Mid term Set-01
oct-2011
solution 2011-12

Symbol of Secularism & National Integration

ANJUMAN-I-ISLAM'S
KALSEKAR TECHNICAL CAMPUS, NEW PANVEL
School of Engineering & Technology

UT-II (Sem I)

Subject: Applied Maths I Class: FE-All Marks:50
Date: 31/10/2011 Time – 2 hrs

Note:1) Q1 is compulsory.
2) Solve any two Questions from remaining Three.

1\. a) If α and β are roots of the equation $Z^2 \sin^2\theta - Z\sin 2\theta + 1 = 0$ then prove that $\alpha^n + \beta^n = 2\cos n\theta \operatorname{cosec}^n\theta$. (5)

b) If $u = \log(x^2 + y^2)$ and $V = y/x$ prove that (5)

$$X\frac{\partial Z}{\partial Y} - Y\frac{\partial Z}{\partial x} = (1+v^2)\frac{\partial Z}{\partial V}$$

2 a) Prove that $\cos^6\theta + \sin^6\theta = 1/8\ [3\cos 4\theta + 5]$. (6)

b) Find the Extreme values of

$$u = x^3 + xy^2 + 21x - 12x^2 - 2y^2$$ (6)

c) If $U = \tan^{-1}\left(\frac{3Y^3 + 2XY^2}{2X + 5Y}\right)$ prove that

$x^2 U_{XX} + 2xy\, U_{XY} + y^2 U_{YY} = 2\sin U \cos 3U$ (8)

3 a) If $F(x, y, z) = x^2 y^3 z^{1/10}$ Find the approximate value of F when $x = 1.99$, $y = 3.01$ and $z = 0.98$. (6)

b) Prove that $\sinh^{-1} X = \log\{X + \sqrt{X^2 + 1}\}$ and hence deduce that (6)

$\sinh^{-1}(\tan\theta) = \log(\sec\theta + \tan\theta)$.

c) Prove that $(u_x)^2 + (u_y)^2 + (u_z)^2 = 2(x u_x + y u_y + z u_z)$ if (8)

$$\frac{x^2}{1+u} + \frac{y^2}{4+u} + \frac{z^2}{9+u} = 1$$

4 a) Evaluate $\lim_{x\to a}\left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ (6)

b) Prove that $\log(\sec x) = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \cdots$ (6)

c) Prove that $\log\{e^{i\alpha} + e^{i\beta}\} = \log\left[2\cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right)$ (8)

Q. 1 (b) Given

$$u = \log (x^2 + y^2)$$

$$v = y/x$$

$$z \longrightarrow f(u, v) \longrightarrow f(x, y)$$

so

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \left(\frac{2x}{x^2 + y^2} \right) + \frac{\partial z}{\partial v} \left(\frac{-y}{x^2} \right) \quad \text{--- (1)}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \left(\frac{2y}{x^2 + y^2} \right) + \frac{\partial z}{\partial v} \left(\frac{1}{x} \right) \quad \text{--- (2)}$$

then consider

$$LHS = x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x}$$

$$= \left\{ \frac{\partial z}{\partial u} \left(\frac{2xy}{x^2 + y^2} \right) + \frac{\partial z}{\partial v} (1) \right\}$$

$$- \left\{ \frac{\partial z}{\partial u} \cdot \left(\frac{2xy}{x^2 + y^2} \right) + \frac{\partial z}{\partial v} \left(-\frac{y^2}{x^2} \right) \right\}$$

$$= \frac{\partial z}{\partial v} \left(1 + \frac{y^2}{x^2} \right)$$

$$= (1 + v^2) \frac{\partial z}{\partial v}$$

(9) $z^2 \sin^2\theta - z \sin 2\theta + 1 = 0$

The roots of this quadratic Eq^n are

$$z = \frac{\sin 2\theta \pm \sqrt{\sin^2 2\theta - 4 \sin^2\theta}}{2 \sin^2\theta}$$

$$= \frac{2 \sin\theta \cos\theta \pm \sqrt{4 \sin^2\theta \cos^2\theta - 4 \sin^2\theta}}{2 \sin^2\theta}$$

$$= \frac{2 \sin\theta \cos\theta \pm 2 \sin\theta \sqrt{\cos^2\theta - 1}}{2 \sin^2\theta}$$

