**Con. 5979-13.** **(OLD COURSE)** **LJ-10180**

(3 Hours) **[Total Marks : 100**

**N.B. :** (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from out of remaining **six** questions.
(3) **Figure** to the **right** indicate **full** marks.

1. (a) Solve $\frac{dy}{dx} = 1 + y^2$ with initial conditions $x_0 = 0, y_0 = 0$ by Taylor's method where $h = 0{\cdot}2$. **3**

(b) Solve $(D^4 + 2 D^2 + 1) y = 0$. **3**

(c) Evaluate $\int_0^{\pi} \int_0^{a \sin \theta} \pi d\pi d\theta$. **3**

(d) $\int_{-1}^{1} \int_{-2}^{2} \int_{-3}^{3} dxdydz$. **3**

(e) Evaluate $\int_0^{\infty} x^{1/4} \; e^{-\sqrt{x}} \; dx$ . **4**

(f) Using Euler's method, find the approximate value of y when $\frac{dy}{dx} = x^2 + y^2$ and = 1 when x = 0 at y = 2 in five steps is h = 0·2, at x = 1. **4**

2. (a) Prove that $\int_0^{\infty} \frac{x}{(1+x^4)^{5/4}} dx \int_0^{\infty} \frac{dx}{\sqrt{1 + x^4}} = \frac{\pi}{2\sqrt{2}}$ . **6**

(b) Solve $\frac{dy}{dx} = xy$ with initial conditions y(1) = 2 and find y at x = 1·2 by Runge-Kutta Method of Fourth order. **6**

(c) Solve $\frac{dy}{dx} = xy + y^2 e^{(-x^2/2)} \log x$. **8**

3. (a) Solve y(x + y(dx – x (y – x) dy = 0 **6**

(b) Prove that $\int_0^{1} \frac{x^a - 1}{\log x} dx = \log(1 + a), a \geq 0$. **6**

(c) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + y = \frac{1}{1 + \sin x}$. **8**

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4. (a) Solve $y(xy + e^x)\,dx - e^x dy = 0$. **6**

(b) Solve $x^2 \frac{dy}{dx} - x \frac{dy}{dx} + 2y = x \log x$. **6**

(c) Solve $\frac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x - \cos 2x$. **8**

5. (a) The Charge q on the plate of a condenser of the capacity C charged through a resistance R by a steady voltage V satisfies the differential equation $R\frac{dq}{dt} + \frac{q}{c} = V$. If $q = 0, t = 0$, show that $q = CV(1 - e^{-t/RC})$. Find also the current flowing into the plate. **6**

(b) Change the order of integration $\int_0^1 \int_{2y}^{2\left(1+\sqrt{1-y}\right)} f(x, y)\, dxdy$. **6**

(c) Evaluate $\iiint xyz\left(x^2 + y^2 + z^2\right) dxdydz$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$. **8**

6. (a) Find the length of the cardiocide $\pi = a(1 - \cos\theta)$ **6**

(b) Change into polar coordinates and evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} e^{-\left(x^2 + y^2\right)} dydx$. **6**

(c) Evaluate $\iint_R xy\,dxdy$ over the region R given by $x^2 + y^2 - 2x = 0$, $y^2 = 2x$, $y = x$. **8**

7. (a) Find the area bounded by $y^2 = 4ax$ and $x^2\ 4\ by$. **6**

(b) Find the mass of Lamina bounded by curve $ay^2 = x^3$ and line $y = x$ if the density at a point varies as the distance of the point from x-axis. **6**

(c) Find the volume bounded by cylinder $x^2 + y^2 = a^2$ and the plane $z = 0$ and $y + z = b$. **8**

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