 MPa

Answer: (a)
Explanation:

$$\tau_{\max} = \frac{16T}{\pi d^3}\,\frac{(16)(1600)(10^3)}{\pi(60)^3}\,\frac{N}{\text{mm}^2} = 37.72\,\text{MPa}$$

Reference: Chapter 2

25. A steel beam of breadth 120 mm and height 750 mm is loaded as shown in Figure 12. Assume E_{Steel} = 200 GPa.

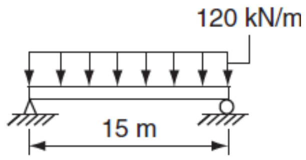


Figure 12 Question 25.

(i) The beam is subjected to a maximum bending moment of
(a) 3375 kN M
(b) 4750 kN m
(c) 6750 kN m
(d) 8750 kN m

(ii) The value of maximum deflection of the beam is:
(a) 93.75 mm
(b) 83.75 mm
(c) 73.75 mm
(d) 63.75 mm

Answer: (i) (a); (ii) (a)
Explanation:
(i) For uniformly distributed load with intensity w_o, we get

$$M_{\max} = \frac{w_o L^2}{8} = \frac{(120)(15)^2}{8}\ \text{kN m} = 3375\ \text{kN m}$$

(ii)

$$\delta_{\max} = \frac{5w_o L^4}{384EI} = \frac{5w_o L^4}{384E \times \left(\frac{1}{12}bh^3\right)} = \frac{5w_o L^4}{32Ebh^3}$$

Reference: (i) Chapter 5; (ii) Chapter 7

26. A uniform stiff rod of length 30 mm and having a weight of 300 N is pivoted at one end and connected to a spring at the other end (Figure 13). For keeping the rod vertical in a stable position, the minimum value of spring constant *k* needed is:

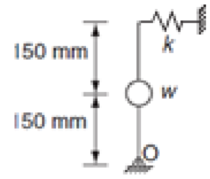


Figure 13 Question26.

(a) 300 N/m
(b) 400 N/m
(c) 500 N/m
(d) 1000 N/m

Answer: (d)
Explanation: For small angle ($\delta\theta$) of tilting from vertical position, we get bending moment about O as:

$$(w)(150\sin\delta\theta)-(k)(300\,\delta\theta)(300)=0$$

or

$$(w)(150)\delta\theta - 300^2 k\cdot\delta\theta = 0 \Rightarrow k=\frac{(300)(150)}{300^2}$$

which gives k = 0.5 N/m = 500 N/mm.

Reference: Chapter 8

Year 2005

27. A uniform, slender cylindrical rod is made of a homogeneous and isotropic material. The rod rests on a frictionless surface and is heated uniformly. If the radial and longitudinal thermal stresses are represented by σ_r and σ_z, respectively, then:

(a) $\sigma_r = 0, \sigma_z = 0$
(b) $\sigma_r \neq 0, \sigma_z = 0$
(c) $\sigma_r = 0, \sigma_z \neq 0$
(d) $\sigma_r \neq 0, \sigma_z \neq 0$

Answer: (a)
Reference: Chapter 1

28. Two identical cantilever beams are supported as shown in Figure 14, with their free ends in contact through a rigid roller. After the load P is applied, the free ends will have

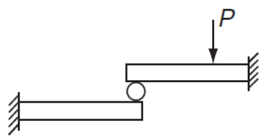


Figure 14 Question 28.

(a) equal deflections but not equal slopes.
(b) equal slopes but not equal deflections.
(c) equal slopes as well as equal deflections.
(d) neither equal slopes nor equal deflections.

Answer: (a)
Reference: Chapter 7

29. Two shafts AB and BC of equal length and diameters d and $2d$ are made of the same material (Figure 15). They are joined at B through a shaft coupling, while the ends A and C are built-in (cantilevered). A twisting moment T is applied to the coupling. If T_A and T_C represent the twisting moments at the ends A and C, respectively, then:

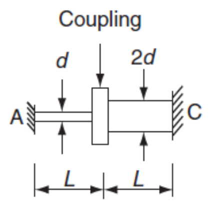


Figure 15 Question 29.