$-(\sin^2\theta)$

$$= \frac{\cos\theta \pm i \sin\theta}{\sin\theta}$$

so

$$\alpha = \frac{\cos\theta + i \sin\theta}{\sin\theta} \qquad \beta = \frac{\cos\theta - i \sin\theta}{\sin\theta}$$

By De Moivre's Th.

$$(\cos\theta \pm i \sin\theta)^n = \cos n\theta \pm i \sin n\theta$$

so LHS =

$$\alpha^n + \beta^n = \frac{1}{\sin^n\theta} \left\{ (\cos\theta + i \sin\theta)^n + (\cos\theta - i \sin\theta)^n \right\}$$

$$= \frac{1}{\sin^n\theta} \left\{ 2 \cos n\theta \right\}$$

$$= 2 \cos n\theta \operatorname{cosec}^n\theta \text{ —— RHS.}$$

— x —

Q. 2 (a)

Let $x = \cos\theta + i\sin\theta$

$\frac{1}{x} = \cos\theta - i\sin\theta$

$x + \frac{1}{x} = 2\cos\theta$ & $x - \frac{1}{x} = 2i\sin\theta$

$x^p + \frac{1}{x^p} = 2\cos p\theta$ } using De Moivre's Theorem.

$x^p - \frac{1}{x^p} = 2i\sin p\theta$

$\longrightarrow 2^6\cos^6\theta = \left(x + \frac{1}{x}\right)^6$

$= x^6 + 6x^4 + 15x^2 + 20 + 15\frac{1}{x^2} + 6\frac{1}{x^4} + \frac{1}{x^6}$

$= \left(x^6 + \frac{1}{x^6}\right) + 6\left(x^4 + \frac{1}{x^4}\right) + 15\left(x^2 + \frac{1}{x^2}\right) + 20$

$= 2\cos 6\theta + 6(2\cos 4\theta) + 15(2\cos 2\theta) + 20$

$\cos^6\theta = \frac{1}{2^5}\left[\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right]$ —— (I)

$\longrightarrow 2^6 i^6 \sin^6\theta = \left(x - \frac{1}{x}\right)^6$

$= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$

$= \left(x^6 + \frac{1}{x^6}\right) - 6\left(x^4 + \frac{1}{x^4}\right) + 15\left(x^2 + \frac{1}{x^2}\right) - 20$

$= 2\cos 6\theta - 6(2\cos 4\theta) + 15(2\cos 2\theta) - 20$

$\sin^6\theta = \frac{-1}{}\left[\cos 6\theta - 6\cos 4\theta + 15\cos 2\theta - 10\right]$

$$u = x^3 + xy^2 + 21x - 12x^2 - 2y^2$$

$$\frac{\partial u}{\partial x} = 3x^2 + y^2 + 21 - 24x$$

$$\frac{\partial u}{\partial y} = 2xy - 4y$$

$$\frac{\partial^2 u}{\partial x^2} = 6x - 24, \quad \frac{\partial^2 u}{\partial x \partial y} = 2y, \quad \frac{\partial^2 u}{\partial y^2} = 2x - 4$$

Now extreme values are given by

$$\frac{\partial u}{\partial x} = 0 \quad \text{and} \quad \frac{\partial u}{\partial y} = 0.$$

so

$$3x^2 + y^2 + 21 - 24x = 0 \quad \text{——— (1)}$$

$$2y(x - 2) = 0 \quad \text{——— (2)}$$

from (2) either $y = 0$ OR $x = 2$

If $y = 0$ then	If $x = 2$ then
from (1)	from (1)
$x^2 + 7 - 8x = 0$	$y^2 = 15$
gives $x = 7$ OR 1	$y = \pm\sqrt{15}$

so the points at which the function can be max or min are

$P_1(7, 0)$ $P_2(1, 0)$ $P_3(2, \sqrt{15})$ $P_4(2, -\sqrt{15})$

	P_1	P_2	P_3	P_4
A	18	−18	~~12~~ −12	−12
B	0	0	$2\sqrt{15}$	$-2\sqrt{15}$
	10	−2	0	0
B^2	+ve	+ve		

