# MPL's Numerator and Denominator Operators

MPL's *Numerator* and *Denominator* operators are defined by the following transformation rules:

**Definition 1** *Let $u$ be an algebraic expression.*

**ND-1**. *If $u$ is a fraction, then*

$$\begin{aligned} Numerator(u) &\rightarrow Operand(u,1), \\ Denominator(u) &\rightarrow Operand(u,2). \end{aligned}$$

**ND-2**. *Suppose $u$ is a power. If the exponent of $u$ is a negative integer or a negative fraction, then*

$$Numerator(u) \rightarrow 1, \qquad Denominator(u) \rightarrow u^{-1},$$

*otherwise*

$$Numerator(u) \rightarrow u, \qquad Denominator(u) \rightarrow 1.$$

**ND-3**. *Suppose $u$ is a product and $v = Operand(u,1)$. Then*

$$Numerator(u) \rightarrow Numerator(v) * Numerator(u/v),$$

$$Denominator(u) \rightarrow Denominator(v) * Denominator(u/v).$$

**ND-4**. *If $u$ does not satisfy any of the previous rules, then*

$$Numerator(u) \rightarrow u, \qquad Denominator(u) \rightarrow 1.$$

**Example 1** Consider the expression $u = (2/3)\,\dfrac{x\,(x+1)}{x+2}\,y^n$. Then

$$Numerator\,(u) \rightarrow 2\,x\,(x+1)\,y^n, \qquad Denominator\,(u) \rightarrow 3\,(x+2).$$

The *Numerator* and *Denominator* operators are defined in terms of the tree structure of an expression and are interpreted in the context of automatic simplification. Although the operators are adequate for our purposes, the next two examples show in some cases they give unusual results.

**Example 2** Consider the expression $\frac{1}{x} + \frac{1}{y}$. Certainly, if we transform the expression to $\frac{x+y}{x\,y}$, it is clear which expression is the numerator and which is the denominator. The definition, however, does not include this transformation as part of the simplification context, and so the numerator is $\frac{1}{x} + \frac{1}{y}$ and the denominator is 1.

**Example 3** Consider the expression $x^{-r^2-4\,r-5}$. In this case, the exponent is negative for all real values of $r$. However, since the exponent of the expression is not a negative integer or fraction, the numerator is $x^{-r^2-4\,r-5}$ and the denominator is 1.

Most computer algebra systems have operators that are similar to MPL's *Numerator* and *Denominator* operators:

| MPL | Maple | Mathematica | MuPAD |
|---|---|---|---|
| *Numerator(u)* | `numer(u)` | `Numerator[u]` | `numer(u)` |
| *Denominator(u)* | `denom(u)` | `Denominator[u]` | `denom(u)` |

(Implementation: Maple (mws), Mathematica (nb), MuPAD (mnb).)

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