(a) $T_C = T_A$
(b) $T_C = 8\,T_A$
(c) $T_C = 16\,T_A$
(d) $T_A = 16\,T_C$

Answer: (c)
Explanation: Clearly, $\theta_{B/A} = \theta_{B/C}$, so

$$\frac{T_A l}{GJ_1} = \frac{T_C l}{GJ_2}$$

or
$$\frac{T_A}{J_1} = \frac{T_C}{J_2} = \frac{T_A + T_C}{J_1 + J_2} = \frac{T}{J_1 + J_2}$$

$$\frac{T_A}{T_C} = \frac{J_1}{J_2} = \frac{\dfrac{\pi d^4}{32}}{\dfrac{\pi(2d)^4}{32}} = \frac{1}{2^4} = \frac{1}{16}.$$

which gives $T_C = 16T_A$.

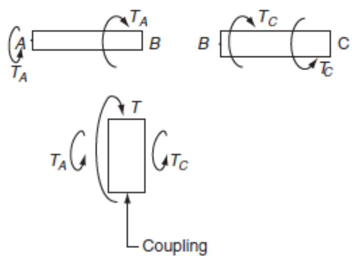


Figure 16 Explanation for Question 29.

Reference: Chapter 2

30. A beam is made up of two identical bars AB and BC by hinging them together at B (Figure 17). The end A is built-in (cantilevered).and end C is simply supported. With the load P acting as shown, the bending moment at A is:

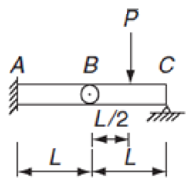


Figure 17 Question 30.

(a) 0
(b) $PL/2$
(c) $3PL/2$
(d) indeterminate

Answer: (a)
Explanation: Let us consider the free-body diagrams of the segments BC and AB in Figure 18.

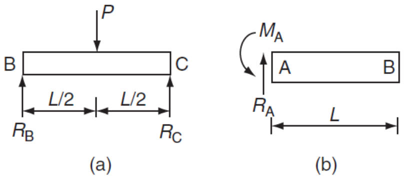


Figure 18 Question 30.

From Figure 18(a), due to the symmetry, we conclude that RB = P/2 and from Figure 18(b), considering $\Sigma M_A = 0$

$$M_A = R_B L \quad \text{or} \quad M_A = PL/2$$

Reference: Chapter 5

31. A cantilever beam carries antisymmetric load shown in Figure 19, where w_o is the peak intensity of the distributed load. Qualitatively, the correct bending moment diagram for this beam is

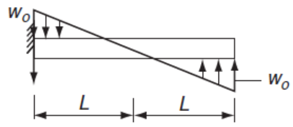
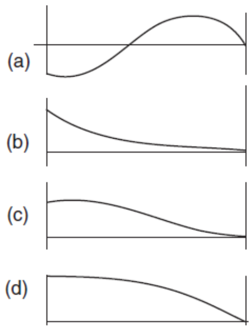


Figure 19 Question 31.



Answer: (c)
Explanation: Shear forces at A and B are 0 and hence bending moment diagram must have horizontal tangents at these ends.
Reference: Chapter 5

32. A cantilever beam has square cross-section of 10 mm× 10 mm. It carries a transverse load of 10 N (Figure 20). Considering only the bottom fibres of the beam, the correct representation of the longitudinal variation of the bending stress is

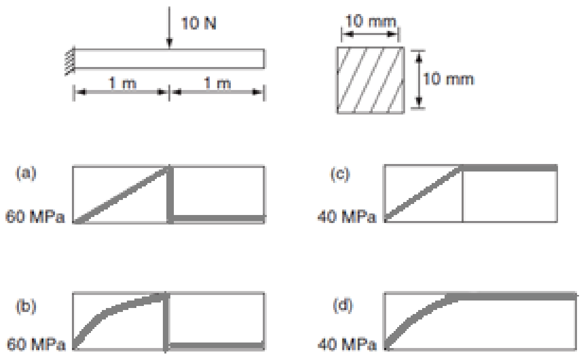


Figure 20 Question 32.