Results

If $F(u)$ is homogeneous function of order n then

① $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{F(u)}{F'(u)} = g(u) \text{ say.}$$

and

② $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = g(u)\left[g'(u)-1\right]$$

Apply these to

$$U = \tan^{-1}\left(\frac{3y^3 + 2xy^2}{2x+5y}\right)$$

$\tan u$ is Homo. of order $n = 2$

so by ①

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\frac{\tan u}{\sec^2 u}$$

$$= 2 \sin u \cos u = \sin 2u$$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

$$= \sin 2u \left[2 \cos 2u - 1\right]$$

$$= 2 \sin u \cos u \left[2 \cos 2u - 1\right]$$

$$= 2 \sin u \left\{2 \cos u \cos 2u - \cos u\right\}$$

$$= 2 \sin u \left\{\cos 3u + \cancel{\cos u} - \cancel{\cos u}\right\}$$

$$= 2 \sin u \cdot \cos 3u.$$

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

$$= (2xy^3 z^{1/10})\,dx + (3x^2y^2 z^{1/10})\,dy$$

$$+ \left(\frac{1}{10}x^2y^3 z^{-9/10}\right)dz$$

put $x = 2$ $\quad dx = -0.01$

$y = 3$ $\quad dy = 0.01$

$z = 1$ $\quad dz = -0.02$

$$dF = (108)(-0.01) + (108)(0.01) + (10.8)(-0.02)$$

$$= -0.216$$

$F = 108$ $\quad F + dF = 107.784$ is the appro. value of F.

(b) Let $\sinh^{-1} x = y$

$$x = \sinh y = \frac{e^y - e^{-y}}{2}$$

$$2x = e^y - e^{-y}$$

$$2x e^y = e^{2y} - 1$$

$$e^{2y} - 2x e^y - 1 = 0$$

$$e^y = \frac{2x \pm \sqrt{4x^2+4}}{2}$$

$$e^y = x \pm \sqrt{x^2+1}$$

$$e^y = x + \sqrt{x^2+1}$$

practically

$$y = \log\left(x + \sqrt{x^2+1}\right)$$

$$\sinh^{-1} x = \log\left(x + \sqrt{x^2+1}\right)$$

Now put $x = \tan\theta$

$$\sinh^{-1}(\tan\theta) = \log\left(\tan\theta + \sqrt{\sec^2\theta}\right)$$

$$= \log(\tan\theta + \sec\theta)$$

(c) Given $\dfrac{x^2}{1+u} + \dfrac{y^2}{4+u} + \dfrac{z^2}{9+u} = 1$ ——— (1)

Differentiate (1) w.r.t x

$$(1+u)(2x) - x^2\frac{\partial u}{\partial x} \quad - y^2\left(\frac{\partial u}{\partial x}\right) \quad - z^2\frac{\partial u}{\partial x} - \ldots$$

2011-12

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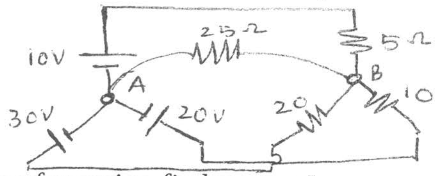
KALSEKAR TECHNICAL CAMPUS, NEW PANVEL
School of Engineering & Technology

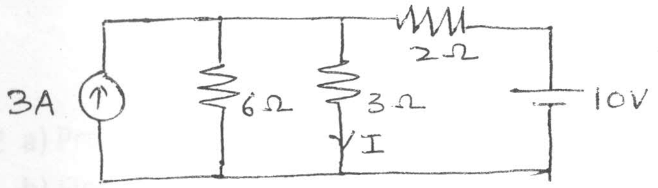
Subject: BEE | Marks:50
Date: 31[st] Oct 2011 | Duration: 2 Hr
Class: FE-EE

Please Note: Q1 is compulsory. Attempt any four out of remaining 6.

