(3 Hours) [Total Marks : 100

N.B. : (1) Question No. 1 is **compulsory**.
(2) Attempt any **four** questions from the remaining questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) If arg $(z + 1) = \frac{\pi}{6}$ and arg $(z - 1) = \frac{2\pi}{3}$, find z, a complex number. 5

(b) Prove that $\tanh^{-1}(\sin\theta) = \cosh^{-1}(\sec\theta)$. 5

(c) Prove that the real part of $\left(1+i\sqrt{3}\right)^{(1+i\sqrt{3})}$ is $2e^{-\pi/\sqrt{3}}\cos\left(\frac{\pi}{3}+\sqrt{3}\cdot\log 2\right)$. 5

(d) Test the convergence of $\frac{x}{1\cdot 2}+\frac{x^2}{3\cdot 4}+\frac{x^3}{5\cdot 6}+\frac{x^4}{7\cdot 8}+\ldots\ldots(x>0, x\neq 1)$. 5

2. (a) Prove that $\frac{a+ib}{1+c}=\frac{1+iz}{1-iz}$, if $a^2 + b^2 + c^2 = 1$ and $b + ic = (1 + a) z$. 6

(b) Find the roots $\alpha, \alpha^2, \alpha^3, \alpha^4$ of the equation $x^5 - 1 = 0$ and show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$ 6

(c) Prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = 0$, if $u = f(e^{y-z}, e^{z-x}, e^{x-y})$. 8

3. (a) Prove that $\arg z_1 - \arg z_2 = \frac{\pi}{2}$, if $|z_1 + z_2| = |z_1 - z_2|$, z_1, z_2 being complex numbers. 6

(b) Prove that $\alpha^n + \beta^n = 2\cos n\theta\ \mathrm{cosec}^n\theta$, if α, β are the roots of the equation $z^2\sin^2\theta - z\sin 2\theta + 1 = 0$ 6

(c) Show that $\tan^{-1} i\left(\frac{x-a}{x+a}\right)=\frac{i}{2}\log\frac{x}{a}$. 8

4. (a) Prove that $x^2 y_{n+2} + (2n + 1) xy_{n+1} + 2n^2 y_n = 0$, if $\cos^{-1}(y/b) = \log\left(\frac{x}{n}\right)^n$. 6

(b) Show that $\frac{\partial^2 z}{\partial x^2} = a^2\frac{\partial^2 z}{\partial y^2}$, if $z = \tan(y + ax) + (y - ax)^{3/2}$. 6

(c) Prove that $2x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = 4x$, if $f(xy^2, z - 2x) = 0$. 8

5. (a) Separate into real and imaginary parts $\cos^{-1}\left(\frac{3i}{4}\right)$. 6

(b) Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z} = 0$, if $u=f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$. 6

(c) Examine the function $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ for extreme values. 8

6. (a) Find x, if $\bar{a} = x i + 12j - k$, $\bar{b} = 2i + 2j + k$, $\bar{c} = i + k$ are coplanar. Also find unit vector in the direction of vector $\bar{a}$. 6

(b) Prove that $\log\sec x = \frac{1}{2}x^2 + \frac{1}{12}x^4 + \frac{1}{45}x^6 + \ldots$ 6

(c) Evaluate $\lim_{x\to 0}\frac{e^x\sin x - x - x^2}{x^2 + x\log(1-x)}$. 8

7. (a) Prove that, $\frac{\partial f}{\partial y}\cdot\frac{\partial \phi}{\partial z}\cdot\frac{dz}{dx}=\frac{\partial f}{\partial x}\cdot\frac{\partial \phi}{\partial y}$, if $f(x,y) = 0$ and $\phi(y,z) = 0$. 6

(b) Find $(1\cdot 04)^{3\cdot 01}$, by using the theory of approximation. 6

(c) Prove that $\left[\bar{b}\times\bar{c}\ \ \bar{a}\times\bar{c}\ \ \bar{a}\times\bar{b}\right]=\left[\bar{a}\ \bar{b}\ \bar{c}\right]^2$ 8
