Con. 7380-13. **GX-10003**

(REVISED COURSE)

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Solve any **three** from the **remaining**.

1. (a) If $\alpha + i\beta = \tanh\left(\chi + i\frac{\pi}{4}\right)$, prove that $\alpha^2 + \beta^2 = 1$. **3**

(b) If $u = x^2y + e^{xy^2}$ show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. **3**

(c) If $u = 1 - x$, $v = x(1 - y)$, $w = xy(1 - z)$ show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = x^2y$. **3**

(d) Prove that $\log(1 - x + x^2) = -x + \frac{x^2}{2} + \frac{2x^3}{3} - - - -$ **3**

(e) Express the relation in $\alpha, \beta, \gamma, \delta$ for which $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary. **4**

(f) Find n^{th} derivative of $2^x \cos^2 x \sin x$. **4**

2. (a) $Z^3 = (z + 1)^3$, then show that $z = \frac{-1}{2} + \frac{i}{2}\cot\frac{\theta}{2}$ where $\theta = 20\frac{\pi}{3}$. **6**

(b) Find the non-singular matrices P and Q such that PAQ is in Normal Form. Also fiind rank of A. **6**

$$A = \begin{bmatrix} 4 & 3 & 1 & 6 \\ 2 & 4 & 2 & 2 \\ 12 & 14 & 5 & 16 \end{bmatrix}$$

(c) State and Prove Euler's theorem for homogeneous functions in two variables **8**

and hence find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ for

$u = e^{x+y} + \log(x^3 + y^3 - x^2y - xy^2)$

3. (a) For what values of λ the system of equations have X non-trivial solution ? Obtain the solution for real values of λ where $3x + y - \lambda x = 0$, $4x - 2y - 3z = 0$, $2\lambda x + 4y - \lambda 2 = 0$. **6**

(b) Find the stationary values of $\sin x \sin y \sin(x + y)$. **6**

(c) If $\cos(x + iy)\cos(u + iv) = 1$, where x, y, u, v are real, then show that $\tanh^2 y \cosh^2 v = \sin^2 u$. **8**

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4. (a) If $ux + vy = a$, $\frac{u}{x} + \frac{v}{y} = 1$, Show that $\frac{u}{x}\left(\frac{\partial x}{\partial u}\right)_v + \frac{v}{y}\left(\frac{\partial y}{\partial v}\right) = 0$. **6**

   (b) If $(1 + i\tan\alpha)^{(1 + i\tan\beta)}$ is real then one of the principal values is $(\sec\alpha)^{\sec^2\beta}$. **6**

   (c) Solve by Crout's Method the system of equaitons $2x + 3y + z = -1$, $5x + y + z = 9$, $3x + 2y + 4z = 11$ **8**

5. (a) If $\sin^4\theta\cos^3\theta = a\cos\theta + b\cos 3\theta + c\cos 5\theta + d\cos 7\theta$ then find a, b, c, d. **6**

   (b) Use Taylor theorem and arrange the equaiton in powers of x. **6**

   $$7 + (x + 2) + 3(x + 2)^3 + (x + 2)^4 - (x + 2)^5$$

   (c) If $y = \cos(m\sin^{-1}x)$ prove that $(1 - x^2)\,y_{n+2} - (2n + 1)\,xy_{n+1} + (m^2 - n^2)\,y_n = 0$. **8**

6. (a) Solve correctly upto three iterations the following equations by Gauss-Seidel method. $10x - 5y - 2z = 3$, $4x - 10y + 3z = -3$, $x + 6y + 10z = -3$. **6**

   (b) If $u = \sin(x^2 + y^2)$ and $a^2x^2 + b^2y^2 = c^2$ find $\frac{du}{dx}$. **6**

   (c) Fit a curve $y = ax + bx^2$ for the data : **8**

x	1	2	3	4	5	6
y	2·51	5·82	9·93	14·84	20·55	27·06

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