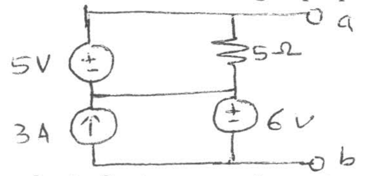
Q.1 Using Norton theorem find the current through 25 ohm resistance for the circuit shown. (10)



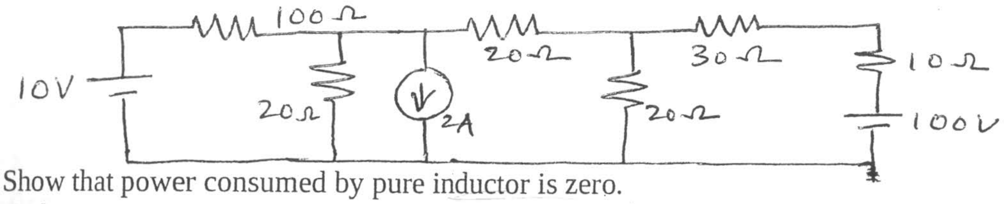
Q.2 a) Using source transformation find current I. (5)



b) Find Vab for the circuit shown using superposition theorem. (5)

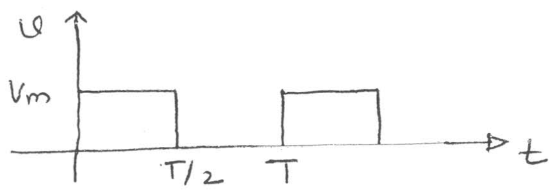


Q.3 Using node analysis find current through 100 ohm resistance for the network shown. (10)



Q.4 a) Show that power consumed by pure inductor is zero. (5)

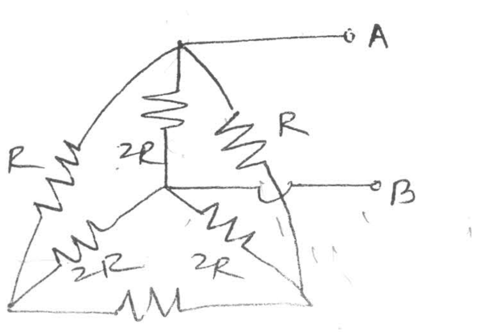
b) Find average & rms value of the following waveform. (5)



Q.5 a) An alternating current is given by i = 141.4 sin 314t.

Find 1) peak value 2) Frequency 3) time period 4) instantaneous value after 3msec. (4)

b) Find resistance between A & B for the circuit shown. (6)



Q.6 a) Three currents i1=1414 sin (wt+TT/4) (6)

i2=30 sin (wt+TT/2)

i3=20 sin (wt-TT/6) find resulting current in polar form.

b)Define RMS & AVERAGE Value of an alternating quantity. (4)

Q.7 a)A voltage of 125v 50Hz is applied across a non inductive resistance connected in series with the capacitance the current is 2.2 A the power loss in resistance is 96.8w find R & C. (6)

b) In RLC series circuit the voltage across R,L,C are 10v,15v,10v find supply voltage & power factor. (4)

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KALSEKAR TECHNICAL CAMPUS, NEW PANVEL

School of Engineering & Technology

2011-12

Subject: Applied Maths I
Date: 31/10/2011

Class: FE-All Marks:50
Time – 2 hrs

Note:1) Q1 is compulsory.
2) Solve any two Questions from remaining Three.