Answer: (a)
Explanation: We note M_{max} = Moment at fixed end = 10 N m. Variation of bending moment is linear up to midpoint and then becomes zero.

$$\sigma_{max} = \frac{6M_{max}}{a^3} = \frac{(6)(10)(10^3)}{(10)^3}\frac{N}{mm^2} = 60 \text{ MPa}$$

Hence (a) is correct.
Reference: Chapter 6

33. The Mohr's circle for plane stress for a point in a body is shown in Figure 21. The design is to be done on the basis of maximum shear stress theory for yielding. Then, yielding will just begin if the designer chooses a ductile material whose yield strength is:

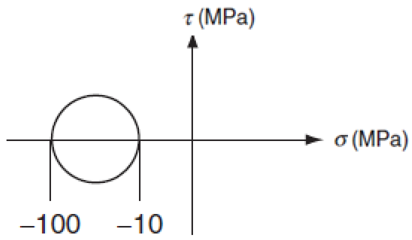


Figure 21 Question 33.

(a) 45 MPa
(b) 50 MPa
(c) 90 MPa
(d) 100 MPa

Answer: (d)
Explanation: Here for plane stress condition the principal stresses are:

$$\sigma_1 = 0 \text{ MPa}, \quad \sigma_2 = -10 \text{ MPa}, \quad \sigma_3 = -100 \text{ MPa}$$

Thus,

$$\tau_{max} = \frac{\tau_{max} - \tau_{min}}{2} = 50 \, \text{MPa}$$

According to maximum shear stress theory, yielding initiates when

$$\tau_{max} = \frac{\sigma_{yp}}{2} \Rightarrow \sigma_{max} - \sigma_{min} = \sigma_{yp}$$

so σ_{yp} = 100 MPa.
Reference: Chapter 4

Year 2006

34. For a circular shaft of diameter *d* subjected to torque *T*, the maximum value of the shear stress is:

(a) $\frac{64T}{\pi d^3}$

(b) $\frac{32T}{\pi d^3}$

(c) $\frac{16T}{\pi d^3}$

(d) $\frac{8T}{\pi d^3}$

Answer: (c)
Reference: Chapter 2

35. A pin-ended column of length *L*, modulus of elasticity *E* and second moment of area of the cross-section *I* is loaded centrically by a compressive load *P*. The critical buckling load (P_{Cr}) is given by:

(a) $P_{Cr} = \frac{EI}{\pi^2 L^2}$

(b) $P_{\mathrm{Cr}} = \dfrac{\pi^2 EI}{3L^2}$

(c) $P_{\mathrm{Cr}} = \dfrac{\pi^2 EI}{L^2}$

(d) $P_{\mathrm{Cr}} = \dfrac{\pi^2 EI}{4L^2}$

Answer: (c)
Reference: Chapter 8

36. According to von Mises' distortion energy theory, the distortion energy density under three-dimensional stress state is represented by

(a) $\dfrac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$

(b) $\dfrac{1-2\nu}{6E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2(\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_3\sigma_1)\right]$

(c) $\dfrac{1+\nu}{3E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)\right]$

(d) $\dfrac{1}{3E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$

Answer: (c)
Reference: Chapter 11

37. A steel bar of 40 mm × 40 mm square cross-section is subjected to an axial compressive load of 200 kN. If the length of the bar is 2 m and E = 200 GPa, the compression of the bar will be

(a) 1.25 mm
(b) 2.70 mm
(c) 4.05 mm
(d) 5.40 mm

Answer: (a)
Explanation:

$$\delta = \frac{PL}{AE} = \frac{(200)(10^3)(2)(10^3)}{(40)^2(200)(10^3)}\,\mathrm{mm} = 1.25\,\mathrm{mm}$$

Reference: Chapter 1

38. A bar having a cross-sectional area of 700 mm^2 is subjected to axial loads at the position indicated in Figure 22. The value of stress in the segment QR is

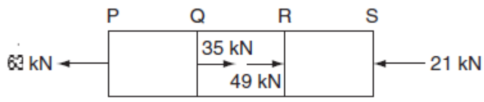


Figure 22 Question 38.

