

(OLD COURSE) Q.P. NO : 11895

(3 Hours)

[Total Marks : 100

- N.B. :** (1) Question No. 1 is compulsory.
 (2) Attempt any **four** questions from remaining.
 (3) Answer to **subquestions** should be written together.

1. (a) P.T. $\frac{1 + \cos \alpha + i \sin \alpha}{1 - \cos \alpha + i \sin \alpha} = \cot\left(\frac{\alpha}{2}\right) e^{i(\alpha - \pi/2)}$ 3

(b) If $y = x^2 e^x \cos x$: find y_n 3

(c) P.T. $\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots$ 3

(d) If $u = \frac{e^{x+y+z}}{e^x + e^y + e^z}$ S.T. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 2u$ 3

(e) Find $[(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})] + [(\vec{b} \times \vec{c}) \times (\vec{a} \times \vec{d})] + [(\vec{c} \times \vec{a}) \times (\vec{b} \times \vec{d})]$ 4

(f) If the temp T at any point (x, y, z) on the surface of the sphere $x^2 + y^2 + z^2 = 1$ is $T = 400xyz^2$. Find highest temperature. 4

2. (a) Find z if $\arg(z+1) = \frac{\pi}{6}$ & $\arg(z-1) = \frac{2\pi}{3}$ 6

(b) If $\sin(\theta + i\phi) = \tan \alpha + i \sec \alpha$ then S.T. $\cos 2\theta \cosh 2\phi = 3$. 6

(c) If $u = \frac{x^3 y^3 z^3}{x^3 + y^3 + z^3} + \cos \left[\frac{x^2 + y^2 + z^2}{xy + yz + zx} \right]$ 8

find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$

3. (a) If $f(x) = x(x+1)(x+2)(x+3)$; S.T. $f'(x) = 0$ has at least three real roots in $[-3, 0]$. 6

(b) Verify the formula that $\frac{d}{dt}(\vec{A} + \vec{B}) = \vec{A} \times \frac{d\vec{B}}{dt} + \frac{d\vec{A}}{dt} \times \vec{B}$ for 6

$\vec{A} = 5t^2 \vec{i} + t \vec{j} - t^3 \vec{k}$ & $\vec{B} = \sin t \vec{i} - \cos t \vec{j}$ 8

(c) Expand $\sin^{-1} x$ in ascending powers of x upto the term x^7 .

Correction