

Time : 03 hours

Max. marks :100

N.B. 1. Question No.1 is compulsory and attempt any four questions from Q.No.2 to 7

2. Figures to the right indicate full marks.

Q.1 a) Determine value of a such that  $\vec{F} = (x + 3y)\mathbf{i} + (y - 2z)\mathbf{j} + (az + x)\mathbf{k}$  is

Solenoidal.

05

b) Determine l, m, n and find inverse of matrix A if  $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$

is orthogonal.

05

c) X is a continuous random variable with probability distribution  $f(x) = \frac{x}{6} + k$

when x belongs to [0,3] and is zero elsewhere. Find k.

05

d) Two lines of regression are given by  $3x + 2y = 26$  and  $6x + y = 31$ . Find mean values of x and y and coefficient of correlation between x and y.

05

Q.2 a) Prove that  $\vec{F} = (y^2 \cos x + z^3)\mathbf{i} + (2y \sin x - 4)\mathbf{j} + (3xz^2 + 2)\mathbf{k}$  is

Irrrotational. Find scalar potential of  $\vec{F}$  and work done in moving an object

In this field from (0,1,-1) to  $(\frac{\pi}{2}, -1, 2)$ .

08

b) Find Adj A if  $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

06

c) Seven dice are thrown 729 times. How many times do you expect at least four dice to show number 3 or 5?

06

Q.3 a) A car hire firm has 2 cars which it hires out day by day. The number of

demands for a car on each day is distributed as a Poisson variable with mean = 1.5. Calculate proportion of days on which i) neither car is used

ii) some demand is refused.

06

b) Show that  $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$  is nonderogatory.

06

c) If  $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$  find  $A^{50}$ .

08

Q.4 a) If  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$  find matrix represented by

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$$

06

b) Verify Stoke's theorem for  $\vec{F} = (x+y)i + (y+z)j - xk$  over surface S of the plane  $2x + y + z = 2$  in first quadrant.

06

c) Find values of a and b so that the system  $x + y + z = 6$ ;  $x + 2y + 3z = 10$   
 $x + 2y + az = b$  have i) no solution ii) unique solution

iii) infinite no. of solutions.

08

Q.5 a) Reduce to normal form and find rank if  $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}$

06

b) Marks scored by students in a college are normally distributed with

mean = 65 and variance = 25. If 3 students are selected at random

what is the probability that at least one of them has scored more

than 75 marks.

06

c) Verify Green's theorem for  $\int_C \frac{dx}{y} + \frac{dy}{x}$  where C is the boundary of the region

given by  $x=1, x=4, y=1, y=\sqrt{x}$ . 08

Q.6 a) Evaluate  $\iiint_S (4xi - 2y^2j + z^2k) \cdot d\vec{S}$  where S is the region bounded by

$Z=0, z=3, x=1, y^2=4x$ . 06

b) Find eigen values and eigen vectors of  $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$  06

c) Show that matrix  $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$  is diagonalizable

. Find the transforming matrix. 08

Q.7 a) Reduce the following quadratic form to normal form using congruent

Transformations and find rank, signature and value class.

$Q = 2x^2 + y^2 - 3z^2 - 8yz - 4zx + 12xy$  08

b) Calculate coefficient of correlation from the following data

X : 12 17 22 27 32

Y : 113 119 117 115 121 06

c) Prove that every square matrix can be uniquely expressed as sum of

a symmetric and a skew symmetric matrix. 06

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