B

AM-III-ME-SEM-III 20th NOV 2014

(OLD COURSE) **QP Code :12198**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No.**1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.

1. (a) Find the constants a, b, c, d, & e **5**
if $f(z) = (ax^3 + bxy^2 + 3x^2 + cy^2 + x) + i(dx^2y - 2y^3 + exy + y)$ is analytic.
(b) Prove that the circle $|z| = 1$ in the z-plane is mapped onto the cordioide in the w-plane under the transformation $w = z^2 + 2z$. **5**
(c) Find the laplace transform of $\sin^3(t)$ **5**
(d) If A is no-singular square matrix of order n then show that adj. $adj(A) = |A|^{n-2} \cdot A$ **5**

2. (a) Find the eigen values and eigon vectors corresponding to the following matrix A **6**

Where $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

(b) Find an analytic function f(z) whose imaginary part is $\frac{\sinh(2y)}{\cos(2x)+\cosh(2y)}$ **6**

(c) Find the Laplace Transform of the following : **8**

(i) $\frac{\sin^2(2t)}{t}$ (ii) $\int_0^t ue^{-3u}\cos^2(2u)du$

3. (a) Determine λ & μ if the system **6**
$3x - 2y + z = \mu$, $5x - 8y + 9z = 3$ $2x + y + \lambda z = -1$ have
(i) no solution (ii) unique solution (iii) an infinite number of solutions.
(b) Find the bilinear transform which maps the points 2, i, -2 onto the point 1, i, -1. **6**
(c) Find inverse laplace transform of the following : **8**

(i) $\log\left(1 + \frac{1}{s^2}\right)$ (ii) $\frac{(s+1)}{(s^2+s+1)}$

4. (a) Find non-singular matrices P & Q such that : **6**

$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ is reduced to normal from. Also find its rank.

(b) Evaluate $\int_0^\infty \frac{dx}{x^4+1}$ **6**

(c) (i) Find $L^{-1}\left\{\frac{(s+3)}{(s^2+6s+13)^2}\right\}$ (ii) Evaluate $\int_0^\infty e^{-t}\frac{\sin^2(t)}{t}dt$ 8

5. (a) Evaluate $\int_c \frac{(z-1)}{((z+1)^2(z-2))}dt$ where 'C' encloses both poles of f(z) 6

(b) Find the orthogonal trajectories of the family of curves $e^{-x}\cos(y)+xy=\alpha$ 6

(c) Using laplace transform solve the following differential equation with given conditions. $(D^2 + 9)\, y = 18t$ with $y(0) = 0$ & $y\left(\frac{\pi}{2}\right)=0$ 8

6. (a) Examine whether the vectors $X_1 = (1, 2, 4)$ $X_2 = (2, -1, 3)$ & $X_3 = (0, 1, 2)$ are Linearly independent. 6

(b) Find the image of real axis in the z-plane on to the w-plane under the transformation $w=\frac{1}{z+i}$. 6

(c) Expand $f(z)=\frac{1}{(z+1)\,(z-2)}$ in the regions 8

(i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$

7. (a) Evaluate : $\int_0^{2\pi}\frac{d\theta}{5+4\cos(\theta)}$ 6

(b) If $A=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$ find A^{50} 6

(c) Find the characteristic equation of 8

Matrix $A=\begin{bmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}$

and verify that it satisfies Cayley - Hamilton theorem hence find the matrix represented by $A^7 - 4A^6 - 20A^5 - 34A^4 - 4A^3 - 20A^2 - 33A + I$
