4. (a) Express $\frac{\sin 7\theta}{\sin\theta}$ in powers of $\sin\theta$ only. 6

(b) Test the convergence of $\Sigma\frac{3^n+4^n}{4^n+5^n}$ 6

(c) If $y=\sin\left(m\sin^{-1}x\right)$ S.T. 8

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2-m^2\right)y_n=0$$

5. (a) S.T. $\frac{d^n}{dx^n}\left(\tan^{-1}x\right)=\frac{(-1)^{n-1}(n-1)!\sin\left(n\tan^{-1}1/x\right)}{\left(x^2+1\right)^{n/2}}$ 6

(b) find a, b if $\lim_{x\to 0}\frac{a\sin x-\sin 2x}{\tan^3 x}=b$ 6

(c) If $\overline{A}=\nabla(xy+yz+zx)$ find $\nabla\cdot\overline{A}$ & $\nabla\times\overline{A}$. 8

6. (a) If $u=x^2-y^2$, $v=2xy$ & $z=f(u,v)$ P.T. 6

$$\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=4\sqrt{u^2+v^2}\left[\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2\right]$$

(b) Find directional derivative of $\phi=e^{2x-y-z}$ at (1,1,1) in the direction of tangent to the curve $x=e^{-t}$, $y=2\sin t+1$ $z=t-\cos t$, at $t=0$. 6

(c) Find the prinicipal value of $(x+iy)^i$ and show that it is enrirely real if $\frac{1}{2}\log\left(x^2+y^2\right)$ is multiple of π. 8

7. (a) If $u(x+y)=x^2+y^2$ then S.T. $\left(\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right)^2=4\left[1-\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\right]$ 6

(b) Find the maximum and minimum values of $x^3+3xy^2-3x^2-3y^2+4$ 6

(c) Separate into real and imaginary parts of $\left(\sqrt{i}\right)^{\sqrt{i}}$ 8
