

(Revised course)

(3 Hours)

[Total Marks : 80

- N.B. :** (1) Question No. 1 is **compulsory**.
 (2) Answer any **three** questions from question nos. 2 to 6.
 (3) **Figures** to the **right** indicate **full marks**.
 (4) Programming Calculators are **not allowed**.

1. (a) Evaluate $\int_0^8 x^2 7^{-4x^2} dx$ 3
 (b) Solve $(D^4+4)y = 0$ 3
 (c) Prove that $E \nabla = \Delta = \nabla E$ 3
 (d) Solve $(x + 2y^3) \frac{dy}{dx} = y$. 3
 (e) Evaluate $\iint_R r^3 dr d\theta$ over the region between the circles $r = 2 \sin \theta$, $r = 4 \sin \theta$. 4
 (f) Evaluate $\int_0^1 \int_y^{\sqrt{y}} \frac{x}{(1-y)\sqrt{y-x^2}} dy dx$ 4
2. (a) Solve $:- (x^3y^4 + x^2y^3 + xy^2 + y) dx + (x^4y^3 - x^3y^2 - x^2y + x) dy = 0$ 6
 (b) Change the order of integral and hence evaluate $\int_0^5 \int_{2-x}^{2+x} dx dy$ 6
 (c) Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \int_0^1 \frac{dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}$ 8
3. (a) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x dx dy dz$. 6
 (b) Find the area of one loop of the lemniscate $r^2 = a^2 \cdot \cos 2\theta$ 6
 (c) Solve $(D^3+2D^2+D)y = x^2 e^{3x} + \sin^2 x + 2^x$. 8
4. (a) Show that the length of arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$ is $a \left(\log 2 + \frac{15}{16} \right)$ 6
 (b) Using the method of variation of parameters solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$. 6
 (c) Compute $y(0.2)$ given $\frac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$ by taking $h = 0.1$ using Runge-Kutta method of fourth order correct to 4 decimals. 8

5. (a) Solve $\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$. 6
- (b) Solve $\frac{dy}{dx} - 2y = 3e^x$, $y(0) = 0$ using Taylor series method. Find approximate value of y for $x = 1$ and 1.1 . 6
- (c) Evaluate $\int_0^6 \frac{dx}{1+x}$ using 8
- (i) Trapezoidal rule
- (ii) Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule and
- (iii) Simpson's $\left(\frac{3}{8}\right)^{\text{th}}$ rule.
- Compare result with exact values.
6. (a) The current in a circuit containing an inductance L , resistance R and voltage $E \sin wt$ is given by 6
- $$L \frac{di}{dt} + Ri = E \sin wt$$
- If $i = 0$ at $t = 0$, find i .
- (b) Evaluate $\iint_R e^{2x-3y} dx dy$ over the triangle bounded by $x + y = 1$, $x = 1$, $y = 1$. 6
- (c) (i) Find the volume of solid bounded by the surfaces $y^2 = 4ax$, $x^2 = 4ay$ and the planes $Z = 0$, $Z = 3$. 4
- (ii) Change to polar co-ordinates and evaluate 4
- $$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dx dy}{\sqrt{a^2 - x^2 - y^2}}$$
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