AM-III - ET - SEM-III, 20th NOV 2014 (16)

(OLD COURSE) **QP Code :12213**

(3 Hours) **[Total Marks : 100**

N.B.: (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of remaining **six** question.

1. (a) Find Laplace transform of $\{\cos(t)\cos(2t)\cos(3t)\}$. 5
(b) Show that every square matrix can be uniquely expressed as the sum of a symmetric matrix and skew symmetric matrix. 5
(c) Obtain complex from of fourier series $f(x) = e^{ax}$ in $(-\ell, \ell)$. 5
(d) Find z-transform of $\{a^{|k|}\}$. 5

2. (a) Find Laplace transform of $\left\{\frac{1-\cos(t)}{t^2}\right\}$ 6

(b) If $A = \frac{1}{9}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix}$ is orthogonal, find a,b & c and A^{-1}. 7

(c) Obtain Forier series of $f(x) = \sqrt{1-\cos(x)}$ in $(-\pi, \pi)$. 7

3. (a) Solve using Laplace transform $\frac{d^2y}{dt^2} + 9y = 18t$ 6

(b) Obtain Fourier Series for $f(x) = \dot{x}\sin(x)$ in $(0, 2\pi)$. 7
(c) Obtain z-transform of $C^k \sin h(\alpha k)$, $k \geq 0$ 7

4. (a) Obtain Fourier series of $f(x) = 9 - x^2$ in $(-3, 3)$. 6
(b) Test for consistency and solve if consistant 7

$$x_1 - 2x_2 + x_3 - x_4 = 2$$
$$x_1 + 2x_2 + 2x_4 = 1$$
$$4x_2 - x_3 + 3x_4 = -1$$

(c) Find inverage z-transform of $f(x) = \frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}$, $3 < z < 4$ 7

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LM-Con.:6428-14.

5. (a) Find Fourier integral representation for $f(x) = 1-x^2 \quad |x| \le 1$ 6
$= 0 \quad |x| > 1$

(b) Obtain the expansion of $f(x) = x(\pi - x)$, $0 < x < \pi$ as a half range cosine series then 7
show that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

(c) Find the non-singular matrices P and Q such that $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ is reduced to 7
the normal form PAQ. Also find its rank.

6. (a) Using Laplace transform evaluate $\int_0^{\infty} e^{-t}\left(1+2t-t^2+t^3\right)H(t-1)\,dt$. 6

(b) Evaluate using Laplace transform $\int_0^{\infty} e^{-t}\left\{\int_0^t u\cos^2(u)\,du\right\}dt$ 7

(c) Find inverse Laplace of $\tan^{-1}\left(\frac{2}{s^2}\right)$. 7

7. (a) Using convolution theorem find the Inverse Laplace transform of $\left\{\frac{1}{\left(s^2+4s+13\right)^2}\right\}$. 6

(b) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$ then show that $(I-N)(I+N)^{-1}$ is a unitary matrix. 7

(c) Obtain Fourier series for the function $f(x) = \begin{cases} 1+\frac{2x}{\pi} & -\pi \le x \le 0 \\ 1-\frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$ 7

Reduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{--}$.

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5. (a) Find Fourier integral representation for $f(x) = 1-x^2 \quad |x| \le 1$ $= 0 \quad |x| > 1$ 6

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6. (a) Using Laplace transform evaluate $\int_0^{\infty} e^{-t}\left(1+2t-t^2+t^3\right)H(t-1)\,dt$. 6

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AM-III - ET-SEM-III, 20th NOV 2014

(OLD COURSE) **QP Code :12211**

(3 Hours) **[Total Marks : 100**

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   (b) If $A = \frac{1}{9}\begin{bmatrix} a & 1 & b \\ c & b & 7 \\ 1 & a & c \end{bmatrix}$ is orthogonal, find a,b & c and A^{-1}.

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