Time : 03 hours

Max. marks :100

N.B. 1. Question No.1 is compulsory and attempt any four questions from Q.No.2 to 7

   2. Figures to the right indicate full marks.

Q.1 a) Determine value of k such that $w = \dfrac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\dfrac{kx}{y}$ is analytic.   05

  b)Find the Laplace transform of $e^{4t}Sin^3 t$.   05

  c)Show that the set of functions $\{1, x, \dfrac{3x^2 - 1}{2}\}$ is orthogonal over (-1,1).   05

  d)Find the image of $|z - 2| = 3$ under the transformation $w = \dfrac{1}{z}$.   05

Q.2 a) Find the bilinear transformation which maps the points 1,i ,-1 in Z-plane

    onto the points 0,1, ∞ in W-plane.   06

  b)Using Laplace transforms evaluate $\displaystyle\int_0^\infty e^{-t}\dfrac{Sin^2 t}{t}dt$ 06

  c) Find half range sine series for the function $f(x) = \dfrac{\pi}{4}; 0< x < \pi$.

    Hence deduce that $\dfrac{\pi}{4}(\dfrac{\pi}{2} - x) = \dfrac{Cosx}{1^2} + \dfrac{Cos3x}{3^2} + \dfrac{Cos5x}{5^2} + \ldots\ldots$ 08

Q.3 a)Find orthogonal trajectory of family of curves $3x^2 y - y^3 = const.$ 06

  b)Using convolution theorem find inverse Laplace transform of

    $\dfrac{s^2}{(s^2 + a^2)^2}$

  c)Find fourier series for the function $f(x) = x^2$ over (0,2π).   08

TURN OVER

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Q.4 a) State true or false with proper justification. 'There does not exist

     An analytic function whose real part is $x^3 - 3x^2 y - y^3$ 06

     b)Obtain complex form of fourier series for $f(x) = Coshax$ in $(-\pi, \pi)$.   06

     c) Using Laplace transforms Solve $\dfrac{d^2 y}{dt^2} + y = t$ with y(0)=1 and y'(0)=0.   08

Q.5 a)Find inverse Laplace transform of i) $cot^{-1}(s+1)$ ii) $\log(\dfrac{s+a}{s+b})$ 06

     b) Express $f(x) = 1; -1 \prec x \prec 1$ and is zero otherwise as a fourier integral.   06

     c)Obtain all possible Laurent's series expansions of $f(z) = \dfrac{z-1}{z^2 - 2z - 3}$

     about z=0.

Q.6 a)Construct analytic function $f(z) = u + iv$ where $u + v = e^x(\cos y + \sin y)$ 06

     b)Find Laplace transform of $(1 + 2t - 3t^2 + 4t^3)H(t-2)$.   06

     c)Stata Dirichlet's conditions for a function f(x) to have fourier series over

     (a,b).Obtain fourier series for $f(x) = e^{-x}$ over 0 <x <2π. $f(x + 2\pi) = f(x)$.   08

Q.7 a)Find fourier series for $f(x) = x$ 0 <x ≤π

              = $2\pi - x$ π≤x <2π   06

     b) State and prove Cauchy's integral formula for $f(z) = u + iv$ 06

     c)Using residue theorem evaluate i) $\displaystyle\int_0^{2\pi} \dfrac{d\theta}{5 + 3Sin\theta}$ 08

                  ii) $\displaystyle\int_{-\infty}^{\infty} \dfrac{x^2 + x + 2}{x^4 + 10x^2 + 9} dx$