(a) 40 MPa
(b) 50 MPa
(c) 70 MPa
(d) 120 MPa

Answer: (a)
Explanation: Let us draw the free-body diagram of different segments as shown in Figure 23:

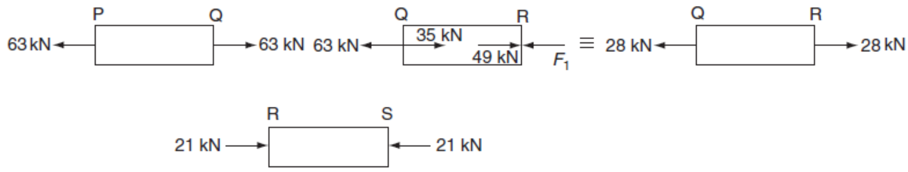


Figure 23 Question 38.

$$\sigma_{QR} = \frac{P_{QR}}{A_{QR}} = \frac{28(10^3)}{700}\frac{\text{N}}{\text{mm}^2} = 40\frac{\text{N}}{\text{mm}^2} = 40 \text{ MPa}$$

Reference: Chapter 1

39. A simply supported beam of span length 6 m and 75 mm diameter carries a uniformly distributed load of 1.5 kN/m.

(i) What is the maximum value of bending moment?
(a) 9 kN m
(b) 13.5 kN m
(c) 6.75 kN m
(d) 8.1 kN m

(ii) What is the maximum value of bending stress?
(a) 162.98 MPa
(b) 325.95 MPa
(c) 625.95 MPa
(d) 651.90 MPa

Answer: (i) (c); (ii) (a)
Explanation:
(i)

$$M_{\max} = \frac{w_o L^2}{8} = \frac{(1.5)(6)^2}{8} \text{kN m} = 6.75 \text{ kN m}$$

(ii)

$$\sigma_{\max} = \frac{32 M_{\max}}{\pi d^3} = \frac{(32)(3.75)(10^6)}{\pi(75)^3}\frac{\text{N}}{\text{mm}^2} = 162.98\frac{\text{N}}{\text{mm}^2} = 162.98 \text{MPa}$$

Reference: (i) Chapter 5; (ii) Chapter 6

Year 2007

40. In a simply supported beam loaded as shown in Figure 24, the maximum bending moment in N m is:

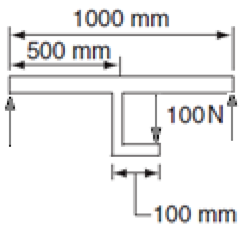


Figure 24 Question 40.

(a) 25
(b) 30
(c) 35
(d) 60

Answer: (d)
Explanation: Let us draw the free-body diagram of the beam as shown in Figure 25:

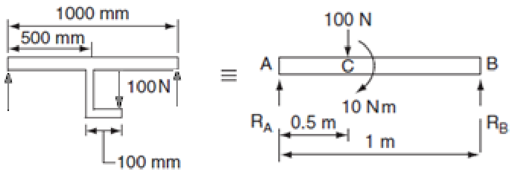


Figure 25 Explanation for Question 40.

Therefore, from $\overset{+}{\uparrow}\sum M_B = 0 \Rightarrow -R_A + (100)(0.5) - 10 = 0 \Rightarrow R_A = 40\,\text{N m}$. Now, taking a section just right to point C gives us the following free-body diagram in Figure 26:

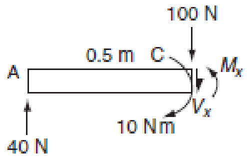


Figure 26 Free-body diagram.

Taking

$$\overset{+}{\uparrow}\sum M_C = 0 \Rightarrow M_x - (40)(0.5) - 10 = 0 \Rightarrow M_x = 30\ \text{N m}.$$

Reference: Chapter 5

41. A steel rod of length *L* and diameter *D*, fixed at both ends, is uniformly heated to a temperature rise of ΔT The Young's modulus is *E* and the coefficient of linear expansion is α. The thermal stress in the rod is:
(a) 0
(b) $\alpha \Delta T$
(c) $E \alpha \Delta T$
(d) $E \alpha \Delta T L$

Answer: (c)
Explanation: We know that (assuming the rod-axis to be along the *x*-axis):

$$\epsilon_{xx} = \frac{1}{E}\left[\sigma_{xx}\right] + \alpha \Delta T$$

So the rod is fixed at both ends, $\epsilon_{xx} = 0$, thus, $\sigma_{xx} = -E \alpha \Delta T$.