1. a) If α and β are roots of the equation $Z^2 \sin^2\theta - Z\sin 2\theta + 1 = 0$ then prove that $\alpha^n + \beta^n = 2\cos n\theta \operatorname{cosec}^n\theta$. (5)

b) If $u = \log(x^2 + y^2)$ and $V = y/x$ prove that (5)

$$X\frac{\partial Z}{\partial Y} - Y\frac{\partial Z}{\partial x} = (1+v^2)\frac{\partial Z}{\partial V}$$

2 a) Prove that $\cos^6\theta + \sin^6\theta = 1/8\ [3\cos 4\theta + 5]$. (6)

b) Find the Extreme values of

$$u = x^3 + xy^2 + 21x - 12x^2 - 2y^2$$ (6)

c) If $U = \tan^{-1}\left(\frac{3Y^2 + 2XY^2}{2X + 5Y}\right)$ prove that

$x^2 U_{XX} + 2xy\, U_{XY} + y^2 U_{YY} = 2\sin U \cos 3U$ (8)

3 a) If $F(x, y, z) = x^2 y^3 z^{1/10}$ Find the approximate value of F when $x = 1.99$, $y = 3.01$ and $z = 0.98$. (6)

b) Prove that $\sinh^{-1} X = \log\{X + \sqrt{X^2 + 1}\}$ and hence deduce that $\sinh^{-1}(\tan\theta) = \log(\sec\theta + \tan\theta)$. (6)

c) Prove that $(u_x)^2 + (u_y)^2 + (u_z)^2 = 2(x u_x + y u_y + z u_z)$ if (8)

$$\frac{x^2}{1+u} + \frac{y^2}{4+u} + \frac{z^2}{9+u} = 1$$

4 a) Evaluate $\lim_{x\to a}\left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ (6)

b) Prove that $\log(\sec x) = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \cdots$ (6)

c) Prove that $\log\{e^{i\alpha} + e^{i\beta}\} = \log\left[2\cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right)$ (8)

Sem-I/2011

2011-12

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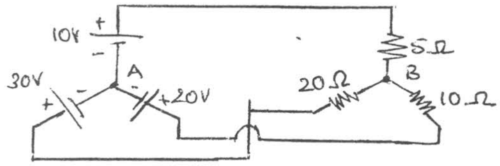
ANJUMAN-I-ISLAM'S
KALSEKAR TECHNICAL CAMPUS, NEW PANVEL
School of Engineering & Technology

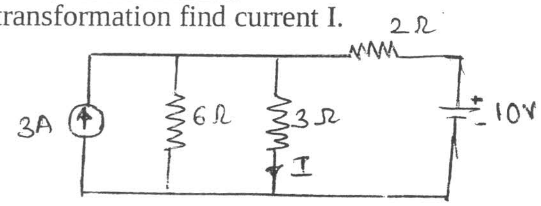
Subject: BEE | Marks:50
Date: 31st Oct 2011 | Duration: 2 Hr
Class: FE-ET/CE/ME/CO

Please Note: Q1 is compulsory. Attempt any four out of remaining 6.

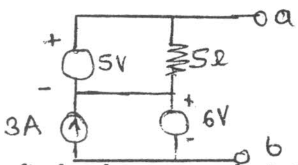
Q.1 Using Norton theorem find the current through 25 ohm resistance for the circuit shown. (10)



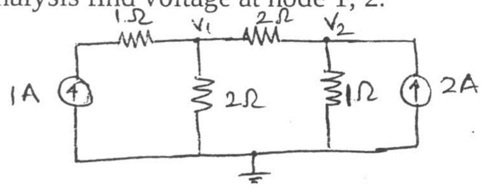
Q.2 a) Using source transformation find current I. (5)



