LIB

FE - Sem II 10/8/13

AM-II (Rev) (2012-13)

Maths.

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Con. 6887–13. (REVISED COURSE) GS–5427

(3 Hours) [Total Marks : 80

N.B. : (1) Question No. **1** is **compulsory**.
(2) Answer any **three** questions from Question Nos. **2** to **6**.
(3) **Figures** to the **right** indicate **full** marks.
(4) **Programmable** calculators are not **allowed**.

1. (a) Evaluate $\int_0^1 (x \log x)^4 \, dx$. 3

(b) Solve $(D^2 - 1)(D - 1)^2 y = 0$. 3

(c) prove that $E = 1 + \Delta = e^{hD}$. 3

(d) Solve $\frac{dy}{dx} = \frac{y+1}{(y+2)\, e^y - x}$ 3

(e) Change into Polar co-ordinates and Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\, dy\, dx$. 4

(f) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dx\, dy}{1+x^2+y^2}$ 4

2. (a) Solve $(x^3 y^3 - xy)\, dy = dx$. 6

(b) Change the order of Integration and Evaluate $\int_0^1 \int_x^{2-x} \frac{x}{y}\, dy\, dx$. 6

(c) (i) P.T. $\int_0^{\pi/2} \tan^n x \, dx = \frac{\pi}{2} \sec \left[\frac{n\pi}{2}\right]$. 4

(ii) Evaluate $\int_0^\infty \frac{\log(1+ax^2)}{x^2}\, dx$, $a > 0$ 4

3. (a) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz\, dy\, dx}{(1+x+y+z)^3}$. 6

(b) Find the area using Double integration where the regionof integration is bounded by the curves $9xy = 4$ and $2x + y = 2$. 6

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(c) (i) Solve $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + 4y = \cos(\log y)$. 4

(ii) Solve the equation by method of variation of parameters 4

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}.$$

4. (a) Show that for the parabola $r = \dfrac{2a}{1+\cos\theta}$ for $\theta = 0$ to $\dfrac{\pi}{2}$ length is $a\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$. 6

(b) Solve $\dfrac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x - \cos 2x$. 6

(c) Apply Runge-Kutta method of fourth order to find an approximation value of y at $x = 0{\cdot}2$ if $\dfrac{dy}{dx} = x + y^2$ given $y = 1$ when $x = 0$ in steps of $h = 0{\cdot}1$. 8

5. (a) Solve $(2xy^4e^y + 2xy^3 + y)\,dx + [x^2y^4e^y - x^2y^2 - 3x]\,dy = 0$. 6

(b) Solve $\dfrac{dy}{dx} = 2x + y$ with initial conditions $x_0 = 0$, $y_0 = 0$ by Taylor's method obtain y as series in powers of x. 6
Find approximation value of y for $x = 0{\cdot}2, 0{\cdot}4$. Compare your result with exact values.

(c) Evaluate $\displaystyle\int_{-1}^{1} \frac{dx}{1+x^2}$ by : 8

(i) Trapizoidal method (ii) Simpson's $1/3^{rd}$ method and (iii) Simpsons $3/8^{th}$ method. Compare result with exact values.

6. (a) In a circuit containing inductance L, resistance R and voltage E. The current i is given by 6

$L\dfrac{di}{dt} + Ri = E$. Find current i at time t if $t = 0$, $i = 0$ and L, R, E are constants.

(b) Evaluate $\displaystyle\iint_R xy\,dx\,dy$ where R is the region bounded by $x^2 + y^2 - 2x = 0$, $y = x$ and $y^2 = 2x$. 6

(c) (i) Find volume of tetrahedron bounded by plane $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$. 4

(ii) Find volume bounded by cone $z^2 = x^2 + y^2$ and Paraboloid $z = x^2 + y^2$. 4