Reference: Chapter 1

42. A uniformly loaded propped cantilever beam and its free-body diagram are shown in Figure 27. The reactions are:

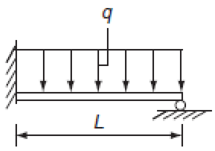


Figure 27 Question 42.

(a) $R_1 = \frac{5qL}{8}, R_2 = \frac{3qL}{8}, M = \frac{qL^2}{8}$

(b) $R_1 = \frac{3qL}{8}, R_2 = \frac{5qL}{8}, M = \frac{qL^2}{8}$

(c) $R_1 = \frac{5qL}{8}, R_2 = \frac{3qL}{8}, M = 0$

(d) $R_1 = \frac{3qL}{8}, R_2 = \frac{5qL}{8}, M = 0$

Answer: (a)
Explanation: We note from Figure 28 that:

$$R_1 + R_2 = qL \text{ and } M + R_2 L = \frac{qL^2}{2}$$

Bending-moment at any section at a distance *x* from left-end is given by:

$$M_x + M + \frac{qx^2}{2} = R_1 x \Rightarrow M_x = R_1 x - M - \frac{qx^2}{2}$$

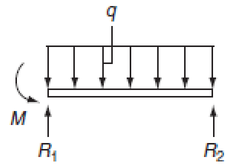


Figure 28 Explanation for Question 42.

Now,

$$(EI)y_2 = -M_x = -R_1 x + M + \frac{qx^2}{2}.$$

Thus,

$$(EI)y_1 = -\frac{R_1 x}{2} + Mx + \frac{qx}{6} + c_1$$

at $x = 0, y_1 = 0 \Rightarrow c_1 = 0$. so

$$(EI)y_1 = -\frac{R_1 x^2}{2} + Mx + \frac{qx^3}{6}$$

or $$(EI)y = -\frac{R_1 x^3}{6} + \frac{Mx^2}{2} + \frac{qx^4}{24} + c_2$$

Again at $x = 0, y = 0 \Rightarrow c_2 = 0$, so

$$(EI)y = -\frac{R_1 x^3}{6} + \frac{Mx^2}{2} + \frac{qx^4}{24}$$

$$(EI)y\big|_{x=L} = -\frac{R_1 L^3}{6} + \frac{ML^2}{2} + \frac{qL^4}{24} = 0$$

Thus,

$$\frac{R_1 L}{6} - \frac{M}{2} = \frac{qL^2}{24} \Rightarrow 4R_1 L - 12M = qL^2 \tag{1}$$

Again,

$$M + (qL - R_1) = \frac{qL^2}{2} \Rightarrow -R_1 L + M = -\frac{qL^2}{2}$$

or, $$-4R_1 L + 4M = -2qL^2 \tag{2}$$

So from Eqs. (1) and (2),

$$-8M = -qL^2 \Rightarrow M = \frac{qL^2}{8} \Rightarrow R_2 = \frac{3qL}{8} \Rightarrow R_1 = \frac{5q}{8}$$

Reference: Chapter 7

43. A 200 × 100 × 500 mm steel block is subjected to a hydrostatic pressure of 15 MPa. The Young's modulus and Poisson's ratio of the material are 200 GPa and 0.3, respectively. The change in the volume of the block in mm^3 is:
(a) 85
(b) 90
(c) 100
(d) 110

Answer: (b)
Explanation: We know that:

$$\epsilon_V = \frac{3(1-2\nu)}{E}\sigma_h$$

where σ_h is the hydrostatic stress.

$$\frac{\Delta V}{V} = \frac{3\sigma_h(1-2\nu)}{E} \Rightarrow \Delta V = \frac{3\sigma_h(1-2\nu)}{E}V$$

$$\Delta V = \frac{(3)(15)(1-2\times 0.3)}{(200)(10^3)}(200\times 100\times 500)\text{mm}^3 = 90\text{mm}^3$$

Reference: Chapter 9

44. A stepped steel shaft shown in Figure 29 is subjected to 10 N m torque. If the modulus of rigidity in 80 GPa, the strain energy in the shaft in N m:
(a) 4.12
(b) 3.46
(c) 1.73
(d) 0.86

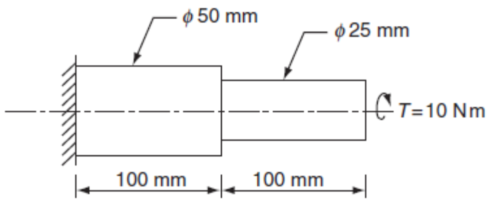


Figure 29 Question 44.