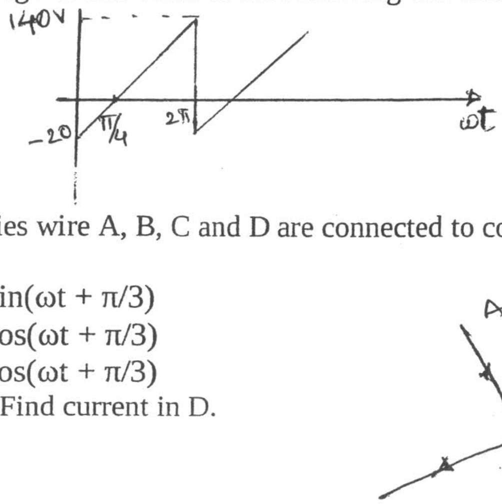
b) Find Vab for the circuit shown using superposition theorem. (5)



Q.3 Using node analysis find voltage at node 1, 2. (10)



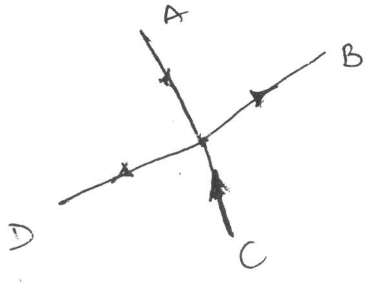
Q.4 a) Show that power consumed by pure capacitor is zero. (5)
b) Find average & rms value of the following waveform. (5)



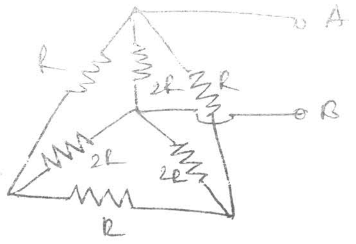
Q.5 a) Four series wire A, B, C and D are connected to common point the current in lines A, B & C are,

$A = 6 \sin(\omega t + \pi/3)$
$B = 5 \cos(\omega t + \pi/3)$
$C = 3 \cos(\omega t + \pi/3)$

Find current in D. (4)



Find resistance between A & B for the circuit shown (6)



Q.6 a) A Series RLC circuit has following parameters R=10Ω, L=0.01mH, C=100µf voltage (6)
v =10sin1000t. Find a) Circuit Impedance b) Power Dissipated in Circuit. c) Resonant Frequency d) B.W and Quality Factor.

b)Define RMS & AVERAGE Value of an alternating quantity. (4)

Q.7 a)Explain FWR with Center Tap Transformer. (6)

b) Find applied voltage Vab that 10A current may flow through capacitor. Assume frequency of 50Hz. (4)

Sem-I/2011

2011-12

Symbol of Secularism
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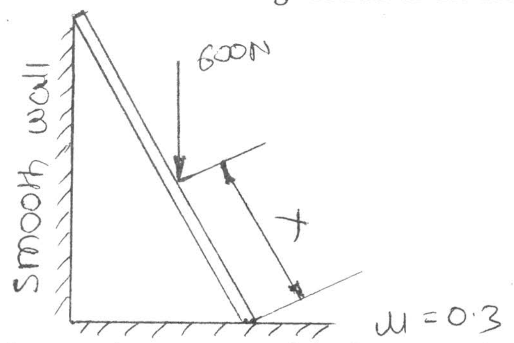
KALSEKAR TECHNICAL CAMPUS, NEW PANVEL

School of Engineering & Technology

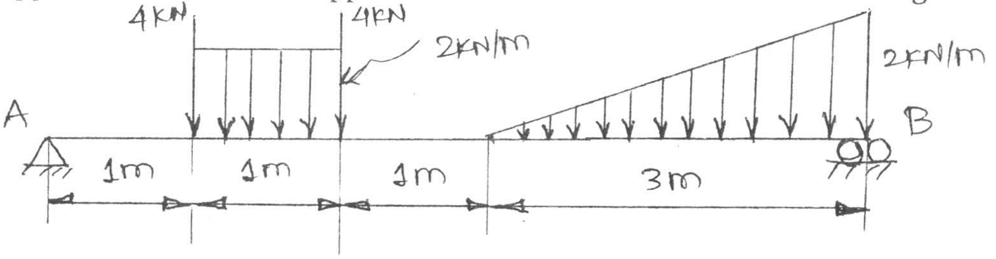
Subject: EM	Marks:50
Date: Oct 2011	Duration: 2 Hr
	Class: FE (EXTC)

