

3. (a) Solve $[1 + \log(xy)] dx + \left(1 + \frac{x}{y}\right) dy = 0$ 6

(b) Find by double integration the mass of a thin plate bounded by $y^2 = x$ and $y = x^3$. If the density of any point varies as the square of its distance from the origin. 6

(c) Solve $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = -x^4 \sin x$. 8

4. (a) Solve $\frac{dy}{dx} + y = y^2 (\cos x - \sin x)$ 6

(b) Change the order of integration and evaluate 6

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{\cos^{-1} x}{\sqrt{1-x^2} \sqrt{1-x^2-y^2}} dx dy$$

(c) Use Euler's method to find an approximate value of y correct to 4 decimal places 8

for $x = 0.1$ given $\frac{dy}{dx} = x - y^2$ at $x = 0, y = 1$. Take $h = 0.02$.

5. (a) Find the length of the astroid $x = a \cos^3 t, y = a \sin^3 t$. 6

(b) Evaluate $\iiint \frac{dx dy dz}{x^2 + y^2 + z^2}$ throughout the volume of the sphere 6

$$x^2 + y^2 + z^2 = a^2.$$

(c) Solve $\frac{dy}{dx} = xy$ with initial conditions $y(1) = 2$ and find y at $x = 1.2, 1.4$ by 8
Runge Kutta method of fourth order.