Answer: (c)
Explanation:

$$U = \int_0^{0.1} \frac{T^2}{2GJ_1}dx + \int_0^{0.1} \frac{T^2 1}{2GJ_2}dx = \frac{T^2 L}{2G} = \left(\frac{1}{J_1} + \frac{1}{J_2}\right)$$

$$\Rightarrow U = \frac{16T^2 L}{\pi G}\left(\frac{1}{d_1^4} + \frac{1}{d_2^4}\right)$$

$$\Rightarrow U = \frac{(16)(10^2)(0.1)}{(\pi)(80)(10^9)}\left(\frac{1}{0.05^4} + \frac{1}{0.025^4}\right)\text{N m} = 1.73\times 10^{-3}\ \text{N m} = 1.73\ \text{N mm}$$

Reference: Chapter 10

45. A machine frame shown in Figure 30 is subjected to a horizontal force of 600 N parallel to the *z*-direction.

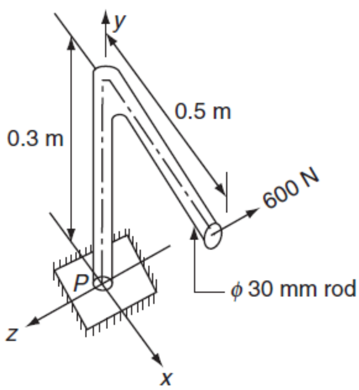


Figure 30 Question 45.

(i) The normal and shear stresses in MPa at point P are, respectively,

(a) 67.9 and 56.6
(b) 56.6 and 67.9
(c) 67.9 and 0.0
(d) 0.0 and 56.6

(ii) The maximum principal stress in MPa and the orientation of the corresponding principal plane in degree are respectively
(a) – 32.0 and – 29.52
(b) 100.0 and 60.48
(c) – 32.0 and 60.48
(d) 100 and – 29.52

Answer: (i) (a); (ii) (d)
Explanation:
(i) Bending moment is

$$M = (600)\cdot(0.3)\ \mathrm{N\,m} = 180\ \mathrm{N\,m}$$

and torsional moment is

$$T = (600)(0.5)\ \mathrm{N\,m} = 300\ \mathrm{N\,m}$$

so normal stress at P is

$$\frac{32M}{\pi d^3} = \frac{(32)(180)(10^3)}{(\pi)(30)^3}\frac{\mathrm{N}}{\mathrm{mm}^2} = 67.9\ \mathrm{MPa}$$

and shear stress at P is

$$\frac{16T}{\pi d^3} = \frac{(16)(300)(10^3)}{\pi(30)^3}\frac{\mathrm{N}}{\mathrm{mm}^2} = 56.6\ \mathrm{MPa}$$

So, $\sigma_{\mathrm{P}} = 67.9\ \mathrm{MPa},\ \tau_{\mathrm{P}} = 56.6\ \mathrm{MPa}$
(ii) Consider Figure 31 and Mohr's circle in Figure 32.

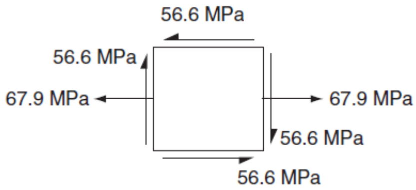


Figure 31 Explanation for Question 45.

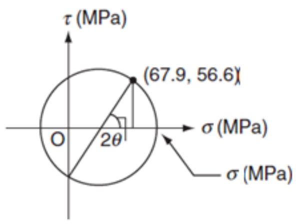


Figure 32 Mohr's circle.

