E.E. & EXTC / S.E. / Sem.IV / AM-IV

27/11/14

QP Code 12440

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **1** is **compulsory.**
(2) Solve any **three** questions from the **remaining.**

1. (a) Find the value of μ which satisfy the equation. 5
$A^{100} x = \mu X$. where

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -2 & -2 \\ 1 & 1 & 0 \end{bmatrix}$$

(b) Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along 5

$y = x$ and $y = x^2$.

(c) Find the extermal of the function. 5

$$\int_{x_1}^{x_2} \left[y^2 - y'^2 - 2y \cosh x \right] dx$$

(d) Verify Cauchy–Schwartz inequality for the vectors. 5
$u = (-4, 2, 1)$ & $v = (8, -4, -2)$

2. (a) Determine the function that gives the shortest distance between two given points. 6
(b) Find eigen values and eigen vectors of— 6

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

(c) Obtain Taylor's and two distinct Laurent's series expansion of $f(z) = \dfrac{z-1}{z^2 - 2z - 3}$ 8

about $z = 0$ indicating the region of convergence.

3. (a) Verify Caley. Hamilen theorem for 6

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$ hence find A^{-2}.

(b) Evaluate by using Residue theorem. 6

$$\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2}$$

(c) Solve the boundary value problem. 8

$$I = \int_0^1 \left(2xy - y^2 - y^{1^2} \right) dx$$

given y (0) = y (1) = 0 by Rayleigh–Ritz method.

4. (a) Reduce the following Quadrate form 6

$$Q = 3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_3x_1$$

into cunmical form. Hence find its rank, index and signature.

(b) Show that the matrix $A = \begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$ is derogatory. 6

(c) (i) Show that the set $W = \{(1, x) \mid x \in R\}$ is a subspace of R^2 under operations $[1, x] + [1, y] = [1, x + y]$; $k\,[1, x] = [1, kx]$; k is any scalar.

(ii) Is the set $W = \{ [a, 1, 1] \mid a \in R \}$ a subspace of R^3 under the usual addition and scalar multiplication ?

5. (a) Find the plane curve of fixed Perimeter and maximum area.

(b) Construct an orthonormal basis of R^2 by applying Gram schmidt orthogonalization to $S = \{ [3, 1], [2, 2] \}$

(c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonable. Also find diagonal form and diagonalising matrix.

6. (a) Evaluate $\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2+1)(x^2+4)}\,dx$ using Cauchy Residue Theorem. 6

(b) If $\phi(\alpha) = \oint_c \frac{ze^z}{z-\alpha}\,dz$ where c is $|z - 2i| = 3$ 6

find $\phi(1), \phi'(2), \phi(3), \phi'(4)$

(c) Show that the set V of positive real numbers with operations. 8

Addition : $x + y = xy$

Scalar multiplication : $kx = x^k$.

is a vector space

where x, y are any two real numbers and k is any scalar.
