Con. 8897-13. **LJ-10448**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question no. 1 is **compulsory.**

(2) Answer any **four** questions out of remaining **six** questions.

1. (a) Find fourier exansion of $f(x) = 4-x^2$ in the ingterval (0, 2) **5**

(b) Find the probability that atmost 5 defective fuses will be found in a box of 200 fuses if experience shows that 2% of such fuses are defective. **5**

(c) Given $6y = 5x + 90$, $15x = 18y + 130$, $6x^2 = 16$. Find **5**

(i) $\bar{x}$ and $\bar{y}$ (ii) r and (iii) $6y^2$

(d) Solve the two dimensional heat equation $\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}=0$ which satisfies the conditions **5**

$u(0,y)=u(\ell,y)=u(x,0)=0$ and $u(x,a)=\sin\frac{n\pi x}{\ell}$.

2. (a) Obtain fourier series for $f(x) = x - x^2$, $-\pi < x < \pi$ Hence deduce that **7**

$$\frac{\pi^2}{12}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+.....$$

(b) Seven dice are thrown 729 times. How many times do you expect atleast 4 dice to show three or five? **7**

(c) A continuous random variable X has p.d.f. $f(x) = kx^2 e^{-x}$, $x \geq 0$. Find K, mean and variance. **6**

3. (a) Using normal distribution find the probability that in a group of 100 persons there will be 55 males assuming that the probability of a person being male is $\frac{1}{2}$. **7**

(b) Derive wave equation for vibration of string **7**

(c) Obtain fourier expansion of $f(x) = \sin ax$ in the interval $(-\ell, \ell)$ where a is not an integer. **6**

4. (a) Calculate correlation coefficient from the foll : data **7**

x :	23	27	28	29	30	31	33	35	36	39
y :	18	22	23	24	25	26	28	29	30	32

(b) A die was thrown 132 times and the foll : frequencies were observed : **7**

No. obtained :	1	2	3	4	5	6
Frequency :	15	20	25	15	29	28

Test the hypothesis that the die is unbiased.

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(c) Obtain complex form of fourier series for f(x) = cash3x + sinh3x in (-3, 3). **6**

5. (a) A homogeneous rod of conducting material of length ℓ has ends kept at zero temperature and the temperature at the centre is T and falls uniformly to zero at the two ends. Find the temperature u(x, t) at any time. **7**

(b) Obtain half range sine series for f(x) when **7**

$f(x) = x, \qquad 0 < x < \frac{\pi}{2}$

$= \pi - x, \quad \frac{\pi}{2} < x < \pi$ Hence deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots$

(c) Two independent samples of sizes 8 and 7 gave the foll : results **6**

Sample 1	:	19	17	15	21	16	18	16	14
Sample 2	:	15	14	15	19	15	18	16	

is the difference between the sample means significant ?

6. (a) Find the expansion of $f(x) = x(\pi - x)$, $0 < x < \pi$ as a half range cosine series. **7**

Hence S. T

(i) $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$ (ii) $\sum_1^\infty \frac{(-1)^{n+1}}{n} = \frac{\pi^2}{12}$

(b) The diameter of a semicircular plate of radius a is kept at 0^0C and the temperature at the semicircular boundary is T^0C. Find the steady state temperature $u(r, \theta)$. **7**

(c) The average of marks scored by 32 boys is 72 with standard deviation 8 while that of 36 girls is 70 with standard deviation 6. Test at 1% level of significance. Whether the boys perform better than the girls? **6**

7. (a) S. T the functions $f_1(x) = 1$, $f_2(x) = x$ are orthogonal on (-1, 1). Determine the constants a and b such that the function $f_3(x) = -1 + ax + bx^2$ is orthogonal to both f_1 and f_2 on that internal. **7**

(b) Find fourier integral representation of **7**

$f(x) = x, \; 0 < x < a$

$= 0, \quad x > a, \; f(-x) = f(x)$

(c) If u = x-y , v = x+y and if x, y are uncorrelated, **6**

P.T r uv = $\frac{6x^2 - 6y^2}{6x^2 + 6y^2}$