Therefore, from the above Mohr's circle, we get

$$\sigma_{\max} = \frac{67.9}{2} + \sqrt{\left(\frac{67.9}{2}\right)^2 + 56.6^2} = 100.0\mathrm{MPa}$$

$$\tan 2\theta = \frac{56.6}{67.9 - \dfrac{67.9}{2}} \Rightarrow \theta = 29.52^\circ$$

So, $\sigma_{\max}$ = 100 MPa and θ = –29.52°.

Reference: (i) Chapter 4; (ii) Chapter 12.

Year 2008

46. The strain energy stored in the beam with flexural rigidity *EI* and loaded as shown in Figure 33 is

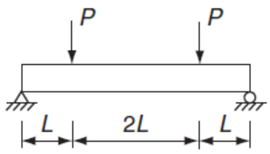


Figure 33 Question 46.

(a) $\frac{P^2L^3}{3EI}$

(b) $\frac{2P^2L^3}{3EI}$

(c) $\frac{4P^2L^3}{3EI}$

(d) $\frac{8P^2L^3}{3EI}$

Answer: (c)

Explanation: From the given beam loading (Figure 34), we can show the free-body diagram as

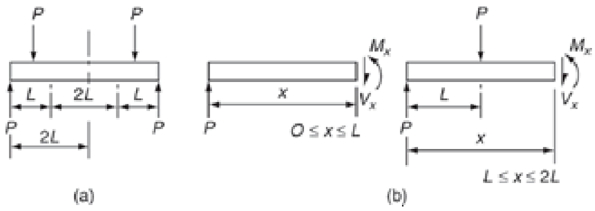


Figure 34 Explanation for Question 46

The rest of the beam loading is symmetric. So, we consider one-half of the beam shown by the chain line in Figure 34(a).

$$M_x = \begin{Bmatrix} Px; \quad 0 \le x \le L \\ Px - P(x - L) \\ PL; \quad L \le x \le 2L \end{Bmatrix}$$

Therefore, U_b, that is, strain energy due to bending is

$$U_b = 2\left[\int_0^L \frac{M_x^2 dx}{2EI} + \int_L^{2L} \frac{M_x^2 dx}{2EI}\right] = \frac{1}{EI}\left[\int_0^L P^2x^2 dx + \int_L^{2L} P^2L^2 dx\right]$$

$$= \frac{1}{EI}\left[\frac{P^2L^3}{3} + P^2L^3\right] = \frac{4P^2L^3}{3EI}$$

Reference: Chapter 10

47. For the component loaded with a force *F* as shown in Figure 35, axial stress at corner point P is

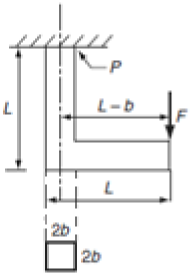


Figure 35 Question 47.

(a) $\dfrac{F(3L-b)}{4b^3}$

(b) $\dfrac{F(3L+b)}{4b^3}$

(c) $\dfrac{F(3L+4b)}{4b^3}$

(d) $\dfrac{F(3L-2b)}{4b^3}$

Answer: (d)
Explanation: Stress at a point P is caused due to direct tensile stress and due to the bending caused by the bending moment, $M = F(L - b)$. Thus,

$$\sigma_{\mathrm{P}} = \frac{F}{4b^2} + \frac{F(L-b)\cdot b}{(1/12)(2b)(2b)^3}$$
$$= \frac{F}{4b^2} + \frac{3F(L-b)}{4b^3}$$

Thus,

$$\sigma_{\mathrm{P}} = \frac{F}{4b^3}\{b + 3L - 3b\} = \frac{F(3L-2b)}{4b^3}$$

Reference: Chapter 12

48. The rod PQ of length L and with flexural rigidity EI is hinged at both ends (Figure 36). For what minimum force F is it expected to buckle?

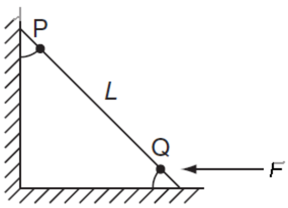


Figure 36 Question 48.