Note: Q1 is compulsory.
Attempt any four out of remaining six questions.
Numbers to the right indicate the marks

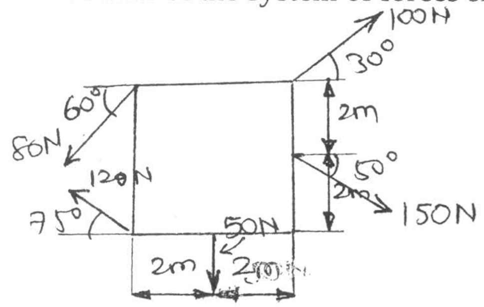
Q.1 (a) A person of weight 600N ascends the 5m ladder of weight 400 N as shown. How far up the ladder the person climbs before sliding motion of the ladder takes place. (5)



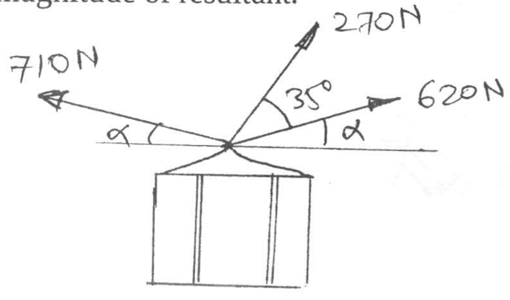
(b)Find the support reactions at the supports of the beam AB loaded as shown in fig. (5)



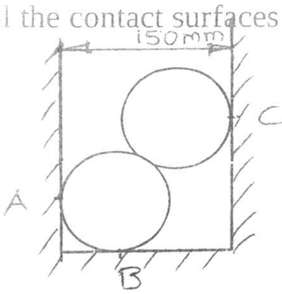
Q.2 (a) Determine the resultant of the system of forces shown in fig. (5)



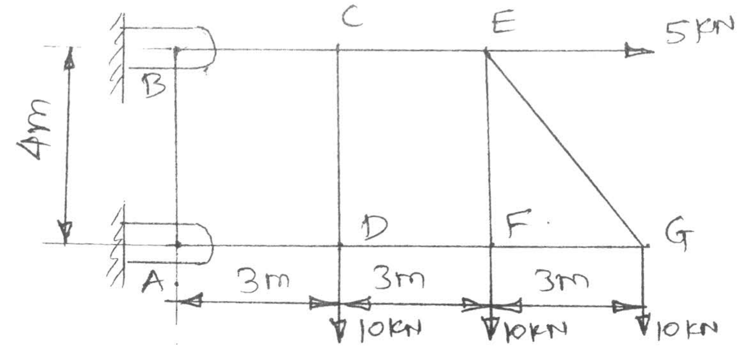
(b) Determine the required value of Θ if the resultant of three forces is to be vertical. Also find the corresponding magnitude of resultant. (5)



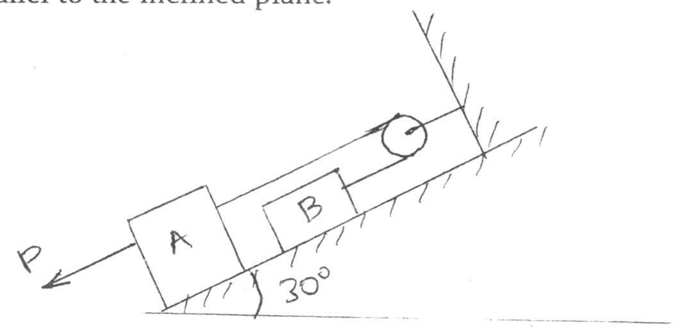
Q.3 (a) Two cylinders each of diameter 100mm and each weighing 200N are placed as shown in fig. Assuming that all the contact surfaces are smooth ,find the reactions at A,B and C. (10)