(a) $\dfrac{\pi^2 EI}{L^2}$

(b) $\dfrac{\sqrt{2}\pi^2 EI}{L^2}$

(c) $\dfrac{\pi^2 EI}{\sqrt{2}L^2}$

(d) $\dfrac{\pi^2 EI}{2L^2}$

Answer: (c)

Explanation: Clearly, $F_{PQ}\cos 45^{\circ} = F$

or
$$F_{PQ} = \sqrt{2}F = \frac{\pi^2 EI}{L^2}$$

so,
$$F = \frac{\pi^2 EI}{\sqrt{2}L^2}$$

Reference: Chapter 8

Year 2009

49. A solid circular shaft of diameter *d* is subjected to a combined bending moment *M* and torque, *T*. The material property to be used for designing the shaft using the following relation is

$$\frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

(a) ultimate tensile strength (σ_u)

(b) tensile yield strength (σ_y)

(c) torsional yield strength (τ_y)

(d) endurance strength (σ_e)

Answer: (c)
Reference: Chapter 12

50. A solid shaft of diameter *d* and length *L* is fixed at both ends. A torque T_o is applied at a distance *L*/4 from the left-end as shown in Figure 37:

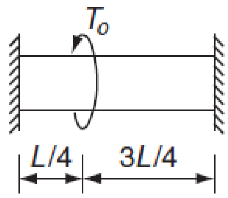


Figure 37 Question 50.

The maximum shears stress in the shaft is:

(a) $\frac{16T_o}{\pi d^3}$

(b) $\frac{12T_o}{\pi d^3}$

(c) $\frac{8T_o}{\pi d^3}$

(d) $\frac{4T_o}{\pi d^3}$

Answer: (b)
Explanation: Let us consider the free-body diagram of the shaft and its segments as shown in Figure 38:

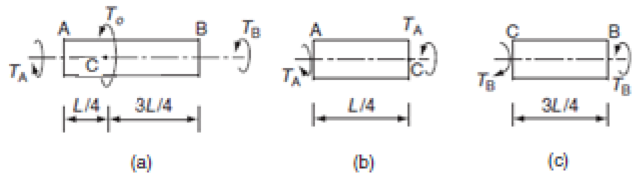


Figure 38 Explanation for Question 50.

From Figure 38(a) above:

$$-T_A + T_o + T_B \Rightarrow T_A - T_B = T_o \tag{1}$$

From Figures 38(b) and (c):

$$\theta_{C/A} = \theta_C - \theta_A = \theta_C = \frac{T_A L}{4GJ} \tag{2}$$

and

$$\theta_{C/B} = \theta_C - \theta_B = \theta_C = \frac{3T_B L}{4GJ} \tag{3}$$

But considering angular twist directions of point C in Figure 38(ii) and (iii) above, we get:

$$\frac{T_A L}{4GJ} = -\frac{3T_B L}{4GJ} \Rightarrow T_A + 3T_B = 0 \tag{4}$$

From Eqs. (1) and (4):

$$T_B = -\frac{T_o}{4}, \quad T_A = \frac{3T_o}{4}$$

so maximum torque is carried by the segment AC of the shaft, and is given by:

$$\tau_{max} = \frac{16\left(\frac{3T_o}{4}\right)}{\pi d^3} = \frac{12T_o}{\pi d^3}$$

Reference: Chapter 2

51. A frame of two arms of equal length *L* is shown in Figure 39. The flexural rigidity of each arm of the frame is *EI*. The vertical deflection at the point of application of load *P* is:

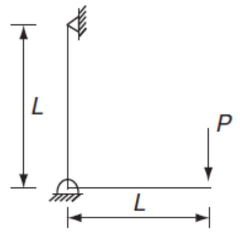


Figure 39 Question 51.

(a) $\frac{PL^3}{3EI}$

(b) $\frac{2PL^3}{3EI}$

(c) $\frac{PL^3}{EI}$

(d) $\frac{4PL^3}{3EI}$

Answer: (b)

Explanation: Total strain energy due to bending of the frame is:

$$U = 2\int_0^L \frac{M_x^2}{2EI} \text{ where } M_x = Px$$

$$\Rightarrow U = \left(\frac{P^2}{EI}\right)\int_0^L x^2 dx = \frac{P^2 L^3}{3EI}$$

By Castigliano's second theorem,

$$\delta = \frac{\partial U}{\partial P} = \frac{2PL^3}{3EI}$$

Reference: Chapter 10