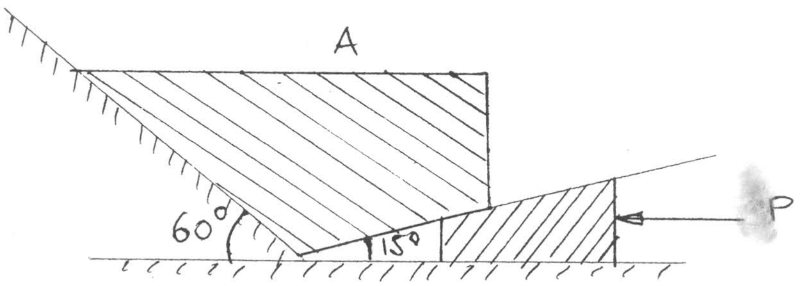
Q.4 (a) For the truss loaded as shown in fig. Find the forces in members CE and CF by method of sections only. Also find the forces in other members by method of joints. (10)



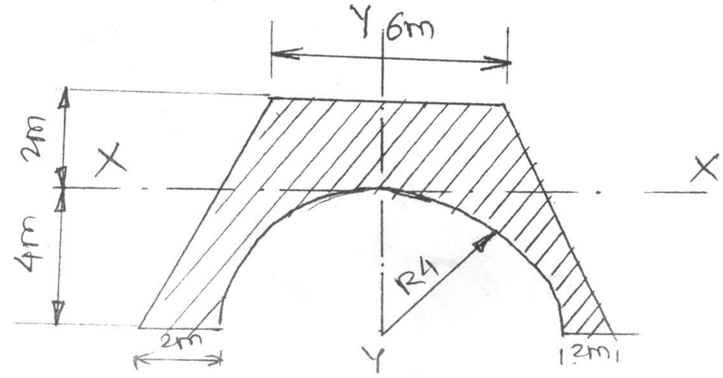
Q.5 (a) Determine the force P to cause the motion to impend. Take masses of blocks A and B as 8Kg and 4 Kg respectively.The coefficient of sliding friction is 0.3.The force P and the rope are parrallel to the inclined plane. (5)



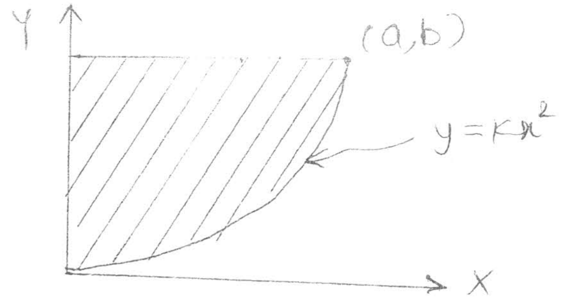
(b) Determine the force P required to move the block A of weight 5000N up the inclined plane.Coefficient of friction between all the surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15^0. (5)



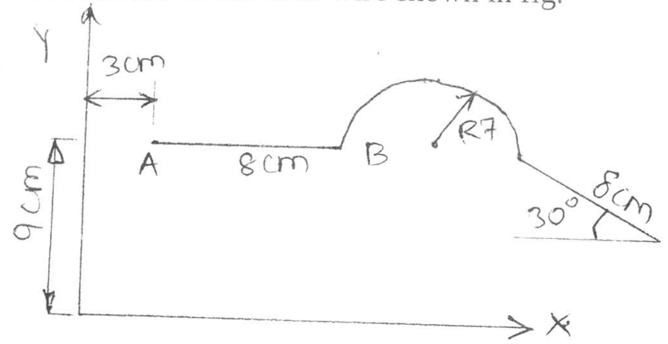
Q.6(a) Determine the moment of inertia of the shaded area about XX and YY axis as shown in fig. (5)



) Determine the moment of inertia of the shaded area with respect to X axis. (5)



7 (a) Find the centroid of the bent wire shown in fig. (5)



Determine the centroid of the shaded area as shown in the fig. (5)